Name:

Level 2 Further Maths

Equation of a Normal

Corbettmaths

Ensure you have: Pencil or pen

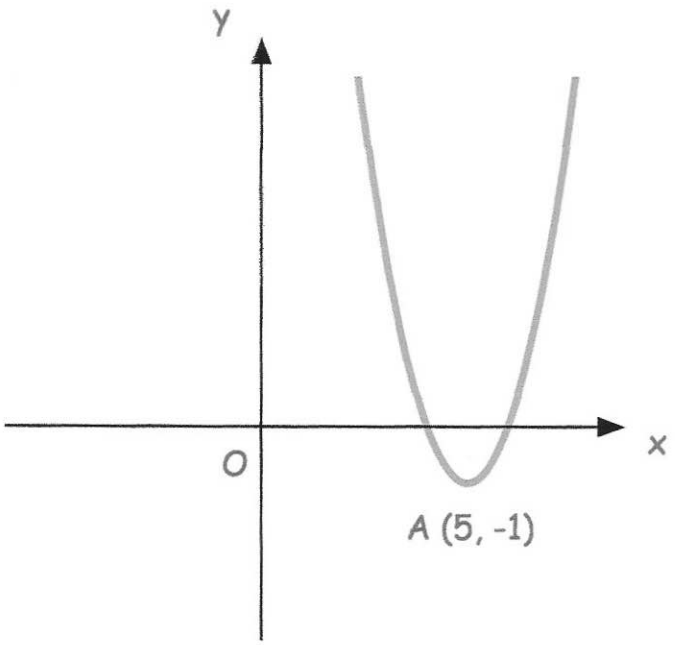
Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic

www.corbettmaths.com/more/further-maths/

1. Below is a sketch of $y = f(x)$



There is a minimum point at A $(5, -1)$

(a) Write down the equation of the tangent at the point A

$y = -1$

(1)

(a) Write down the equation of the normal at the point A

$x = 5$

(1)

2. A curve has equation $y = x^2 + 8x$

(a) Find $\frac{dy}{dx}$

$\frac{dy}{dx} = 2x + 8$

.....

(2)

$x = 1$

(b) Find the gradient of the curve at the point $(1, 9)$

$(2 \times 1) + 8 = 10$

10

.....

(2)

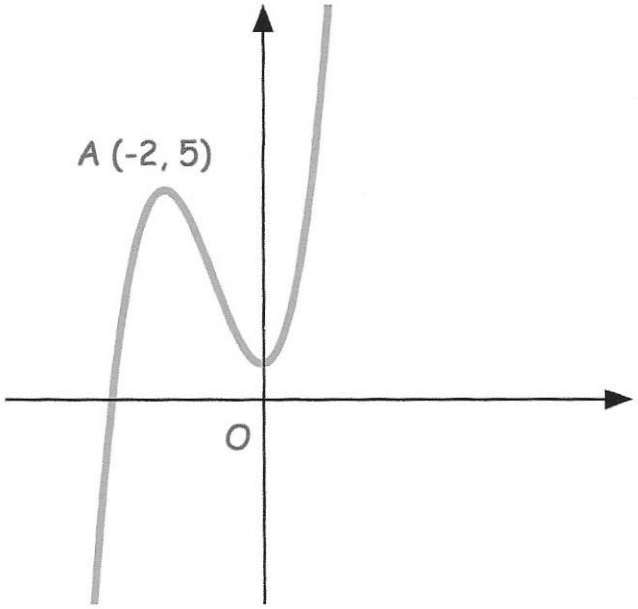
(c) Find the gradient of the normal to the curve at the point $(1, 9)$

$-\frac{1}{10}$

.....

(1)

3. Here is a sketch of $y = f(x)$



There is a maximum point at A $(-2, 5)$

(a) Write down the equation of the normal to the curve at A

$x = -2$

.....

