Equipment

1. A black ink ball-point pen.
2. A pencil.
3. An eraser.
4. A ruler.
5. A pair of compasses.
6. A protractor.
7. A calculator

Guidance

1. Read each question carefully.
2. Don’t spend too long on one question.
3. Attempt every question.
4. Check your answers seem right.
5. Always show your workings

Information

1. Time: 1 hour 30 minutes
2. The maximum mark for this paper is 80.
3. You may use tracing paper.
1. \( \xi = \{\text{odd numbers less than 32}\} \)

A = multiples of 3
B = multiples of 5

(a) Complete the Venn diagram

![Venn Diagram](image)

One of the numbers is selected at random.

(b) Write down \( P(A \cap B) \)

\[ \frac{1}{16} \]

2. Solve the simultaneous equations

\[ \begin{align*}
3x - y &= 23 \\
2x + 3y &= 8
\end{align*} \]

Do not use trial and improvement

\[ \begin{align*}
\text{①} \times 3 & \Rightarrow 9x - 3y = 69 \\
\text{②} & \quad + \quad + \\
\hline
11x &= 77 \\
x &= 7
\end{align*} \]

Substituting

\[ \begin{align*}
3x - y &= 23 \\
21 - y &= 23 \\
y &= -2
\end{align*} \]

\[ x = \qquad \text{y} = \qquad \]

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3. The table shows the ages of an under-21 rugby squad.

<table>
<thead>
<tr>
<th>Age</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>5</td>
</tr>
<tr>
<td>19</td>
<td>10</td>
</tr>
<tr>
<td>20</td>
<td>9</td>
</tr>
<tr>
<td>21</td>
<td>4</td>
</tr>
</tbody>
</table>

(a) Find the median age.

median player is number \( \frac{23+1}{2} = 12 \)

the 12th oldest is 20

(b) Is Courtney correct?

You must give a reason for your answer.

mean \( \frac{18 \times 5 + 19 \times 5 + 20 \times 9 + 21 \times 4}{23} \)

\( = \frac{449}{23} \)

\( = 19.52 \ldots \)

median \( = 20 \)

mean \( = 19.52 \)

No
4. Harley sold 380 ice creams. He sold only vanilla, chocolate, strawberry and honeycomb ice creams. 45% of the ice creams are chocolate.

The ratio of vanilla ice creams to strawberry ice creams to honeycomb ice creams is 1:2:8.

Work out how many more chocolate ice creams are sold than honeycomb ice cream.

\[ 55\% \text{ are not chocolate} = 0.55 \times 380 = 209 \]

honeycomb is \[ \frac{8}{11} \times 209 = 152 \]
chocolate = 380 - 209 = 171

\[ 171 - 152 = 19 \]

5. Shown below are two identical regular polygons and an equilateral triangle.

![Diagram of two regular polygons and an equilateral triangle]

Calculate the number of sides each regular polygon has.

\[ 2x + 60 = 360 \]
\[ \therefore x = 150^\circ \text{ (interior angle)} \]
\[ \therefore \text{ exterior angle} = 30^\circ \]

\[ 360 \div 30 = 12 \]
6. Material A has a density of 5.8g/cm³. 
   Material B has a density of 4.1g/cm³.

   377g of Material A and 1.64kg of Material B form Material C.

   Work out the density of Material C.

\[
A: \quad V_A = \frac{377}{5.8} = 65\text{cm}^3 \\
B: \quad V_B = \frac{1640}{4.1} = 400\text{cm}^3 \\
\therefore \quad V_C = 465\text{cm}^3 \\
M_C = 1640 + 377 = 2017\text{g} \\
D_C = \frac{2017}{465} = 4.3376\ldots
\]

\[\text{Density of Material C} = \frac{2017}{465} = 4.34\text{g/cm}^3 \]

(4)

7. 

[Diagram of a triangle with sides 12cm, 13cm, hyp = hypotenuse, opp = opposite]

Calculate the size of angle BAC.

\[\sin BAC = \frac{12}{13}\]

\[\therefore \quad BAC = \sin^{-1}\left(\frac{12}{13}\right) = 67.38\ldots\]

(3)
8. Shown is a square and a circle.

