Name:

Level 2 Further Maths

Using Differentiation to Solve Problems

Corbettmaths

Ensure you have: Pencil or pen

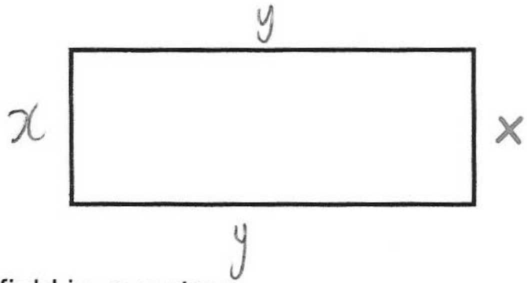
Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic

www.corbettmaths.com/more/further-maths/

1. A farmer creates a rectangular pen for his chickens.



The width of the field is x metres.
The perimeter of the field is 100 metres.

(a) Show that the length of the rectangle is $50 - x$ metres

$x + x + y + y = 100$

$2x + 2y = 100$

$2y = 100 - 2x$

$y = 50 - x$

(1)

(b) Show that the area of the field is $A = 50x - x^2$

$A = x(50 - x)$

$A = 50x - x^2$

(1)

(c) Use differentiation to find the value of x for which A is a maximum

$\frac{dA}{dx} = 50 - 2x$

$50 - 2x = 0$

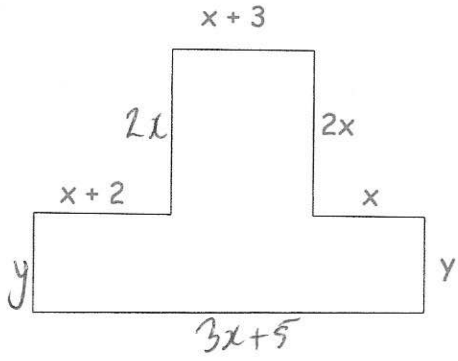
$2x = 50$

$x = 25$

25

(3)

2. The shape below is made from two rectangle.



The perimeter of the shape is 100cm.

(a) Show that $y = 45 - 5x$

$2y + 3x + 5 + 3x + 5 + 4x = 100$

$2y + 10x + 10 = 100$

$2y + 10x = 90$ $\quad y = 45 - 5x$

$2y = 90 - 10x$

(2)

The area of the shape is Acm^2

(b) Show that $A = 225 + 116x - 13x^2$

$A = (45 - 5x)(3x + 5) + 2x(x + 3)$

$A = 135x + 225 - 15x^2 - 25x + 2x^2 + 6x$

$A = 225 + 116x - 13x^2$

(2)

(c) Use differentiation to find the value of x for which A is a maximum

$\frac{dA}{dx} = 116 - 26x$

$116 - 26x = 0$

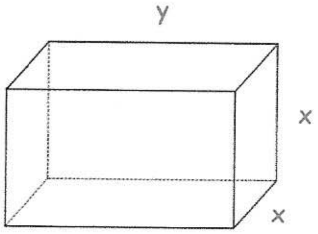
$26x = 116$

$x = \frac{58}{13}$ $(4.461...)$

....

(3)

3. Shown below is a metal box in the shape of a cuboid.



The volume of the box is 80cm³

(a) Show that $y = \frac{80}{x^2}$

$x^2 y = 80$

$y = \frac{80}{x^2}$

(2)

(b) Show that the area of metal to make the box is given by

$$A = 2x^2 + \frac{320}{x}$$

$A = x^2 + x^2 + 4xy$

$A = 2x^2 + 4xy$

$A = 2x^2 + 4x\left(\frac{80}{x^2}\right)$

$A = 2x^2 + \frac{320}{x}$

(2)

(c) Use differentiation to find the value of x for which A is a minimum

$A = 2x^2 + 320x^{-1}$

$\frac{dA}{dx} = 4x - 320x^{-2}$

$= 4x - \frac{320}{x^2}$

$4x - \frac{320}{x^2} = 0$

$4x = \frac{320}{x^2}$

$4x^3 = 320$

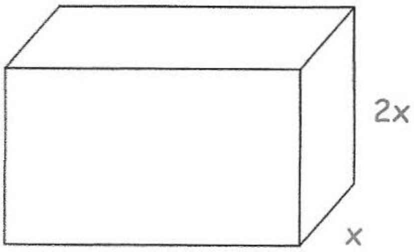
$x^3 = 80$

$x = \sqrt[3]{80}$

4.309

(5)

4. Shown below is a cuboid.



The surface area of the cuboid is 120cm².

