Equipment

1. A black ink ball-point pen.
2. A pencil.
3. An eraser.
4. A ruler.
5. A pair of compasses.
6. A protractor.
7. A calculator

Guidance

1. Read each question carefully.
2. Don’t spend too long on one question.
3. Attempt every question.
4. Check your answers seem right.
5. Always show your workings

Information

1. Time: 1 hour 30 minutes
2. The maximum mark for this paper is 80.
3. You may use tracing paper.
1. There are 50 sweets in a jar. In a trial, a sweet is chosen at random and then it is replaced. The results are recorded after every 20 trials. The graph shows the relative frequency of a blue sweet.

\[
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
\text{Number of trials} & 0 & 20 & 40 & 60 & 80 & 100 \\
\hline
\text{Relative frequency} & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 \\
\hline
\end{array}
\]

In the first forty trials, ten blue sweets were chosen.

(a) Plot this result on the graph.

\[
\frac{\text{10}}{\text{40}} = 0.25
\]

(b) What is the best estimate, from the graph, of the probability of choosing a blue sweet? Explain your answer.

0.3, since this is from the experiment with the most trials.

(c) Use your answer to estimate the number of blue sweets in the jar.

\[0.3 \times 50 = 15\]
2. At Donhampton High School the ratio of boys to girls is 7:5. Each student studies one language, Spanish or German.

\[
\frac{3}{5} \text{ of the girls studied Spanish.}
\]

168 girls studied German.

75% of the boys study Spanish.

How many boys study Spanish?

\[
\frac{5}{12} \text{ of the students are girls}
\]

\[
\frac{2}{5} \text{ of these study German}
\]

\[
\frac{2}{5} \times \frac{5}{12} = \frac{1}{6} \text{ of the total} = 168
\]

\[
6 \times 168 = 1008 \text{ (total number of students)}
\]

\[
\frac{7}{10} \text{ are boys} = \frac{1008}{12} \times 7 = 588
\]

75% of 588 =

\[
441
\]

(4)
3. Here are the front and side elevations of a solid shape.

(a) On the grid, draw the plan view.
4. Lee complete a journey in three stages.

In stage 1 of his journey, he drives at an average speed of 30km/h for 45 minutes.

(a) How far does Lee travel in stage 1 of his journey?

\[ d = s \times t = 30 \times \frac{3}{4} = 22.5 \text{ km} \]

In stage 2 of his journey, Lee drives at an average speed of 50km/h for 2 hours 48 minutes.

Altogether, over all three stages, Lee drives 200 km in 4 hours.

What is his average speed, in km/h, in stage 3 of his journey?

\[ \text{Stage 3:} \]
\[ d = 200 \text{ km} - 140 \text{ km} - 22.5 \text{ km} = 37.5 \text{ km} \]
\[ t = 4 \text{ hr} - 2 \text{ hr} 48 \text{ min} - 45 \text{ min} = 27 \text{ min} \]
\[ s = 37.5 \div 0.45 = 83.33... \text{ km/h} \]
Triangle $ABC$ is similar to triangle $ADE$.

$AB = 6\text{ cm}$
$BC = 8\text{ cm}$
$CE = 6.25\text{ cm}$
$DE = 18\text{ cm}$

(a) Work out the length of $DB$.

S. f. from $ABC \rightarrow ADE = \frac{18}{8} = 2.25$

$AD = 6 \times 2.25 = 13.5$

$DB = AD \cdot AB = 13.5 - 6$

(b) Work out the length of $AC$.

$AC \times 2.25 = AC + 6.25$

$\therefore 1.25AC = 6.25$

$AC = \frac{6.25}{1.25} = 5\text{ cm}$
6. Martyn has some money to invest and sees this advert.

**Bank of Maths**

Double your money in 15 years.

The average annual growth for your investment is 4.5%

Will Martyn double his money in 15 years by investing his money with "Bank of Maths?"

You **must** show your workings.

\[ x \times 1.045^{15} = 1.935x \ldots \]

so it is unlikely he will double his money

---

7. A number, \( y \), is rounded to 1 decimal place.

The result is 8.1

Using inequalities, write down the error interval for \( y \)

\[ 8.05 \leq y < 8.15 \]
8. A university surveyed 60 mathematics graduates on their starting salary. The cumulative frequency graph shows some information about the salaries.

