Level 2 Further Maths

Domains and Ranges
Sketching Functions

Ensure you have: Pencil or pen

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic

www.corbettmaths.com/more/further-maths/
1. A function \( f(x) \) is defined as

\[
\begin{align*}
f(x) &= 5 & 0 \leq x < 2 \\
      &= 6 - \frac{1}{2}x & 2 \leq x < 4 \\
      &= x & 4 \leq x \leq 6
\end{align*}
\]

(a) Draw the graph of \( y = f(x) \) on the axes below.

(b) State the range of \( f(x) \)

\[
4 \leq f(x) \leq 6
\]
2. A function $f(x)$ is defined as

$$f(x) = \begin{cases} 
  x + 3 & -2 \leq x < -1 \\
  2 & -1 \leq x < 1 \\
  2x & 1 \leq x \leq 4 
\end{cases}$$

(a) Draw the graph of $y = f(x)$ on the axes below.

(b) State the range of $f(x)$

$$1 \leq f(x) \leq 8$$

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3. A function \( f(x) \) is defined as

\[
f(x) = 9 - 3x \\
= (5 - x)(x - 1)
\]

\( 0 \leq x < 2 \)
\( 2 \leq x \leq 5 \)

\( x = 2 \quad 3 \times 1 = 3 \)
\( x = 3 \quad 2 \times 2 = 4 \)
\( x = 4 \quad 1 \times 3 = 3 \)

(a) Draw the graph of \( y = f(x) \) on the axes below.

(b) State the range of \( f(x) \)

\( 0 < f(x) < 9 \)
4. A function \( f(x) \) is defined as

\[
 f(x) = \begin{cases} 
 -x & -4 \leq x < 0 \\ 
 x^2 & 0 \leq x < 2 \\ 
 10 - 3x & 2 \leq x \leq 4 
\end{cases}
\]

(a) Draw the graph of \( y = f(x) \) on the axes below.

(b) How many solutions are there to \( f(x) = 0 \)

(c) State the range of \( f(x) \)

\(-2 \leq f(x) \leq 4\)
Here is a graph of $y = f(x)$
It consists of three straight lines.

Define $f(x)$, stating clearly the domain for each part.

$$f(x) = \begin{cases} 2x + 4 & -4 \leq x \leq 0 \\ 4 & 0 \leq x \leq 2 \\ 6 - x & 2 < x \leq 4 \end{cases}$$
Here is a graph of $y = f(x)$
It consists of a quadratic curve and two straight lines.

Define $f(x)$, stating clearly the domain for each part.

$$f(x) = \begin{cases} 
-x^2 + 3 & -2 \leq x < 2 \\
-4 & -2 < x < -1 \\
-2x + 5 & -1 \leq x \leq 2 \\
-1 & 2 \leq x \leq 4 
\end{cases}$$
7. A function \( f(x) \) is defined as

\[
f(x) = 10 + x - x^2 \quad -3 \leq x < -1
\]

\[
e = x^2 + 7 \quad -1 \leq x < 1
\]

\[
= \frac{8}{x} \quad 1 \leq x \leq 4
\]

(a) Draw the graph of \( y = f(x) \) on the axes below.

(b) State the range of \( f(x) \)

\( -2 \leq f(x) \leq 8 \)
8. \( f(x) = x^2 - 2 \) for all values of \( x \)

(a) Find the value of \( f(-4) \)

\[
f(-4) = (-4)^2 - 2 = 16 - 2 = 14
\]

(b) What is the range of \( f(x) \)?

\[ f(x) \geq -2 \]

9. \( f(x) = 8 - 2x^2 \) for all values of \( x \)

Write down the range of \( f(x) \)

\[ f(x) \leq 8 \]

10. \( f(x) = (x + 1)(x + 3) \) for all values of \( x \)

Write down the range of \( f(x) \)

Minimum when \( x = -2 \)

\[
f(-2) = (-1) \times (1) = -1
\]

\[ f(x) \geq -1 \]
11. The graph of \( y = f(x) \) is shown below.

(a) Write down the domain of \( f(x) \)

\[-4 \leq x \leq 3\]  

(1)

(b) Write down the range of \( f(x) \)

\[-3 \leq f(x) \leq 3\]  

(1)

(c) Solve \( f(x) = 1 \)

\[x = 2\]  

(1)
12. Shown below is the graph of \( y = f(x) \)

(a) Write down the domain of \( f(x) \)

\[ -4 \leq x \leq 9 \]

(b) Write down the range of \( f(x) \)

\[ -2 \leq f(x) \leq 14 \]
13. \( f(x) = 5 - 2x \) for \(-4 \leq x \leq 3\)

Work out the range of \( f(x) \)

when \( x = -4 \) \quad f(x) = 13

\( x = 3 \) \quad f(x) = -1

\[-1 \leq f(x) \leq 13\]

(2)

14. \( g(x) = x^3 - 5 \) for \(-2 \leq x \leq 3\)

Work out the range of \( g(x) \)

\( x = -2 \) \quad g(x) = -13

\( x = 3 \) \quad g(x) = 22

\[-13 \leq g(x) \leq 22\]

(2)

15. \( f(x) = 9x - 2 \)

The range of \( f(x) \) is \(-38 \leq f(x) \leq 61\)

Work out the domain of \( f(x) \)

\( 9x - 2 = -38 \)

\( 9x = -36 \)

\( x = -4 \)

\( 9x - 2 = 61 \)

\( 9x = 63 \)

\( x = 7 \)

\(-4 \leq x \leq 7\)

(2)
16. \( g(x) = 3 - 2x \)

The range of \( g(x) \) is \(-7 \leq g(x) \leq 6\)

Work out the domain of \( g(x) \)

\[
\begin{align*}
3 - 2x &= -7 & 3 - 2x &= 6 \\
-2x &= -10 & -2x &= 3 \\
\Rightarrow x &= 5 & \Rightarrow x &= -1.5
\end{align*}
\]

\(-1.5 \leq x \leq 5\) \hspace{1cm} (2)

17. The function \( f(x) \) is defined as

\[ f(x) = 14 - 3x \quad \text{for} \quad p \leq x < 8 \]

The range of \( f(x) \) is \(-10 \leq f(x) \leq 30.5\)

Work out the value of \( p \)

\[
\begin{align*}
14 - 3x & \quad \text{when} \quad x = 8 \\
14 - 24 &= -10 \\
-3x &= 16.5 \\
\Rightarrow x &= -5.5
\end{align*}
\]

\( p = -5.5 \) \hspace{1cm} (2)
18. \( f(x) = \cos x \) for \( 90^\circ \leq x \leq 120^\circ \)

Work out the range of \( f(x) \)

\[
\begin{align*}
f(90^\circ) &= 0 \\
f(120^\circ) &= -0.5
\end{align*}
\]

\(-0.5 \leq f(x) \leq 0\) (2)