

November 19th

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = ?$$

$$S = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$$

$$\frac{1}{2}S = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots$$

Hence

$$S - \frac{1}{2}S = \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots\right) - \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots\right)$$

(All but the first term cancel to 0)

Giving

$$\frac{1}{2}S = \frac{1}{2}$$

Hence

$$S = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = \mathbf{1}$$