October 26th

Show that every odd square leaves a remainder of 1 when divided by 8.

Show that every even square leaves a remainder of 0 or 4 when divided by 8.

Every odd square is of the form

$(2n + 1)^2 = 4n^2 + 4n + 1 =$

$4n(n + 1) + 1$

For integer n, n(n + 1) must be even, ∴ 4n(n + 1) must be a multiple of 8

So, every odd square leaves a remainder of 1 when divided by 8.

Every even square is of the form

$(2n)^2 = 4n^2$

If n is even, n^2 is even, ∴ $4n^2$ must be a multiple of 8

If n is odd, n^2 is odd, ∴ $4n^2$ must be an odd multiple of 4, hence leaving a remainder of 4 when divided by 8.