Name:

Level 2 Further Maths

Equation of a Tangent

Corbettmaths

Ensure you have: Pencil or pen

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic

www.corbettmaths.com/more/further-maths/

1. A curve has equation $y = x^2 + 6x - 3$

(a) Find the value of $\frac{dy}{dx}$ when $x = 1$

$\frac{dy}{dx} = 2x + 6$

When $x = 1$ $\frac{dy}{dx} = 8$

8

(2)

(b) Work out the equation of the tangent to the curve $y = x^2 + 6x - 3$ at the point $(1, 4)$

x y

$y = 8x + c$

$4 = 8 + c$

$c = -4$

$y = 8x - 4$

(4)

2. A curve has equation $y = x^3 + 4x^2 + x$

(a) When $x = -1$, show that the value of $\frac{dy}{dx}$ is -4

$\frac{dy}{dx} = 3x^2 + 8x + 1$

when $x = -1$

$\frac{dy}{dx} = 3 + (-8) + 1 = -4$ ✓

(2)

(b) Work out the equation of the tangent to the curve $y = x^3 + 4x^2 + x$ at the point $(-1, 2)$
$x \quad y$

$y = -4x + c$

$2 = 4 + c$

$c = -2$

$y = -4x - 2$

(4)

3. A curve has equation $y = 2x^2 - 3x + 1$

(a) Work out $\frac{dy}{dx}$

$\frac{dy}{dx} = 4x - 3$

$\frac{dy}{dx} = 4x - 3$

(2)

(b) Line L is the tangent to the curve $y = 2x^2 - 3x + 1$ at the point $(3, 10)$ (x, y)
Work out the equation of L

$x = 3$

$\frac{dy}{dx} = 9$

$y = 9x + c$

$10 = 27 + c$

$c = -17$

$y = 9x - 17$

(4)

4. A curve has equation $y = (x - 7)(x - 3)$

(a) When $x = -2$, show that the value of $\frac{dy}{dx}$ is -14

$y = x^2 - 10x + 21$

$\frac{dy}{dx} = 2x - 10$

when $x = -2$ $\frac{dy}{dx} = -14$ ✓

(2)

(b) Work out the equation of the tangent to the curve $y = (x - 7)(x - 3)$ at the point where $x = -2$

$x = -2$

$y = (-9)(-5) = 45$

$y = -14x + c$

$45 = 28 + c$

$c = 17$

$y = -14x + 17$

(4)

5. A curve has equation $y = x^4 - 3x^3 + x$

(a) Work out $\frac{dy}{dx}$

$\frac{dy}{dx} = 4x^3 - 9x^2 + 1$

(2)

(b) Work out the equation of the tangent to the curve at the point where $x = -1$

$x = -1$

$y = 1 - 3(-1) + (-1)$

$y = 3$

$\frac{dy}{dx} = -12$

$y = -12x + c$

$3 = 12 + c$

$c = -9$

$y = -12x - 9$

(4)

6. A curve has equation $y = x^2(3 - x)$

Work out the equation of the tangent to the curve at the point $(3, 0)$ (x, y)

$y = 3x^2 - x^3$

$\frac{dy}{dx} = 6x - 3x^2$

When $x = 3$

$\frac{dy}{dx} = 18 - 3 \times 9$

$= -9$

$y = -9x + c$

$0 = -27 + c$

$c = 27$

$y = -9x + 27$

(5)

7. A curve has equation $y = 4x^3 - 7x^2 + 12$

Work out the equation of the tangent to the curve at the point where $x = 2$

$\frac{dy}{dx} = 12x^2 - 14x$

$x = 2$

$\frac{dy}{dx} = 48 - 28 = 20$

$x = 2 \quad y = 16$

$y = 20x + c$

$16 = 40 + c$

$c = -24$

$y = 20x - 24$

(6)

8. The equation of a curve is $y = x^2 - 3x - 8$

(a) Work out $\frac{dy}{dx}$

$\frac{dy}{dx} = 2x - 3$

(2)

P is a point on the curve.
The tangent to the curve at P has gradient 5

(b) Work out the coordinates of P

$2x - 3 = 5$

$2x = 8$

$x = 4$

$(4, -4)$

(2)

---

9. The equation of a curve is $y = (x - 2)(x + 6)$

P is a point on the curve.
The tangent to the curve at P has gradient –2

Work out the coordinates of P

$y = x^2 + 4x - 12$

$\frac{dy}{dx} = 2x + 4$

$2x + 4 = -2$

$2x = -6$

$x = -3$

$(-3, -15)$

(4)

10. A curve has equation $y = 3x^2 - x + 7$

At the point P on the curve, the tangent is parallel to the line $y = 2x - 8$

Work out the coordinates of P

$\frac{dy}{dx} = 6x - 1$

$6x - 1 = 2$

$6x = 3$

$x = \frac{1}{2}$

$\left(\frac{1}{2}, \frac{29}{4}\right)$

(4)

---

11. A curve has a gradient function $\frac{2x^3 - 9}{10}$

The point P is a point on the curve.

The tangent to the curve at the point P is perpendicular to the line $2x - 5y + 3 = 0$

Work out the x-coordinate of P

$5y = 2x + 3$

$y = \frac{2}{5}x + \frac{3}{5}$

$m = -\frac{5}{2}$

$\frac{2x^3 - 9}{10} = -\frac{5}{2}$

$2x^3 - 9 = -25$

$2x^3 = -16$

$x^3 = -8$

$x = -2$

$x = -2$

(5)

12. Show that the tangents to the curve $y = x^3 - 4x^2 - 4x + 4$ at $x = -\frac{1}{3}$ and $x = 3$ are parallel.

$\frac{dy}{dx} = 3x^2 - 8x - 4$

when $x = 3$

$= 3(3)^2 - 8(3) - 4$

$= -1$

$x = -\frac{1}{3}$ $= 3(-\frac{1}{3})^2 - 8(-\frac{1}{3}) - 4$

$= -1$

$\therefore$ parallel

(5)

13. The curve C has equation $y = \frac{1}{2}x^4 - 3x^2$

The point P on the curve C has x-coordinate 2.

The tangent at P meets the x-axis at the point $(k, 0)$

Find the value of k

$\frac{dy}{dx} = 2x^3 - 6x$

$x = 2$

$\frac{dy}{dx} = 16 - 12 = 4$

$x = 2$ $\quad$ $x \quad y$

$y = -4$ $\quad$ $P(2, -4)$

$y = 4x + c$

$-4 = 8 + c$

$c = -12$

$y = 4x - 12$

$0 = 4x - 12$

$4x = 12$

$x = 3$

$k = 3$

$(3, 0)$

.....

(6)

14. The curve C has equation $y = \frac{1}{3}x^3 - 2x^2 - 10x + 4$

The point P has coordinates $(\overset{x}{-3}, \overset{y}{7})$

(a) Find the equation of the tangent to C at P.

$\frac{dy}{dx} = x^2 - 4x - 10$

$\frac{dy}{dx} = 9 - (-12) - 10$

$= 21 - 10$

$= 11$

$y = 11x + c$

$7 = -33 + c$

$c = 40$

$y = 11x + 40$

.....

(5)

Another point Q also lies on C.
The tangent to C at Q is parallel to the tangent to C at P.

(b) Find the x-coordinate of Q

$x^2 - 4x - 10 = 11$

$x^2 - 4x - 21 = 0$

$(x - 7)(x + 3) = 0$

$x = 7$ or $x = -3$ ✓

$x = 7$

.....

(5)