March 16th

Four consecutive odd numbers:

$2n-3, 2n-1, 2n+1, 2n+3$ (where n is an integer, $n>1$)

Product:;

$(2n - 3)(2n - 1) = 4n^2 - 8n + 3$

$(2n + 3)(2n + 1) = 4n^2 + 8n + 3$

$(4n^2 - 8n + 3)(4n^2 + 8n + 3)$

$= 16n^4 + 32n^3 + 12n^2 - 32n^3 - 64n^2 - 24n + 12n^2 + 24n + 9$

$= 16n^4 -40n^2 + 9$

Completing the square gives

$(4n^2 - 5)^2 - 16$

Hence adding **16** will make the product of 4 consecutive odd numbers square.

If we follow the same process with even numbers:

$2n-2, 2n, 2n+2, 2n+4$

$2n(2n+4) = 4n^2 + 8n$

$(2n -2)(2n+2) = 4n^2 - 4$

$(4n^2 + 8n)(4n^2 - 4) = 16n^4 - 16n^2 + 32n^3 - 32n$

$=16(n^4 +2n^3 - n^2 - 2n)$

Adding 16 gives

$16(n^4 +2n^3 - n^2 - 2n + 1)$

$=16(n^2 + n - 1) (n^2 + n - 1)$

which is also a perfect square