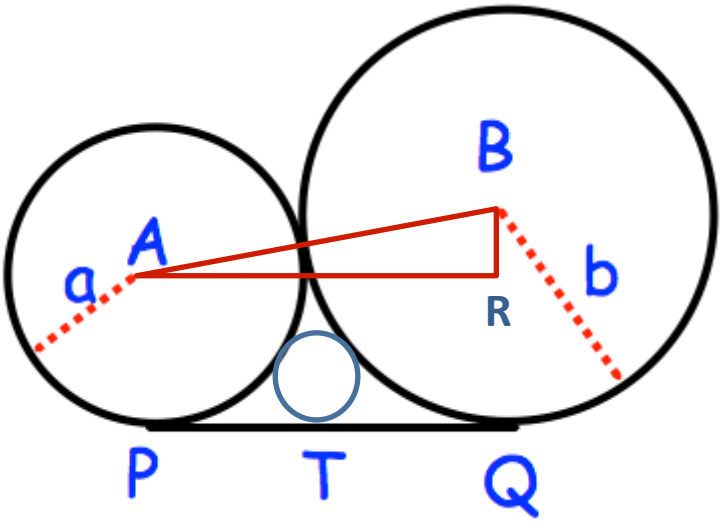
June 2nd

Two circles, with centres A and B have radii a and b. The circle touch each and also the line PTQ at P and Q.

Find the length of PQ.

A third circle touches both circles and the line PTQ at T.

If its radius is x, find PT and QT in terms of x, a and b.



In the triangle ARB

R is a right angle

AB = b + a

BR = b – a

AR = PQ

Hence

$PQ^2 = (a + b)^2 – (b – a)^2 = a^2 + 2ab + b^2 – (b^2 – 2ab + a^2) = 4ab$

Therefore $\mathbf{PQ = 2\sqrt{ab}}$

Using the same argument, $\mathbf{PT = 2\sqrt{ax}}$ and $\mathbf{TQ = 2\sqrt{bx}}$