Use the graph to find an estimate for the interquartile range.

\[ IQR = \text{£35,000} - \text{£25,000} \]

\[ \text{£10,000} \ldots \ldots \ldots \ldots \]

(2)

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Describe fully the single transformation that maps triangle A onto triangle B.

**rotation 90° clockwise about (0, 0)**
10. \[ M = 4ac^2 \]

\[ a = 9.2 \times 10^{-6} \quad \text{and} \quad c = 7.8 \times 10^4 \]

(a) Work out the value of \( M \)

Give your answer in standard form correct to 2 significant figures.

\[ = 223891.2 \]

\[ M = 2.24 \times 10^5 \]  

(2)

a is doubled

and c is doubled

Jordan says,

"The value of \( M \) will be four times larger because both \( a \) and \( c \) are doubled."

(b) Explain why Jordan is wrong.

\[ \text{as \( c \) is squared, \( M \) would actually be} \]

\[ \times 8 \quad (x^2 \text{ for } a, \ x^2 \text{ for } c, \ x2 \text{ for } c) \]  

(1)

11. Solve

\[ \frac{10x - 3}{3} + \frac{5x + 2}{4} = 5 \quad \text{both sides by 12} \]

\[ \frac{4(10x-3)+3(5x+2)}{12} = 60 \]

\[ 40x - 12 + 15x + 6 = 60 \]

\[ 55x = 66 \quad x = \frac{66}{55} \]

\[ \frac{6}{5} = 1.2 \]

\[ x = \text{..........................} \]  

(4)
12. Jennifer is playing darts. She throws two darts aiming for a Bullseye.

The probability Jennifer hits the Bullseye on her first throw is \(\frac{1}{4}\). The probability she hits the Bullseye on her second throw \(\frac{1}{3}\).

(a) Complete the tree diagram.

(b) Work out the probability Jennifer hits the Bullseye at least once.

\[
P(\text{miss, miss}) = \frac{3}{4} \times \frac{2}{3} = \frac{6}{12}
\]

\[
P(\text{at least one hit}) = 1 - \frac{5}{12} = \frac{6}{12}
\]
Below is a histogram showing information about the weight of 66 parcels.

\[
\begin{array}{c|c|c|c|c}
\text{Weight (kilograms)} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
\text{Frequency density} & 0 & 5 & 10 & 20 & 15 & 10 & 0 \\
\end{array}
\]

\( \frac{1}{3} \) of the parcels which weighed between 2.5kg and 4kg were sent to Scotland.

Work out an estimate for the number of parcels sent to Scotland.

\[
\text{area } A = 0.5 \times 18 = 9
\]
\[
\text{area } B = 1 \times 12 = 12
\]
\[
\text{total} = 21
\]

\[
\frac{1}{3} \times 21 = 7
\]

(3)
14. Match each graph to the correct equation

Graph A

Graph B

Graph C

Graph D

$y = x^2$ is graph A

$y = x^3$ is graph B

$y = 2^x$ is graph C

$y = \frac{1}{x}$ is graph D

(2)
A and B are points on the circumference of a circle, centre O.
CA is a tangent to the circle.
Angle CAB = 2x

Prove that angle AOB = 4x
Give reasons for each stage of your working.

extend A & B to make triangle APB

\[ \angle APB = \angle CAB = 2x \text{ (Alternate angle theorem)} \]

hence
\[ \hat{A}OB = 4x \text{ (Angle at the centre is twice the angle at the circumference)} \]
16. Show, using algebra, that \( 1.0\overline{24} = 1 \frac{4}{165} \)

\[
0.0\overline{24} = 0.0242424\ldots
\]

\[
x = 0.02424\ldots
\]

\[
10x = 0.242424\ldots
\]

\[
1000x = 24.2424\ldots \quad \text{subtracting}
\]

\[
990x = 24
\]

\[
\therefore 0.0\overline{24} = x = \frac{24}{990} = \frac{4}{165}
\]

\[
\therefore 1.0\overline{24} = 1 \frac{4}{165}
\]
AOC is an equilateral triangle of side length 14 cm.

OBD is a sector of a circle with centre O and radius 11 cm.

Calculate the area of the shaded region as a percentage of the area of triangle AOC.

