Name:

Level 2 Further Maths

Completing the Square

Corbettmaths

Ensure you have: Pencil or pen

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic

www.corbettmaths.com/more/further-maths/

1. Work out the values of a and b such that

$$x^2 + 8x + 3 \equiv (x + a)^2 + b$$

$(x+4)^2 - 16 + 3$

$(x+4)^2 - 13$

$a =$ 4 $b =$ -13

(2)

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2. Write $x^2 + 4x + 20$ in the form $(x + a)^2 + b$, where a and b are constants

$(x+2)^2 - 4 + 20$

$(x+2)^2 + 16$

.....

(2)

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3. Write $x^2 - 6x - 10$ in the form $(x + a)^2 + b$, where a and b are constants

$(x-3)^2 - 9 - 10$

$(x-3)^2 - 19$

.....

(2)

4. Write $x^2 + x - 8$ in the form $(x + a)^2 + b$, where a and b are constants

$\left(x + \frac{1}{2}\right)^2 - \frac{1}{4} - 8$

$\left(x + \frac{1}{2}\right)^2 - 8\frac{1}{4}$

$\left(x + \frac{1}{2}\right)^2 - \frac{33}{4}$

.....

(2)

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5. Write $x^2 - 9x - 1$ in the form $(x + a)^2 + b$, where a and b are constants

$\left(x - \frac{9}{2}\right)^2 - \frac{81}{4} - 1$

$\left(x - \frac{9}{2}\right)^2 - \frac{85}{4}$

.....

(2)

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6. Work out the values of *a* and *b* such that

$$x^2 + 11x + 3 \equiv (x + a)^2 + b$$

$\left(x + \frac{11}{2}\right)^2 - \frac{121}{4} + 3$

$\left(x + \frac{11}{2}\right)^2 - \frac{109}{4}$

$a =$ $\frac{11}{2}$ $b =$ $-\frac{109}{4}$

(2)

7. $(x + a)^2 + 11 \equiv x^2 - 10x + b$

Work out the values of a and b

$x^2 + 2ax + a^2 + 11 \equiv x^2 - 10x + b$

$2a = -10$

$a = -5$

$25 + 11 = b$

$36 = b$

$a =$ -5 $b =$ 36

(3)

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8. $x^2 + 4ax + b \equiv (x + 8)^2 - 3a$

Work out the values of a and b

$x^2 + 4ax + b \equiv x^2 + 16x + 64 - 3a$

$4a = 16$ $b = 64 - 12$

$a = 4$ $b = 52$

$a =$ 4 $b =$ 52

(3)

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9. Write $2x^2 + 8x + 2$ in the form $a(x + b)^2 + c$, where a, b and c are constants

$2[x^2 + 4x + 1]$

$2[(x+2)^2 - 4 + 1]$

$2[(x+2)^2 - 3]$

$2(x+2)^2 - 6$

(4)

10. Write $2x^2 + 12x - 3$ in the form $a(x + b)^2 + c$, where a, b and c are constants

$2\left[x^2 + 6x - \frac{3}{2}\right]$

$2\left[(x+3)^2 - 9 - \frac{3}{2}\right]$

$2\left[(x+3)^2 - \frac{21}{2}\right]$

$2(x+3)^2 - 21$

.....

(4)

11. Write $3x^2 - 12x + 2$ in the form $a(x + b)^2 + c$, where a, b and c are constants

$3\left[x^2 - 4x + \frac{2}{3}\right]$

$3\left[(x-2)^2 - 4 + \frac{2}{3}\right]$

$3\left[(x-2)^2 - 3\frac{1}{3}\right]$

$3\left[(x-2)^2 - \frac{10}{3}\right]$

$3(x-2)^2 - 10$

.....

(4)

12. Write $4x^2 + 12x - 5$ in the form $a(x + b)^2 + c$, where a, b and c are constants

$4\left[x^2 + 3x - \frac{5}{4}\right]$

$4\left[\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - \frac{5}{4}\right]$

$4\left[\left(x + \frac{3}{2}\right)^2 - \frac{14}{4}\right]$

$4\left(x + \frac{3}{2}\right)^2 - 14$

.....

(4)

13. Write $2x^2 - 17x + 1$ in the form $a(x + b)^2 + c$, where a, b and c are constants

$2\left[x^2 - \frac{17}{2}x + \frac{1}{2}\right]$

$2\left[\left(x - \frac{17}{4}\right)^2 - \frac{289}{16} + \frac{1}{2}\right]$

$2\left[\left(x - \frac{17}{4}\right)^2 - \frac{289}{16} + \frac{8}{16}\right]$

$2\left[\left(x - \frac{17}{4}\right)^2 - \frac{281}{16}\right]$

$2\left(x - \frac{17}{4}\right)^2 - \frac{281}{8}$

.....

(4)

14. Write $8x^2 - 56x + 5$ in the form $a(bx + c)^2 + d$ where a, b, c and d are integers.

$8\left[x^2 - 7x + \frac{5}{8}\right]$

$8\left[\left(x - \frac{7}{2}\right)^2 - \frac{49}{4} + \frac{5}{8}\right]$

$8\left[\left(x - \frac{7}{2}\right)^2 - \frac{93}{8}\right]$

$8\left(x - \frac{7}{2}\right)^2 - 93$

$2(2x - 7)^2 - 93$

$2(2x-7)^2 - 93$

(5)

15. (a) Work out the values of a and b such that

$$x^2 + 10x + 7 \equiv (x + a)^2 + b$$

$(x+5)^2 - 25 + 7$

$(x+5)^2 - 18$

$a =$ 5 $b =$ -18

(2)

(b) Write down the coordinates of the minimum point on the curve $y = x^2 + 10x + 7$

(-5, -18)

(1)

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16. (a) Work out the values of a and b such that

$$x^2 - 6x - 20 \equiv (x + a)^2 + b$$

$(x-3)^2 - 9 - 20$

$(x-3)^2 - 29$

$a =$ -3 $b =$ -29

(2)

(b) Write down the coordinates of the minimum point on the curve $y = x^2 - 6x - 20$

(3, -29)

(1)

17. By using completing the square, find the coordinates of the minimum point on the curve $y = x^2 + 3x - 5$

$(x + \frac{3}{2})^2 - \frac{9}{4} - 5$

$(x + \frac{3}{2})^2 - \frac{29}{4}$

$(-\frac{3}{2}, -\frac{29}{4})$

.....

(4)

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18. The nth term of a sequence is $n^2 - 4n + 5$

By using completing the square, show that every term is positive.

$(n-2)^2 - 4 + 5$

$(n-2)^2 + 1$

Since $(n-2)^2$ is always greater than, or equal to 0,

$(n-2)^2 + 1$ is always positive

(3)

19. The nth term of a sequence is $n^2 - 10n + 30$

By using completing the square, show that every term is positive.

$(n-5)^2 - 25 + 30$

$(n-5)^2 + 5$

since $(n-5)^2 \geq 0$

then $(n-5)^2 + 5 > 0$

(3)