

December 31st

Choose any four digit number, where the first digit is larger than the last.

Reverse your number and subtract it from your original.

Taking your answer, add it to it's reverse.

Note your final answer and repeat this process five more times.

$$4542 - 2454 = 2088$$

$$5634 - 4365 = 1269$$

$$2088 + 8802 = 10890$$

$$1269 + 9621 = 10890$$

Why does this always happen?

In general, "abcd" with  $a > d$  and  $b > c$

$$(1000a + 100b + 10c + d) - (1000d + 100c + 10b + a) =$$

$$1000(a-d) + 100(b-c) + 10(c-b) + (d-a)$$

But since  $a > d$

In the units column, when subtracting we need to "borrow" from the 10s column.

Also, since  $b > c$

In the 10s column we have to "borrow" from the 100s column.

Leaving

$$1000(a - d) + 100(b - c - 1) + 10(c - b + 9) + (d - a + 10)$$

Reversing

$$1000(d - a + 10) + 100(c - b + 9) + 10(b - c - 1) + (a - d)$$

Adding these together gives

$$1000 \times 10 + 100 \times 8 + 10 \times 8 + 10 =$$

$$10890$$