(a) Show that $y = \frac{20}{x} - \frac{2x}{3}$

$2x^2 + 2x^2 + xy + xy + 2xy + 2xy = 120$

$4x^2 + 6xy = 120$

$6xy = 120 - 4x^2$

$y = \frac{120}{6x} - \frac{4x^2}{6x}$

$y = \frac{20}{x} - \frac{2x}{3}$

(3)

(b) Show that the volume of the cuboid is given by

$\cancel{V = 120x - 4x^3}$ $V = 40x - \frac{4}{3}x^3$

$V = 2x^2 y$

$V = 2x^2\left(\frac{20}{x} - \frac{2x}{3}\right)$

$V = \frac{40x^2}{x} - \frac{4x^3}{3}$

$V = 40x - \frac{4}{3}x^3$

(2)

(c) Use differentiation to find the value of x for which V is a maximum

$\frac{dV}{dx} = 40 - 4x^2$

$0 = 40 - 4x^2$

$4x^2 = 40$

$x^2 = 10$

$x = \sqrt{10}$

3.162

(4)

(d) Use your answer to (c) to find the maximum volume of the cuboid

$\sqrt{10} \times 2\sqrt{10} \times \frac{4\sqrt{10}}{3}$

$= \frac{80\sqrt{10}}{3}$

84.33 ....cm²
to 2dp **(2)**

---

5. The volume of a container with a height of x, is given by

$$V = x(x-1)(9-x) \quad \text{where } 1 < x < 9$$

(a) Find $\frac{dV}{dx}$

$V = x(9x - x^2 - 9 + x)$

$V = x(10x - x^2 - 9)$

$V = 10x^2 - x^3 - 9x$

$\frac{dV}{dx} = 20x - 3x^2 - 9$

$20x - 3x^2 - 9$ ....

(3)

(b) Hence find the value of x for which the volume is a maximum.
Give your answer to 1 decimal place.

$20x - 3x^2 - 9 = 0$

$a = -3$
$b = 20$
$c = -9$

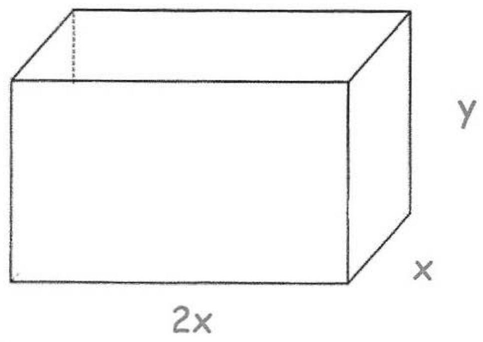
$x = \frac{10 \pm \sqrt{73}}{3}$

$x = 0.5$ or $x = 6.2$
× ✓

6.2 ....

(3)

6. An open-topped tank in the shape of a cuboid is shown below.



The surface area of the cuboid is 300cm²

(a) Show that $y = \dfrac{50}{x} - \dfrac{x}{3}$

$xy + xy + 2x^2 + 2xy + 2xy = 300$

$2xy + 2x^2 + 4xy = 300$

$2x^2 + 6xy = 300$

$6xy = 300 - 2x^2$

$y = \dfrac{50}{x} - \dfrac{x}{3}$

(3)

(b) Show that the volume of the tank is $V = 100x - \dfrac{2}{3}x^3$

$v = 2x^2\left(\dfrac{50}{x} - \dfrac{x}{3}\right)$

$= 100x - \dfrac{2x^3}{3}$

$= 100x - \dfrac{2}{3}x^3$

(3)

(c) Use differentiation to find the value of x for which V is a maximum

$\dfrac{dv}{dx} = 100 - 2x^2$

$0 = 100 - 2x^2$

$2x^2 = 100$

$x^2 = \dfrac{100}{2}$

$x^2 = 50$

$x = \sqrt{50}$

$5\sqrt{2}$ ....

(3)

(d) Find the maximum volume of the tank

$$2\sqrt{50} \times \sqrt{50} \times \frac{10\sqrt{2}}{3}$$

471.4 cm^3

(2)