Video 365

Textbook Answers

1a) $(n + 7)^2 - (n + 1)^2$ $= n^2 + 14n + 49 – (n^2 + 2n + 1)$

$= n^2 + 14n + 49 - n^2 – 2n - 1$

$= 12n + 48$

$= 12(n + 4)$

b) $(n + 1)^2 - (n – 3)^2$ $= n^2 + 2n + 1 – (n^2 - 6n + 9)$

$= n^2 + 2n + 1 - n^2 + 6n - 9$

$= 8n - 8$

$= 8(n - 4)$

c) $(n + 1)^2 + (n + 5)^2 - (n + 9)^2$ $= n^2 + 2n + 1 + n^2 + 10n + 25 -(n^2 + 18n + 81)$

$= n^2 + 2n + 1 + n^2 + 10n + 25 - n^2 - 18n - 81$

$= n^2 – 6n - 55$

$= (n + 5)(n – 11)$

2a) $(n + 4)^2 - (n + 2)^2$ $= n^2 + 8n + 16 – (n^2 + 4n + 4)$

$= n^2 + 8n + 16 - n^2 – 4n - 4$

$= 4n + 12$

$= 4(n + 3)$ *…which is a multiple of 4 for all positive integer n*

b) $(n + 10)^2 - (n + 2)^2$ $= n^2 + 20n + 100 – (n^2 + 4n + 4)$

$= n^2 + 20n + 100 - n^2 – 4n - 4$

$= 16n + 96$

$= 16(n + 6)$ *…which is a multiple of 16 for all positive integer n*

c) $(2n + 3)^2 - (2n + 1)$ $= 4n^2 + 12n + 9 – (2n + 1)$

$=4n^2 + 12n + 9 -2n - 1$

$= 4n^2 + 10n + 8$

$= 2(2n^2 + 5n + 4)$ *…which is even for all positive integer n*

d) $(5n + 2)^2 - (5n - 1)^2$ $= 25n^2 + 20n + 4 – (25n^2 - 10n + 1)$

$= 25n^2 + 20n + 4 - 25n^2 + 10n - 1$

$= 30n + 3$

$= 3(10n + 1)$ *…which is a multiple of 3 for all positive integer n*

e) $(2n + 9)^2 - (2n + 5)^2$ $= 4n^2 + 36n + 81 – (4n^2 + 20n + 25)$

$= 4n^2 + 36n + 81 - 4n^2 - 20n - 25$

$= 16n + 56$

$= 4(4n + 14)$ *…which is a multiple of 4 for all positive integer n*

f) $(n + 2)^2 – (n – 2)^2 + 3$ $=n^2 + 4n + 4 – (n^2 – 4n + 4) + 3$

$= n^2 + 4n + 4 – n^2 + 4n - 4 + 3$

$=8n + 3$

….which must always be odd since 8n is even

3a) all even numbers are of the form $2n$

b) all odd numbers are of the form $2n + 1$ or $2n – 1$

….for positive integer n

4a) Three consecutive integers can be written n, $n + 1$ and $n + 2$

The sum of these $= n + n + 1 + n + 2 = 3n + 3 = 3(n + 1)$

…which is a multiple of 3 for all positive integer n

b) Three consecutive even numbers can be written $2n, 2n + 2, 2n + 4$

The sum of these $= 2n + 2n + 2 + 2n + 4 = 6n + 6 = 6(n + 1)$

…which is a multiple of 6 for all positive integer n

c) Two consecutive odd numbers can be written $2n + 1$ *and* $2n + 3$

The sum of these $= 2n + 1 + 2n + 3 = 4n + 4 = 2(n + 2)$

…which is even for all positive integer n

d) Three consecutive odd numbers can be written $2n + 1, 2n + 3$ *and* $2n + 5$

The sum of these $= 2n + 1 + 2n + 3 + 2n + 5 = 6n + 9 = 3(2n + 3)$

…which is a multiple of 3 for all positive integer n

e) Four consecutive odd numbers can be written $2n + 1, 2n + 3, 2n + 5$ *and* $2n + 7$

The sum of these $= 2n + 1 + 2n + 3 + 2n + 5 + 2n + 7 = 8n + 16 = 8(n + 2)$

…which is a multiple of 8 for all positive integer n

f) Two consecutive integers can be written n *and* $n + 1$

The sum of these $= n + n + 1 = 2n + 1$

…which is odd for all positive integer n

g) Four consecutive integers can be written $n, n + 1, n + 2$ *and* $n + 3$

The sum of these $= n + n + 1 + n + 2 + n + 3 = 4n + 6 = 4(n + 1) + 2$

…which is always 2 more than a multiple of 4 for all positive integer n

5a) Two odd numbers can be written $2n + 1$ *and* $2m + 1$

Their product $= (2n + 1)(2m + 1)$

$=4nm + 2n + 2m + 1$

$=2(2nm + n + m) + 1$

…which is odd for all positive integer n and m

b) Two consecutive even numbers can be written $2n$ *and* $2n + 2$

Their product $=2n(2n + 2)$

$=4n^2 + 4n$

$=4(n^2 + n)$

…which is a multiple of 4 for all positive integer n

c) The squares of two consecutive integers can be written as n^2 and $(n + 1)^2$

Their difference $= (n + 1)^2 - n^2$

$= n^2 + 2n + 1 – n^2$

$= 2n + 1$

$= n + n + 1$

= the sum of the 2 original integers

d) The squares of two consecutive even numbers can be written $(2n)^2$ *and* $(2n + 2)^2$

Their sum $= (2n)^2 + (2n + 2)^2$

$=4n^2 + 4n^2 + 8n + 4$

$=8n^2 + 8n + 4$

$= 4(2n^2 + 2n + 1)$

…which is a multiple of 4 for all positive integer n

e) The square of an odd integer can be written as $(2n + 1)^2$

$= 4n^2 + 4n + 1$

$=4(n^2 + n) + 1$

…which is one more than a multiple of 4 for positive integer n

Apply

1a) $5n – 3$

b) $(5n – 3)^2 + 1 =$

$25n^2 – 30n + 9 + 1 =$

$25n^2 – 30n + 10 =$

$5(5n^2 – 6n + 2)$ …which is a multiple of 5 for all positive integer n

2a) 1st Term: a

2nd Term: b

3rd Term: a + b

4th Term: a + b + b = a + 2b

b) The sequence continues: 2a + 3b, 3a + 5b, 5a + 8b, 8a + 13b, 13a + 21b, 21a + 34b

Sum of the first 10 terms = 55a + 88b

11 times the 7th term = 11(5a + 8b) = 55a + 88b

3) Start with "abc" (that is 100a + 10b + c with a>c)

Reverse it: "cba"

If we try to subtract these using the column method, in the units column a>c so we would need to "borrow" from b, and then when we come to subtract in the tens column, the top b has become b-1, so we need to borrow from the a to make it b+9

So the subtraction was

100a + 10b + c – 100c – 10b – a

But has become

$100(a-1) + 10(b+9) + c + 10 – 100c – 10b – a =$

$100(a-c-1) + 10x9 + c + 10 – a$

Reverse the digits and add:

$100(a-c-1) + 10x9 + c + 10 – a + 100(10-a + c) + 10x9 + a – c - 1$

$= 100 \times 9 + 20 \times 9 + 9 = 1089$