## Solution 1

(a) $\frac{dy}{dx} = 3x^2.e^x + e^x.6x$ |M1A2

(b) $\frac{dy}{dx} = 0 \Rightarrow 3x^2e^x + 6xe^x = 0$ |M1

$\Rightarrow 3x(x+2) = 0 \Rightarrow x = 0, -2$

$\Rightarrow (0,0), (-2, 12e^{-2})$ |M1A1

(c) $\frac{d^2y}{dx^2} = 3x^2.e^x + e^x.6x + e^x.6x + e^x.6$

$= 3x^2e^x + 12xe^x + 6e^x$ |M1A1

(d) $x = 0 \Rightarrow \frac{d^2y}{dx^2} = 6 > 0 \Rightarrow min$ |A1

$x = -2 \Rightarrow \frac{d^2y}{dx^2} = -6e^{-2} < 0 \Rightarrow max$ |A1

## Solution 2

(a) $y = \ln x \Rightarrow \frac{dy}{dx} = \frac{1}{x}$ |B1

(b) $y = (9+x)\ln x \Rightarrow \frac{dy}{dx} = (9+x)\cdot\frac{1}{x} + \ln x \cdot 1$ |M1A1

(c) $x = 1 \Rightarrow \frac{dy}{dx} = 10$ |M1

gradient of normal $= -\frac{1}{10}$ |M1A1

equation of normal : $y = -\frac{1}{10}(x-1)$ |A1

## Solution 3

(a) $\frac{dy}{dx} = (x^2 - 24)\cdot e^x + e^x \cdot 2x$ |M1A1

(b) $\frac{d^2y}{dx^2} = (x^2 - 24)\cdot e^x + e^x \cdot 2x + e^x \cdot 2 + 2x \cdot e^x$ |M1A1

(c) $\frac{dy}{dx} = 0 \Rightarrow (x^2 - 24)e^x + 2xe^x = 0$ |M1

$\Rightarrow x^2 + 2x - 24 = 0 \Rightarrow (x+6)(x-4) = 0$ |M1

$\Rightarrow x = -6, 4$ |A2

(d) $x = -6 \Rightarrow \frac{d^2y}{dx^2} = -10e^x \Rightarrow$ max |M1

$x = 4 \Rightarrow \frac{d^2y}{dx^2} = 10e^x \Rightarrow$ min |M1

Solution 4

(a) $\frac{dy}{dx} = 7e^{2x} \cdot \sec^2 x + 7\tan x \cdot 2e^{2x}$ |M1A2

$\frac{dy}{dx} = 0 \Rightarrow \sec^2 x + 2\tan x = 0$ |M1

$\Rightarrow \tan^2 x + 2\tan x + 1 = 0$

$\Rightarrow (\tan x + 1)^2 = 0 \Rightarrow \tan x = -1$ |M1A1

(b) $x = 0 \Rightarrow y = 0, \frac{dy}{dx} = 7$

$\Rightarrow$ equation: $y = 7x$ |M1A1

Solution 5

$$\frac{dy}{dx} = x^4 \cdot e^x + e^x \cdot 4x^3 \quad |M2A2$$

$$x = 2 \Rightarrow \frac{dy}{dx} = 2^4 \cdot e^2 + e^2 \cdot 4(2^3)$$

$$\Rightarrow \frac{dy}{dx} = 48e^2 \quad |A1$$

Solution 6

$$\frac{dy}{dx} = x \cdot e^{-\frac{1}{4}x} \cdot -\frac{1}{4} + e^{-\frac{1}{4}x} \quad |B1M1A$$

$$= -\frac{1}{4}xe^{-\frac{1}{4}x} \cdot +e^{-\frac{1}{4}x}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{4}x \cdot e^{-\frac{1}{4}x} \cdot -\frac{1}{4} + e^{-\frac{1}{4}x} \cdot -\frac{1}{4} + e^{-\frac{1}{4}x} \cdot -\frac{1}{4} \quad |B1$$

$$= \frac{1}{16}xe^{-\frac{1}{4}x} - \frac{2}{4}e^{-\frac{1}{4}x}$$

$$\frac{d^2y}{dx^2} = 0 \Rightarrow \frac{1}{16}xe^{-\frac{1}{4}x} - \frac{2}{4}e^{-\frac{1}{4}x} = 0 \quad |M1$$

$$\Rightarrow \frac{1}{16}x - \frac{2}{4} = 0$$

$$\Rightarrow x = 8 \quad |A1$$

$$\Rightarrow y = 8e^{-2} \quad |A1$$

Solution 7

$$\begin{aligned}\frac{dy}{dx} &= x\cdot -\sin 6x\cdot 6+\cos 6x\cdot 1 && |M1B1\\ &= -6x\sin 6x+\cos 6x && |A1\end{aligned}$$

Solution 8

$$\begin{aligned}\frac{dy}{dx} &= x\cdot 7e^{7x}+e^{7x}\cdot 1 && |M1A1\\ \frac{dy}{dx}=0 &\Rightarrow 7xe^{7x}+e^{7x}=0\\ &\Rightarrow 7x+1=0\\ &\Rightarrow x=-\frac{1}{7} && |M1A1\\ &\Rightarrow y=-\frac{1}{7}e^{-1} && |A1\end{aligned}$$

Solution 9

(a)
$$\begin{aligned}\frac{dy}{dx} &= e^{2x}\cdot(2x-2)+(x^2-2x-11)\cdot 2e^{2x} && |M1A1\\ &= e^{2x}(2x^2-2x-24) && |M1A1\\ \frac{dy}{dx}=0 &\Rightarrow 2x^2-2x-24=0\\ &\Rightarrow (x-4)(x+3)=0\\ &\Rightarrow x=4,-3 && |M1A1\end{aligned}$$

(b)
$$\begin{aligned}\frac{d^2y}{dx^2} &= e^{2x}(4x-2)+(2x^2-2x-24)2e^{2x} && |M1A1\\ x=4 &\Rightarrow \frac{d^2y}{dx^2}=e^8(14)>0\Rightarrow \min\\ x=-3 &\Rightarrow \frac{d^2y}{dx^2}=e^{-6}(-14)<0\Rightarrow \max && |M1A1\end{aligned}$$

Solution 10

$$\begin{aligned}\frac{dy}{dx} &= e^{2x}\cdot 4\sec 4x\tan 4x+\sec 4x\cdot 2e^{2x}\\ &= e^{2x}\sec 4x(4\tan 4x+2)\end{aligned}$$

Solution 11

(a) $\sqrt{2+4x^2} = (2+4x^2)^{\frac{1}{2}}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(2+4x^2)^{-\frac{1}{2}}8x$ |M1B1

$= 4x(2+4x^2)^{-\frac{1}{2}}$ |A1

(b) $\frac{dy}{dx} = x \times 4x(2+4x^2)^{-\frac{1}{2}} + \sqrt{2+4x^2} \times 1$ |M1A1

$= 4x^2(2+4x^2)^{-\frac{1}{2}} + \sqrt{2+4x^2}$

$= \frac{4x^2}{\sqrt{2+4x^2}} + \sqrt{2+4x^2}$

$= \frac{4x^2+2+4x^2}{\sqrt{2+4x^2}}$

$= \frac{2+8x^2}{\sqrt{2+4x^2}}$ |M1A1