

August 20th

The radius of a circle is increased from r to $r + a$

By how much is the circumference increased by?

By how much is the area increased by?

What value of a is needed to treble the original circumference?

What value of a is needed to treble the original area?

$$\text{Old circumference} = 2\pi r$$

$$\text{New circumference} = 2\pi(r + a)$$

Which is an increase of **$2\pi a$**

Therefore, to treble the original circumference $2\pi a = 4\pi r$

So we need **$a = 2r$**

$$\text{Old area} = \pi r^2$$

$$\text{New area} = \pi(r + a)^2 = \pi r^2 + 2\pi r a + \pi a^2$$

Which is an increase of **$2\pi r a + \pi a^2$**

Therefore to treble the area $2\pi r a + \pi a^2 = 2\pi r^2$

$$a^2 + 2ar - 2r^2 = 0$$

Solving this gives

$$a = \frac{-2r \pm \sqrt{(2r)^2 - 4 \times 1 \times -2r^2}}{2 \times 1}$$

Hence

$$a = -r \pm r\sqrt{3}$$

Taking the positive root

$$\mathbf{a = (\sqrt{3} - 1)r}$$