Each vertex of the square is on the circumference of the circle.

The area of the circle is $81 \text{ cm}^2$.

Find the area of the shaded region.
Give your answer to 4 significant figures.

\[
\text{area} = \pi r^2 = 81 \quad \therefore r = \sqrt{\frac{81}{\pi}} = 5.0777 \ldots
\]

\[
\text{side length of the square} = \sqrt{r^2 + r^2} = 7.18 \ldots
\]

\[
\therefore \text{shaded area} = 81 - (7.18)^2 = 29.4339 \ldots
\]

\[29.43 \text{ cm}^2\]

(4)
9. The functions \( f(x) \) and \( g(x) \) are given by the following:

\[
f(x) = 8 - 3x \\
g(x) = 4x
\]

(a) Calculate the value of \( g(f(3)) \)

\[
f(3) = 8 - 3 \times 3 = -1 \\
g(-1) = 4 \times -1 = -4
\]

\[
\text{---} \quad 4
\]

(b) Solve the equation \( g(f(x)) = 80 \)

\[
g(f(x)) = g(8 - 3x) \\
= 4(8 - 3x) \\
= 32 - 12x \\
32 - 12x = 80 \Rightarrow 12x = -48
\]

\[
\text{x = -} \quad 4
\]


The bank pays 10% interest for the first year and then \( y\% \) every year after that.

After three years, Charlotte has £5610.55

Calculate \( y \).

\[
5000 \times 1.1 \times x^2 = 5610.55
\]

\[
x^2 = 1.0201 \\
x = 1.01
\]

\[
\therefore y = 1\%
\]

\[
\text{y = \%}
\]
11. A food standards inspector is going to visit 3 establishments in one day. In the town, there are 40 restaurants and 12 cafes.

He writes a list of the three different establishments, and the order will either be:

\[
\begin{array}{|c|}
\hline
\text{Cafe} \\
\text{Restaurant} \\
\text{Restaurant} \\
\hline
\end{array}
\quad \begin{array}{|c|}
\hline
\text{Restaurant} \\
\text{Cafe} \\
\text{Cafe} \\
\hline
\end{array}
\]

How many possible lists could he write?

\[12 \times 40 \times 39 = 18,720\]

or

\[40 \times 12 \times 11 = 52,800\]

\[24,000\]

(3)
12. The points D, E, F and G lie in a straight line.

\[ \text{DE} : \text{EG} = 1 : 4 = 4 : 16 \]
\[ \text{DF} : \text{FG} = 9 : 11 \]

Work out \( \text{DE} : \text{EF} : \text{FG} \)

\[ \text{DE} = \frac{4}{20} \text{ at total length } \text{EG} \]
\[ \text{FG} = \frac{11}{20} \text{ at total length } \text{EG} \]
\[ \text{DF} = \frac{9}{20} \text{ so } \frac{\text{EF}}{\text{EG}} = \frac{9}{20} \frac{4}{20} = \frac{5}{20} \]

\[ \therefore \text{Ratio} = \frac{4}{20} : \frac{5}{20} : \frac{11}{20} \]

\[ 4 : 5 : 11 \]
The region labelled R satisfies three inequalities.

State the three inequalities

- Line A has equation $y = x - 2$
- Line B has equation $y = -2x + 4$ (or $2x + y = 4$)
- Line C has equation $y = -4$

\[
\begin{align*}
&y > -4 \\
&y \leq x - 2 \\
&y \leq -2x + 4 \quad \text{(3)}
\end{align*}
\]

($x$ or $2x + y \leq 4$)
14. (a) Simplify fully

\[
\frac{5x^2 - 13x - 6}{x^2 - 9} = \frac{(5x+2)(x-3)}{(x+3)(x-3)}
\]

(b) Make \( m \) the subject of the formula

\[
y = \frac{m + 4}{m + 5}
\]

\[
y(m+5) = m+4
\]

\[
ym + 5y = m + 4
\]

\[
ym - m = 4 - 5y
\]

\[
m(y - 1) = 4 - 5y
\]

\[
m = \frac{4 - 5y}{y - 1}
\]

