

May 27th

Show that the difference of the squares of any two prime numbers greater than 3 is always a multiple of 24.

This proof relies on the fact that all prime numbers greater than 3 are of the form $6n \pm 1$

Therefore the square of any such prime is

$$\begin{aligned}(6n \pm 1)^2 &= 36n^2 \pm 12n + 1 \\ &= 12n(3n \pm 1) + 1\end{aligned}$$

If n is odd, then $3n \pm 1$ is also odd and therefore $n(3n \pm 1)$ is even

Hence it can be written as $24k + 1$ where k is an integer

If n is even, then it can also be written as $24r + 1$ where r is an integer

Therefore, the difference between any pair of such numbers will always be a multiple of 24.