January 27th

1, 7, 13, 19, ..., ...   and   5, 11, 17, 23, ..., ...

A quick inspection of these two sequences suggests that Kate is correct, but we need to prove it….

Both sequences go up in 6s.

If you divide any of the first sequence by 6, you always get a remainder of 1.

If you divide any of the second sequence by 6, you always get a remainder of 5.

Indeed, if you divide any number by 6 there are six possibilities:

There's no remainder, therefore the number is a multiple of 6

	so can be written as 6n
The remainder is 1	so can be written 6n+1 (sequence 1 above)
The remainder is 2	so can be written 6n+2 (hence is even)
The remainder is 3	so can be written 6n+3 (hence is multiple of 3)
The remainder is 4	so can be written 6n+4 (hence is even)
The remainder is 5	so can be written 6n+5 (sequence 2 above)

Notice unless the remainder is 1 or 5, the number can't be even.

Therefore all primes lie in either one or the other of these sequences.