

July 10<sup>th</sup>

**Find a number, half of which is a square number,  
and one third of which is a cube number.**

Consider the prime factors of such a number.

If it only has 2 distinct prime factors, they must be 2 and 3 (as it must be divisible by both)

For half of it to be square it must be of the form  $2 \times 2^n \times 3^m$

Where n and m are even

For a third of it to be cube it must be of the form  $3 \times 2^j \times 3^k$

Where j and k are both multiples of 3

Therefore

$2^{n+1} \times 3^m = 2^j \times 3^{k+1}$  where n and m are even, and j and k are multiples of 3

The smallest such solution is

$$n=2 \quad j=3$$

$$m=4 \quad k=3$$

Hence the smallest such number is

$$\mathbf{2^3 \times 3^4 = 648}$$

$$(648 = 2 \times 18^2 = 3 \times 6^3)$$