(1)

Given $\frac{dy}{dx} = 3x^2 + 6x$

(b) Work out the equation of the normal to the curve at the point at $(-1, 3)$

x y

At $x = -1$ $\frac{dy}{dx} = 3 - 6 = -3$

$y = \frac{1}{3}x + c$

$3 = -\frac{1}{3} + c$

$c = 3\frac{1}{3}$

$y = \frac{1}{3}x + 3\frac{1}{3}$

.....

(5)

4. Work out the equation of the normal to the curve $y = 2x^2 - 4x + 1$ at the point (2, 1)

Give your answer in the form $y = mx + c$

$\frac{dy}{dx} = 4x - 4$

$x = 2 \quad \frac{dy}{dx} = 4$

$y = -\frac{1}{4}x + c$

$1 = -\frac{1}{2} + c$

$c = 1\frac{1}{2}$

$y = -\frac{1}{4}x + 1\frac{1}{2}$

(5)

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5. Work out the equation of the normal to the curve $y = x^3 - 2x + 1$ at the point where $x = -1$

$x = -1 \quad y = 2$

$\frac{dy}{dx} = 3x^2 - 2$

when $x = -1 \quad \frac{dy}{dx} = 1$

$y = -x + c$

$2 = 1 + c$

$c = 1$

$y = -x + 1$

(5)

6. Work out the equation of the normal to the curve $y = 2x^3 + x^2 - 7x - 5$ at the point $(0, -5)$

x y

Give your answer in the form $y = mx + c$

$\frac{dy}{dx} = 6x^2 + 2x - 7$

$x = 0 \quad \frac{dy}{dx} = -7$

$y = \frac{1}{7}x + c$

$-5 = 0 + c$

$c = -5$

$y = \frac{1}{7}x - 5$

(5)

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7. Work out the equation of the normal to the curve $y = (x + 2)(x + 3)$ at the point where $x = -4$

$y = x^2 + 5x + 6$

$x = -4$

$y = 2$

$\frac{dy}{dx} = 2x + 5$

$x = -4 \quad \frac{dy}{dx} = -3$

$y = \frac{1}{3}x + c$

$2 = -\frac{4}{3} + c$

$c = 3\frac{1}{3}$

$y = \frac{1}{3}x + 3\frac{1}{3}$

(5)

8. The point A lies on the curve $y = x^2 - 2x + 4$

The x-coordinate of A is –1

The normal at A also intersects the curve at B.

Work out the coordinates of point B.

$\frac{dy}{dx} = 2x - 2$

$x = -1 \quad \frac{dy}{dx} = -4$

$y = \frac{1}{4}x + c \qquad x = -1, \; y = 7$

$y = \frac{1}{4}x + c$

$7 = -\frac{1}{4} + c$

$c = 7\frac{1}{4}$

$y = \frac{1}{4}x + 7\frac{1}{4}$

$\frac{1}{4}x + 7\frac{1}{4} = x^2 - 2x + 4$

$x + 29 = 4x^2 - 8x + 16$

$0 = 4x^2 - 9x - 13$

$(4x - 13)(x + 1) = 0$

$x = \frac{13}{4} \qquad x = -1$ ✓

$\left(\frac{13}{4}, \frac{129}{16}\right)$

(8)

9. A curve has equation $y = 4x^2 + 2x - 3$

A normal to the curve is drawn at the point A.
The normal is parallel to the line with equation $x - 6y = 2$

Find the equation of the normal at the point A.
Give your answer in the form $y = mx + c$

$\frac{dy}{dx} = 8x + 2$

$8x + 2 = -6$

$8x = -8$

$x = -1$

$x - 6y = 2$

$6y = x - 2$

$y = \frac{1}{6}x - \frac{1}{3}$

$y = -1$

$y = \frac{1}{6}x + c$

$-1 = -\frac{1}{6} + c$

$-\frac{5}{6} = c$

$y = \frac{1}{6}x - \frac{5}{6}$

.....

(6)