Question 1

The equation $x = \ln(x+4) + 5$ is to be solved.
Use the iterative formula $x_{n+1} = \ln(x_n + 4) + 5$, with $x_1 = 7.5$ to find a solution correct to 3 decimal places.

Question 2

$$f(x) = x^3 - 6x - 11.$$

(a) Show that there is a root α of $f(x) = 0$ for x in the interval $[3, 4]$.
The root α is to be estimated using the iterative formula

$$x_{n+1} = \sqrt{(6 + \frac{11}{x_n})},\ x_0 = 3$$

(b) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 significant figures.
(c) Prove that, to 5 significant figures, α is 3.0916.

Question 3

The iterative formula

$$x_{n+1} = \frac{1}{18}e^{-x_n},\ x_0 = 1$$

is used to find an approximate value for α .
(c) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places.
(d) By considering the change of sign of $f'(x)$ in a suitable interval, prove that $\alpha = 0.0527$ correct to 4 decimal places.

Question 4

$$f(x) = 4x^3 - x - 24.$$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\frac{6}{x} + \frac{1}{4}}$$

The equation $4x^3 - x - 24 = 0$ has a root between 1.85 and 1.9.

(b) Use the interative formula

$$x_{n+1} = \sqrt{\frac{6}{x_n} + \frac{1}{4}}$$

with $x_0 = 1.85$, to find, to 2 decimal places, the values of x_1, x_2 and x_3.

The only real root of $f(x) = 0$ is α.

(c) By choosing a suitable interval, prove that $\alpha = 1.863$, to 3 decimal places.

Question 5

The equation $48k - 12 + 6\sin 8k = 0$ is to be solved using the iterative formula $x_{n+1} = \frac{1}{48}(12 - 6\sin 8x_n), x_0 = 0.1$.

(a) Calculate the values of x_1, x_2, x_3, x_4, giving your answers to 4 decimal places.

(b) Show that $k = 0.138$, correct to 3 significant figures.

Question 6

The equation $\frac{1}{x} = 1.3\sin 1.1x$ has two roots α and β in the interval $0 < x < \pi$

(a) Given that $\alpha \approx 1$, use an interation based on a simple rearrangement of the equation to find α correct to 2 decimal places.

(b) Verify that $\beta = 2.581$ correct to 3 decimal places.

Question 7

The curve $y = x^3 - x - 13$ intersects the x-axis at the point where $x = \alpha$.
(a) Show that α lies between 2.4 and 2.5.
(b) Show that the equation $x^3 - x - 13 = 0$ can be rearranged in the form $x = \sqrt[3]{x + 13}$
(c) Use the interation with $x_1 = 2.4$ to find the values of x_2, x_3 and x_4
giving your answers to 3 significant figures.

Question 8

$$f(x) = -x^3 + 7x^2 - 2.$$

(a) Show that the equation $f(x) = 0$ can be rewritten as

$$x = \sqrt{\frac{2}{7 - x}}$$

(b) Starting with $x_1 = 0.5$, use the iteration

$$x_{n+1} = \sqrt{\frac{2}{7 - x_n}}$$

to calculate the values of x_2, x_3 and x_4,
giving all your answers to 4 decimal places.
(c) Show that $x = 0.557$ is a root of $f(x) = 0$,
correct to 3 decimal places.

Question 9

The curve $y = 4^x$ intersects the line $y = x + 4$ at the point where $x = \alpha$.
(a) Show that α lies between 0.5 and 1.5.
(b) Show that the equation $4^x = x + 4$ can be rearranged into the form

$$x = \frac{\ln(x + 4)}{\ln 4}$$

(c) Use the iteration $x_{n+1} = \frac{\ln(x_n + 4)}{\ln 4}$ with $x_1 = 0.5$ to find x_3 to two significant figures.