

March 10th

Find at least three square numbers which are also triangular numbers.

Square numbers:

1, 4, 9, 16, 25, 36, 49,

Triangle numbers:

1, 3, 10, 15, 21, 28, 36, 45, 55,

By inspection of these

$$S_1 = T_1 = \mathbf{1}$$

$$S_6 = T_7 = \mathbf{36}$$

You have to go looking a lot further to find the next one which is

$$S_{35} = T_{49} = \mathbf{1225}$$

It would appear these are quite hard to find!

It turns out that there is a rule for finding squares that are also primes, which requires a considerable amount of work to prove, however...

$$1^2 \text{ is square and triangular} \quad (1 \times 1)^2$$

$$6^2 \text{ is square and triangular} \quad (2 \times 3)^2$$

$$35^2 \text{ is square and triangular} \quad (5 \times 7)^2$$

The Fibonacci-like sequence generates the next number

$$1+1=2, 1+2=3$$

$$2+3=5, 2+5=7$$

$$5+7=12, 12+5=17$$

Hence the next 2 square/triangular numbers will be

$$(12 \times 17)^2 = 204^2 = \mathbf{41616}$$

$$(29 \times 41)^2 = 189^2 = \mathbf{1,413,721}$$

$$(12+17=29, 29+12=41)$$