

Question 1

- (a) Express $6 \cos \theta + 4 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the values of R and α to 3 significant figures.
- (b) Find the maximum value of $6 \cos \theta + 4 \sin \theta$ and the smallest positive value of θ for which this maximum occurs.
- (c) The temperature, T , of an unheated house is modelled using the equation

$$T = 11 + 6 \cos \frac{\pi t}{12} + 4 \sin \frac{\pi t}{12}, 0 \leq t < 24,$$

where t is the number of hours after 1200. Calculate the maximum temperature predicted by this model and the value of t when this maximum occurs.

Question 2

- (a) Express $(8 \cos \theta - 6 \sin \theta)$ in the form $R \cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90$.
- (b) Hence solve the equation $8 \cos \theta - 6 \sin \theta = 4$, for $0 \leq \theta < 90$, giving your answer to 1 decimal place.
- (c) Solve $10 \cot \theta - 6 \tan \theta = 7$, for $0 \leq \theta < 90$, giving your answer to 1 decimal place.

Question 3

- (a) Express $3.5 \sin 2x + 6 \cos 2x$ in the form $R \sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the values of R and α to 3 decimal places.
- (b) Express $7 \sin x \cos x + 12 \cos^2 x$ in the form $a \cos 2x + b \sin 2x + c$, where a, b and c are constants to be found.
- (c) Hence, using your answer to (a), deduce the maximum value of $7 \sin x \cos x + 12 \cos^2 x$.

Question 4

$$f(x) = 9 \cos x - 4 \sin x.$$

Given that $f(x) = R \cos(x + \alpha)$, where $R \geq 0$ and $0 \leq \alpha \leq 90^\circ$,

(a) find the value of R and the value of α .

(b) Hence solve the equation

$$9 \cos x - 4 \sin x = 9$$

for $0 \leq x < 360^\circ$, giving your answers to 1 dp.

(c) Write down the minimum value of $9 \cos x - 4 \sin x$.

(d) Find, to 2 dp, the smallest positive value of x for which this minimum value occurs.

Question 5

(a) Express $7 \cos x - 4 \sin x$ in the form $R \cos(x + \alpha)$, where $R \geq 0$ and $0 \leq \alpha \leq 90^\circ$,

(b) Hence solve the equation $7 \cos x - 4 \sin x = 4$ giving all solutions for which $-180^\circ \leq x < 180^\circ$.