(or \( \frac{5y - 4}{1 - y} \))
The area of the triangle is \(90\sqrt{3}\, \text{cm}^2\)

Work out the value of \(x\).

\[
\text{area} = \frac{1}{2}ab\sin C
\]
\[
= \frac{1}{2}(2x - 2)(x + 10)\sin 60
\]
\[
= \frac{\sqrt{3}}{2} (x^2 + 9x - 10) = 90\sqrt{3}
\]
\[
\therefore x^2 + 9x - 10 = 180
\]
\[
\therefore x^2 + 9x - 190 = 0
\]
\[
(x + 19)(x - 10) = 0
\]

\[x = 10\]
16. Using \( x_{n+1} = -3 - \frac{2}{x_n^2} \)

with \( x_0 = -3.5 \)

(a) find the values of \( x_1 \), \( x_2 \) and \( x_3 \)

\[
x_1 = -3 - \frac{2}{(-3.5)^2} = -3.163 \ldots
\]

\[
x_2 = \ldots \ldots
\]

\[
x_1 = -3.163 \ldots
\]

\[
x_2 = -3.1998 \ldots
\]

\[
x_3 = -3.1953 \ldots
\]

(b) Explain the relationship between the values of \( x_1 \), \( x_2 \) and \( x_3 \) and the equation \( x^3 + 3x^2 + 2 = 0 \)

They are increasingly accurate solutions to the equation.

\[
\text{(since } x^3 + 3x^2 + 2 = 0 \Rightarrow x^3 = -3x^2 - 2 \text{ )}
\]

\[
\Rightarrow x = -3 - \frac{2}{x^2}
\]

(2)
17. The curved surface area of a cone is given by the formula

\[ A = \pi rl \]

where \( A \) is the curved surface area
\( r \) is the radius of the base of the cone
and \( l \) is the slant height

Given \( A = 220 \text{ cm}^2 \) correct to 3 significant figures,
and \( r = 8 \text{ cm} \) correct to 1 significant figure.

Calculate the upper bound for \( l \).

\[
\ell = \frac{A}{\pi r}
\]

\[
\therefore l_{\text{max}} = \frac{A_{\text{max}}}{\pi r_{\text{min}}} = \frac{220.5}{\pi \times 7.5} = 9.3583 \ldots
\]

9.4 cm

(3)
18. Find the values of $x$ and $y$

**cyclic quadrilateral**: 

\[ x + 32y + 13y = 180 \]

\[ 2x + 45y + x = 180 \]

\[ x + 45y = 180 \]

\[ 3x - 45y = 180 \]

\[ 4x = 360 \]

\[ x = 90, \quad y = 2 \]

\[ \text{19. Solve the inequality} \quad 2x^2 + 9x + 10 > 0 \]

\[ (x+2)(2x+5) > 0 \]

Critical points are $x = -2$, $x = -2.5$

\[ x < -2.5 \text{ or } x > -2 \]
20. The circle C has equation \( x^2 + y^2 = 4 \)

The circle is reflected in the line \( y = 2 \) to give circle D

Circle D is translated by the vector \( \begin{pmatrix} -1 \\ 0 \end{pmatrix} \) to give circle E

(a) Draw a sketch of circle E

\( C \) has centre \( (0,0) \) radius \( 2 \)
\( D \) has centre \( (0,4) \) radius \( 2 \)
\( E \) has centre \( (-1,4) \) radius \( 2 \)

(b) Write down the coordinates of the centre of circle E

\( (-1, 4) \)

(c) Write down the coordinates of points where circle E meets the y-axis

Equation is \( (x+1)^2 + (y-4)^2 = 4 \)

\( x = 0 \) \( \therefore 1 + (y-4)^2 = 4 \)

\( (y-4)^2 = 3 \)

\( y = 4 \pm \sqrt{3} \)

\( (0, 4 - \sqrt{3}) \) and \( (0, 4 + \sqrt{3}) \)