Level 2 Further Maths
Using Differentiation
to Solve Problems

Ensure you have: Pencil or pen

Guidance
1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic
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1. A farmer creates a rectangular pen for his chickens.

![Rectangular Pen Diagram]

The width of the field is \(x\) metres.
The perimeter of the field is 100 metres.

(a) Show that the length of the rectangle is \(50 - x\) metres

\[
\begin{align*}
x + x + y + y &= 100 \\
2x + 2y &= 100 \\
2y &= 100 - 2x \\
y &= 50 - x
\end{align*}
\]

(b) Show that the area of the field is \(A = 50x - x^2\)

\[
\begin{align*}
A &= x(50 - x) \\
A &= 50x - x^2
\end{align*}
\]

(c) Use differentiation to find the value of \(x\) for which \(A\) is a maximum

\[
\frac{dA}{dx} = 50 - 2x
\]

\[50 - 2x = 0\]
\[2x = 50\]
\[x = 25\]
2. The shape below is made from two rectangles.

The perimeter of the shape is 100 cm.

(a) Show that \( y = 45 - 5x \)

\[
2y + 3x + 5 + 3x + 5 + 4x = 100
\]
\[
2y + 10x + 10 = 100
\]
\[
y = 45 - 5x
\]
\[
2y = 90 - 10x
\]

The area of the shape is \( A \) cm\(^2\)

(b) Show that \( A = 225 + 116x - 13x^2 \)

\[
A = (45 - 5x)(3x + 5) + 2x(x + 3)
\]
\[
A = 135x + 225 - 15x^2 - 15x + 2x^2 + 6x
\]
\[
A = 225 + 116x - 13x^2
\]

(c) Use differentiation to find the value of \( x \) for which \( A \) is a maximum

\[
\frac{dA}{dx} = 116 - 26x
\]
\[
116 - 26x = 0
\]
\[
26x = 116
\]
\[
x = \frac{58}{13} \approx 4.461
\]
3. Shown below is a metal box in the shape of a cuboid.

![Cuboid Diagram]

The volume of the box is 80cm³

(a) Show that \( y = \frac{80}{x^2} \)

\[
x^2 y = 80
\]

\[
y = \frac{80}{x^2}
\]

(b) Show that the area of metal to make the box is given by

\[
A = 2x^2 + \frac{320}{x}
\]

\[
A = x^2 + x^2 + 4xy
\]

\[
A = 2x^2 + 4xy
\]

\[
A = 2x^2 + 4x \left( \frac{80}{x^2} \right)
\]

\[
A = 2x^2 + \frac{320}{x}
\]

(c) Use differentiation to find the value of \( x \) for which \( A \) is a minimum

\[
\frac{dA}{dx} = 4x - \frac{320}{x^2}
\]

\[
4x^3 - 320 = 0
\]

\[
4x^3 = 320
\]

\[
x^3 = 80
\]

\[
x = \sqrt[3]{80}
\]

\[
x = 4.309
\]
4. Shown below is a cuboid.

The surface area of the cuboid is 120cm².

(a) Show that \( y = \frac{20}{x} - \frac{2x}{3} \)

\[
\begin{align*}
2x^2 + 2x^2 + xy + xy + 2xy + 2xy & = 120 \\
4x^2 + 6xy & = 120 \\
6xy & = 120 - 4x^2 \\
y & = \frac{120}{6x} - \frac{4x^2}{6x} \\
y & = \frac{20}{x} - \frac{2x}{3}
\end{align*}
\]

(b) Show that the volume of the cuboid is given by

\[
V = 120x - 4x^3
\]

\[
V = 2x^2 y
\]

\[
V = 2x^2 \left( \frac{20}{x} - \frac{2x}{3} \right)
\]

\[
V = \frac{40x^2}{x} - \frac{4x^3}{3}
\]

\[
V = 40x - \frac{4}{3} x^3
\]

(c) Use differentiation to find the value of \( x \) for which \( V \) is a maximum

\[
\frac{dV}{dx} = 40 - 4x^2
\]

\[
0 = 40 - 4x^2
\]

\[
4x^2 = 40
\]

\[
x^2 = 10
\]

\[
x = \sqrt{10}
\]

\[
\frac{d^2V}{dx^2} = -8x
\]

\[
\frac{d^2V}{dx^2} = -8 \sqrt{10} < 0
\]

\[
3.162
\]

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(d) Use your answer to (c) to find the maximum volume of the cuboid

\[ \sqrt[10]{10} \times 2\sqrt[10]{10} \times \frac{4\sqrt[10]{10}}{3} \]

\[ = \frac{80\sqrt[10]{10}}{3} \]

\[ \approx 84.33 \text{ cm}^2 \]

\[ \approx \quad \text{to } 2\text{dp} \quad (2) \]

5. The volume of a container with a height of \( x \), is given by

\[ V = x(x - 1)(9 - x) \quad \text{where } 1 < x < 9 \]

(a) Find \( \frac{dV}{dx} \)

\[ V = x(9x - x^2 - 9 + x) \]

\[ V = x(10x - x^2 - 9) \]

\[ V = 10x^2 - x^3 - 9x \]

\[ \frac{dV}{dx} = 20x - 3x^2 - 9 \]

\[ 20x - 3x^2 - 9 \]

\[ \quad \text{to } 2\text{dp} \quad (3) \]

(b) Hence find the value of \( x \) for which the volume is a maximum.

Give your answer to 1 decimal place.

\[ 20x - 3x^2 - 9 = 0 \]

\[ a = -3 \]

\[ b = 20 \]

\[ c = -9 \]

\[ x = \frac{10 \pm \sqrt{73}}{3} \]

\[ x = 6.2 \text{ or } x = 0.5 \]

\[ x = 6.2 \]

\[ \quad \text{to } 1\text{dp} \quad (3) \]
6. An open-topped tank in the shape of a cuboid is shown below.

The surface area of the cuboid is 300cm²

(a) Show that \( y = \frac{50}{x} - \frac{x}{3} \)

\[
\begin{align*}
xy + xy + 2x^2 + 2xy + 2xy &= 300 \\
2xy + 2x^2 + 4xy &= 300 \\
2x^2 + 6xy &= 300 \\
6xy &= 300 - 2x^2 \\
y &= \frac{50}{x} - \frac{x}{3}
\end{align*}
\]

(b) Show that the volume of the tank is \( V = 100x - \frac{2}{3}x^3 \)

\[
\begin{align*}
V &= 2x^2 \left( \frac{50}{x} - \frac{x}{3} \right) \\
&= 100x - \frac{2x^3}{3} \\
&= 100x - \frac{2}{3}x^3
\end{align*}
\]

(c) Use differentiation to find the value of x for which V is a maximum

\[
\frac{dV}{dx} = 100 - 2x^2
\]

\[
0 = 100 - 2x^2
\]

\[
2x^2 = 100
\]

\[
x^2 = \frac{100}{2}
\]

\[
x^2 = 50
\]

\[
x = \sqrt{50}
\]

\[
x = \frac{5\sqrt{2}}{2}
\]
(d) Find the maximum volume of the tank

\[2 \sqrt{50} \times \sqrt{50} \times \frac{10 \sqrt{2}}{3}\]

\[471.4 \text{ cm}^3\]

(2)