

Solution 1

$$\begin{aligned}
 6x^4 - 4x^2 + x + 1 &\equiv (ax^2 + bx + c)(x^2 - 1) + dx + e && |M1 \\
 (x^4) &\Rightarrow 6 = a && \\
 (x^3) &\Rightarrow 0 = b && |A1 \\
 x = 1 &\Rightarrow 4 = 1d + e && \\
 x = -1 &\Rightarrow 2 = -1d + e && \\
 &\Rightarrow 2e = 6 \Rightarrow e = 3, d = 1 && |A1 \\
 (x^2) &\Rightarrow -4 = -1a + c \Rightarrow c = 2 && |A1
 \end{aligned}$$

Solution 2

$$\begin{aligned}
 \frac{6x^2}{(x+2)(x-4)} &\equiv A + \frac{Bx+C}{(x+2)(x-4)} && \\
 &\Rightarrow 6x^2 \equiv A(x+2)(x-4) + Bx + C && |B1 \\
 (x^2) &\Rightarrow 6 = A \Rightarrow A = 6 && |A1 \\
 x = 0 &\Rightarrow 0 = -8A + C \Rightarrow C = 48 && |A1 \\
 x = 4 &\Rightarrow 96 = 4B + C \Rightarrow B = 12 && |A1 \\
 &\Rightarrow \frac{6x^2}{(x+2)(x-4)} \equiv 6 + \frac{12x+48}{(x+2)(x-4)}
 \end{aligned}$$

Solution 3

$$\begin{aligned}
 \text{(a)} \quad f\left(\frac{2}{5}\right) &= 5\left(\frac{2}{5}\right)^3 + 3\left(\frac{2}{5}\right)^2 - 32\left(\frac{2}{5}\right) + 12 = 0 \\
 \text{(b)} \quad f(x) &= (5x-2)(x^2+1x-6)
 \end{aligned}$$

Solution 4

$$\begin{aligned}
 x^4 - 5x^3 - 4x^2 + 6x + a &\equiv (x^2 + bx + 21)(x^2 + 3x - 1) + cx + 6 \\
 (x^3) &\Rightarrow -5 = b + 3 \\
 &\Rightarrow b = -8 \\
 (\text{constant}) &\Rightarrow a = -21 + 6 = -15 \\
 (x) &\Rightarrow 6 = -1b + 63 + c \\
 &\Rightarrow c = -65
 \end{aligned}$$

Solution 5

$$\begin{aligned}
 \text{(a)} \quad \frac{(6x^2 - 39x - 21)(x+5)}{(3x^2 + 7x - 40)(x-7)} &= \frac{(6x+3)(x-7)(x+5)}{(3x-8)(x+5)(x-7)} && |M1 \\
 &= \frac{6x+3}{3x-8} && |A1 \\
 \text{(b)} \quad x^3 + 3x^2 - 4x - 8 &= (x+a)(x^2 + 2x + 2) + bx + c && |M2 \\
 (x^2) &\Rightarrow 3 = a + 2 \\
 &\Rightarrow a = 1 \\
 (x) &\Rightarrow -4 = 2a + 2 + b \\
 &\Rightarrow b = -8 \\
 (\text{constant}) &\Rightarrow -8 = 2a + c \\
 &\Rightarrow c = -10 \\
 &\Rightarrow \text{quotient} = x + 1, \text{ remainder} = -8x - 10 && |A2
 \end{aligned}$$