Name:

Level 2 Further Maths

Circle Theorems

Corbettmaths

Ensure you have: Pencil or pen

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic

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1. Find the values of $x$ and $y$

\[
3x - 45y = 180 \\
x + 45y = 180 \\
\frac{4x}{4} = 360 \\
x = 90^\circ
\]

\[
90 + 45y = 180 \\
y = 2
\]

\[
x = ............^\circ \\
y = ............^\circ
\]

2. AB is a tangent to the circle.

Calculate the size of angle $x$.

\[
\sin x = \frac{12}{15}
\]

\[x = 53.13^\circ \]

\[x = ............^\circ\]
Given $\angle ODF = 2y - 10^\circ$ and $\angle OFD = y + 22^\circ$

Find $\angle DEF$

\[2y - 10 = y + 22\]

\[y = 32^\circ\]

$\angle OFD = 54^\circ$

\[54 + 54 = 108\]

$\angle ODF = 72^\circ$

$\angle DEF = 72 \div 2$

36°
Work out the size of angle $x^\circ$

$2(2x + 60) = 2x + 60$

$(2x + 60) + (2x - 40) = 360^\circ$

$4x + 20 = 360$

$4x = 340$

$x = 85^\circ$

$x = \frac{85}{2}$

(4)
A and B are points on the circumference of a circle, centre O. CA is a tangent to the circle. Angle CAB = 2x

Prove that angle AOB = 4x
Give reasons for each stage of your working.

\[ \angle OAB = 90 - 2x^\circ \quad (\angle OAC = 90^\circ \text{ as radius and tangent meet at } 90^\circ) \]
\[ \angle OBA = 90 - 2x^\circ \quad (\triangle OAB \text{ is isosceles as } OA = OB = r) \]

As the angles in a triangle add to 180°

\[ y + (90 - 2x) + (90 - 2x) = 180^\circ \]
\[ y + 180 - 4x = 180^\circ \]
\[ y - 4x = 0 \]
\[ y = 4x \]
\[ \angle AOB = 4x \quad \therefore \text{OED} \]
The points D, E and F are points on a circle, centre O.

\[ \text{Angle } DEF = 2x \quad \text{Angle } DOF = 5x - 72^\circ \quad \text{Angle } EDO = y \]

Angle EFO is 14° smaller than angle DEF

Work out the value of \( y \)

\[ (2x) \times 2 = 4x \]

\[ 4x + 5x - 72 = 360^\circ \]

\[ 9x = 432^\circ \]

\[ x = 48^\circ \]

\[ \angle EFO = y - 14^\circ \]

\[ 72 - 72 + y + y - 14 = 360 \]

\[ 3y = 360 \]

\[ y = 55^\circ \]

\[ y = \ldots 55^\circ \]
w : x = 2 : 3
x : y = 6 : 5  \quad 9 : 3

Work out the size of angle z.

\[ w + x = 180 \] (opposite angles in a cyclic quadrilateral)

\[ 2 + 3 = 5 \]
\[ 180 \div 5 = 36 \]
\[ 36 \times 2 = 72 \quad \overline{w} \]
\[ 36 \times 3 = 108 \quad \overline{x} \]

\[ 108 \div 9 = \text{rem} 12 \]
\[ 12 \times 8 = 96 \]
\[ 180 - 96 = 84 \]

\[ z = 84 \degree \]

(5)
8.

AC is the diameter of a circle, centre O.
DE is the tangent to the circle.
BCD is a straight line.

Angle BAC = x

Express angle COD in terms of x.

\[
\begin{align*}
\angle ABC &= 90^\circ \quad \text{(angle in semi-circle)} \\
\angle ACB &= 90^\circ - x \quad \text{(angles in } \triangle ABC \text{ add to } 180^\circ) \\
\angle OCD &= 90^\circ + x \quad \text{(BCD is a straight line)} \\
\angle OED &= 90^\circ \quad \text{(radius/tangent meet at } 90^\circ) \\
180 - (90 + x) &= 90 - x \\
180 - 90 - x &= 90 - x \\
90 - x &= 90 - x \\
\frac{90 - x}{2} &= 45 - \frac{1}{2}x
\end{align*}
\]

(5)
9.

Prove that the angle in a semi-circle is always 90°

\[OA = OB = OC = \text{radius}\]
\[\angle OAB = \angle ABO = x^\circ\] \{\text{Angles in an isosceles triangle}\}
\[\angle OCB = \angle OBC = y^\circ\] \{\text{are equal}\}

Angles in a triangle add to 180°
\[\therefore x + (x+y) + y = 180\]
\[2x + 2y = 180\]
\[x + y = 90^\circ\]

\[\text{As } \angle ABC = \angle AOB \times y\]
\[\text{it equals } 90^\circ\]
EF is a tangent to a circle, centre O.

\[ \angle DAF = 2x - 30^\circ \]

\[ \angle ABC = 8x \]

Find the size of angle \( \angle DAF \)

\[ \angle ACD = 2x - 10 \] (Alternate segment theorem)

\[ \angle ADC = \frac{180 - (2x - 10)}{2} = \frac{190 - 2x}{2} = 95 - x \]

\[ 6x + (95 - x) = 180 \]

\[ 5x + 95 = 180 \]

\[ 5x = 85 \]

\[ x = 17 \]

\[ 2 \times 17 - 10 = 24^\circ \]