Level 2 Further Maths

Equation of a Normal

Ensure you have: Pencil or pen

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic

www.corbettmaths.com/more/further-maths/
1. Below is a sketch of \( y = f(x) \)

\[ \text{There is a minimum point at } A (5, -1) \]

(a) Write down the equation of the tangent at the point A

\[ y = -1 \]

(1)

(b) Write down the equation of the normal at the point A

\[ x = 5 \]

(1)
2. A curve has equation \( y = x^2 + 8x \)

(a) Find \( \frac{dy}{dx} \)

\[
\frac{dy}{dx} = 2x + 8
\]

(b) Find the gradient of the curve at the point (1, 9)

\[
(2 \times 1) + 8 = 10
\]

(c) Find the gradient of the normal to the curve at the point (1, 9)

\[
-1 \quad \frac{-1}{10}
\]
3. Here is a sketch of \( y = f(x) \)

There is a maximum point at A \((-2, 5)\)

(a) Write down the equation of the normal to the curve at A

Given \( \frac{dy}{dx} = 3x^2 + 6x \)

(b) Work out the equation of the normal to the curve at the point at \((-1, 3)\)

\[
\begin{align*}
\frac{dy}{dx} &= 3 - 6 = -3 \\
y &= \frac{1}{3}x + c \\
\frac{-3}{3} &= c \\
c &= 3\frac{1}{3} \\
y &= \frac{1}{3}x + 3\frac{1}{3}
\end{align*}
\]
4. Work out the equation of the normal to the curve \( y = 2x^2 - 4x + 1 \) at the point \( (2, 1) \)

Give your answer in the form \( y = mx + c \)

\[
\frac{dy}{dx} = 4x - 4
\]

\( x = 2 \)

\[
\frac{dy}{dx} = 4.
\]

\[y = -\frac{1}{4}x + c\]

\[1 = -\frac{1}{4} + c\]

\[c = \frac{5}{4}\]

\[
y = -\frac{1}{4}x + \frac{5}{4}
\]

5. Work out the equation of the normal to the curve \( y = x^3 - 2x + 1 \) at the point where \( x = -1 \)

\[
\frac{dy}{dx} = 3x^2 - 2
\]

\[x = -1 \quad y = 2\]

when \( x = -1 \)

\[
\frac{dy}{dx} = 1
\]

\[y = -x + c\]

\[2 = 1 + c\]

\[c = 1\]

\[
y = -x + 1
\]
6. Work out the equation of the normal to the curve \( y = 2x^3 + x^2 - 7x - 5 \) at the point \((0, -5)\).

Give your answer in the form \( y = mx + c \).

\[
\frac{dy}{dx} = 6x^2 + 2x - 7
\]

\[x = 0, \quad \frac{dy}{dx} = -7\]

\[y = \frac{1}{2}x + c\]

\[-5 = 0 + c\]

\[c = -5\]

\[
\text{.........................}
\]

\[
y = \frac{1}{2}x - 5
\]

(5)

7. Work out the equation of the normal to the curve \( y = (x + 2)(x + 3) \) at the point where \( x = -4 \).

\[
y = x^2 + 5x + 6
\]

\[
\frac{dy}{dx} = 2x + 5
\]

\[x = -4, \quad \frac{dy}{dx} = -3\]

\[y = \frac{1}{2}x + c\]

\[2 = \frac{4}{3} + c\]

\[c = 3\frac{1}{3}\]

\[
\text{.........................}
\]

\[
y = \frac{1}{3}x + 3\frac{1}{3}
\]

(5)

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8. The point A lies on the curve \( y = x^2 - 2x + 4 \)

The x-coordinate of A is \(-1\)

The normal at A also intersects the curve at B.

Work out the coordinates of point B.

\[
\frac{dy}{dx} = 2x - 2
\]

\( x = -1 \)
\[
\frac{dy}{dx} = -4
\]

\( y = \frac{1}{4}x + c \)

\( x = -1 \)
\( y = 7 \)

\( 7 = -\frac{1}{4} - c + c \)

\( c = 7 + \frac{1}{4} \)

\( y = \frac{1}{4}x + 7 + \frac{1}{4} \)

\[
\frac{1}{4}x + 7 + \frac{1}{4} = x^2 - 2x + 4
\]

\( x + 29 = 4x^2 - 8x + 16 \)

\( 0 = 4x^2 - 9x - 13 \)

\( (4x - 13)(x + 1) = 0 \)

\( x = \frac{13}{4} \quad x = -1 \)
9. A curve has equation \( y = 4x^2 + 2x - 3 \)

A normal to the curve is drawn at the point A.
The normal is parallel to the line with equation \( x - 6y = 2 \)

Find the equation of the normal at the point A.
Give your answer in the form \( y = mx + c \)

\[
\frac{dy}{dx} = 8x + 2
\]

\[
x - 6y = 2
6y = x - 2
y = \frac{1}{6}x - \frac{1}{3}
\]

\[
8x + 2 = -6
8x = -8
x = -1
y = -1
\]

\[
\frac{dy}{dx} = \frac{1}{6}
-1 = -\frac{1}{6} + c
-\frac{5}{6} = c
y = \frac{1}{6}x - \frac{5}{6}
\]

(6)