March 2nd

$$\sqrt{3+\sqrt{3+\sqrt{3+\sqrt{3+\dots\dots\dots\dots}}}}$$

This is an infinite square root, and with a calculator you can start to find values for

$\sqrt{3+\sqrt{3}}$, $\sqrt{3+\sqrt{3+\sqrt{3}}}$, $\sqrt{3+\sqrt{3+\sqrt{3+\sqrt{3}}}}$, etc

which, as approximate decimals are

2.175, 2.275, 2.297, 2.301,….

….which suggests if we go far enough, this will reach a limit:

Proof:

Let x = $\sqrt{3+\sqrt{3+\sqrt{3+\sqrt{3+\dots}}}}$

Now realise that the section highlighted must be equal to x

$$\sqrt{3+\underbrace{\sqrt{3+\sqrt{3+\sqrt{3+\dots}}}}_{x}}$$

Leading to

$x = \sqrt{3+x}$

Hence

$x^2 = 3 + x$ so $x^2 - x - 3 = 0$

Solving this gives

$x = \frac{1 + \sqrt{13}}{2}$ or 2.302…….