Give your answer correct to 3 significant figures.

\[
\text{area of triangle} = \frac{1}{2} \times 14 \times 14 \times \sin 60^\circ = 49\sqrt{3}
\]

\[
\text{area of sector} = \frac{1}{6} \times \pi \times 11^2 = \frac{121\pi}{6}
\]

\[
\therefore \text{shaded region as a } \% \text{ is}
\]

\[
= \frac{49\sqrt{3} - \frac{121\pi}{6}}{49\sqrt{3}} \times 100 = 25.3504\ldots
\]

\[
25.3 \ldots\% \quad (5)
\]
18. \[ \frac{81^y}{3^{y-5}} = 3\sqrt{3} \]

Work out the exact value of \( y \)

\[ 3\sqrt{3} = 3^1 \times 3^{\frac{1}{2}} = 3^{\frac{3}{2}} \]
\[ 81^y = (3^4)^y = 3^{4y} \]

Now, \( \frac{3^{4y}}{3^{y-5}} = 3^{\frac{3}{2}} \Rightarrow 4y - (y-5) = \frac{3}{2} \)
\[ 3y + 5 = \frac{3}{2} \]
\[ 3y = -\frac{7}{2} \]
\[ y = -\frac{7}{6} \] (3)

19. Simplify fully

\[ \frac{x^3 - x}{x + 2} \div \frac{x^2 - x}{x^2 - 5x - 14} \]

\[ \frac{x(x^2 - 1)}{x + 2} \div \frac{x(x - 1)}{(x - 7)(x + 2)} \]

\[ = \frac{x(x + 1)(x - 1)}{(x + 2)x(x - 1)} \times \frac{(x - 7)(x + 2)}{x^2} = (x + 1)(x - 7) \] (3)
20. The diagram shows part of the graph of \( y = x^2 - x - 2 \)

(a) By drawing an appropriate straight line, use your graph to find estimates for the solutions of \( x^2 - 2x - 1 = 0 \)

\[
\begin{align*}
\frac{y = x^2 - x - 2}{x = 0 - 2x - 1} & \quad y = x - 1 \\
\end{align*}
\]

\[x = -0.4 \text{ or } x = 2.4 \quad \text{(2)}\]

(b) Calculate an estimate for the gradient of the graph \( y = x^2 - x - 2 \) at the point where \( x = 1 \)

\[
\frac{3}{3} \quad \text{or} \quad 1
\]

\[\text{..........................} \quad \text{(3)}\]
21. Shown below is a parallelogram.

\[ \text{area} = 2 \times \frac{1}{2} \times 7 \times 10 \times \cos 50^\circ \]
\[ = 53.62311 \ldots \]

22. Here are the first five terms of a quadratic sequence.

\[ \begin{array}{cccccc}
3 & 12 & 25 & 42 & 63 \\
+9 & +13 & +17 & +21 \\
\end{array} \]

Find an expression, in terms of \( n \), for the \( n \)th term of this sequence.

\[ 4 \div 2 = 2 \]

\[ 2n^2: 2 \quad 8 \quad 18 \quad 32 \quad 50 \]
\[ +1 \quad +4 \quad +7 \quad +10 \quad +13 \]

\[ 3n - 2 \]

\[ 2n^2 + 3n - 2 \]
23. A circle has an equation of \( x^2 + y^2 = 5 \)
\[ Q \left( \frac{4}{3}, \frac{\sqrt{29}}{3} \right) \] is a point on the circle.

Find the equation of the tangent to the circle at the point \( Q \).

\[
\text{gradient of radius } OQ = \frac{\sqrt{29}}{\frac{4}{3}} = \frac{\sqrt{29}}{4}
\]

\[
\therefore \text{gradient of tangent} = -\frac{4}{\sqrt{29}}
\]

\[
y = -\frac{4}{\sqrt{29}} x + C
\]

\[
\frac{\sqrt{29}}{3} = -\frac{4}{\sqrt{29}} \times \frac{4}{3} + C
\]

\[
C = \frac{\sqrt{29}}{3} + \frac{16}{3 \sqrt{29}} = \frac{15 \sqrt{29}}{29}
\]

\[
y = -\frac{4}{\sqrt{29}} x + 15 \frac{\sqrt{29}}{29}
\]

\[
0 = y = \frac{\sqrt{29}}{29} (-4x + 15)
\]