Level 2 Further Maths

Equation of a Circle

Ensure you have: Pencil or pen

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic

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1. The equation of a circle \( C \), with centre \( O \), is:

\[ x^2 + y^2 = 289 \]

(a) Find the coordinates of the centre \( O \).

\[ (\ldots, \ldots) \]  

(b) Find the radius of \( C \).

\[ \sqrt{289} = 17 \]

(c) Show the point \((8, 15)\) lies on \( C \).

\[ 8^2 + 15^2 = 289 \]
2. The circle below has centre (0, 0).
The point (−24, 7) is a point on the circle.

Find the equation of the circle.

\[
24^2 + 7^2 = 625
\]

\[
\sqrt{625}
\]

\[
2^2 + y^2 = 25^2
\]

\[
25
\]

3. The circle shown has \(x^2 + y^2 = 42.25\)

Find the circumference of the circle.
Give your answer in terms of \(\pi\)

\[
\sqrt{42.25} = 6.5
\]

\[
6.5 \times 2 = 13
\]

\[
13\pi
\]
4. A circle has the equation \( x^2 + y^2 = 400 \)

Find the area of the circle.
Give your answer in terms of \( \pi \)

\[
\begin{align*}
  r &= 20 \\
  A &= \pi r^2 \\
  &= \pi \times 20^2 \\
  &= 400\pi \\
\end{align*}
\]

5. A circle has equation \( x^2 + y^2 = 25 \)

A straight line meets the circle at the points A and B.

(a) Write down the equation of the straight line.

\[
y = 2
\]

(b) Find the distance AB

\[
\begin{align*}
  x^2 + z^2 &= 25 \\
  x^2 &= 21 \\
  x &= \pm \sqrt{21}
\end{align*}
\]

\[
2\sqrt{21}
\]
6. The circle below has equation \( x^2 + y^2 = 4 \)
The line has equation \( y = x - 4 \)

(a) Find the area of the shaded region.

\[
\text{Area of triangle } OAB = \frac{1}{2} \times 4 \times 4 = 8
\]
\[
\text{Area of sector } OCD = \frac{1}{4} \left( \pi \times 2^2 \right) = \frac{\pi}{2}
\]

\[
8 - \frac{\pi}{2}
\]

(b) Find the perimeter of the shaded region.

\[
AD = 2 \quad \text{Arc } DC = \frac{1}{4} \left( \pi \times 4 \right) = \pi
\]
\[
BC = 2 \quad \text{Line } AB = \sqrt{34} \approx 4.52
\]

\[
4 + \pi + 4.52 \approx 12.798
\]
7. The equation of a circle \( C \), with centre \( A \), is:

\[(x - 3)^2 + (y + 2)^2 = 25\]

(a) Find the coordinates of the centre \( A \).

\[ (3, -2) \]  

(1)

(b) Find the radius of \( C \).

\[ 5 \]

(1)

(c) Show the point \((6, 2)\) lies on \( C \).

\[
\begin{align*}
(6 - 3)^2 + (2 + 2)^2 &= 9 + 16 = 25 \\
\therefore \text{it is on } C
\end{align*}
\]

(2)

8. A circle has centre \((5, 2)\) and radius 4.

(a) Write down the equation of the circle.

\[
(x - 5)^2 + (y - 2)^2 = 4^2
\]

or \[
(x - 5)^2 + (y - 2)^2 = 16
\]

(2)

(b) Does the point \((7, 4)\) lie on the circle?

\[
\begin{align*}
(7 - 5)^2 + (4 - 2)^2 &= 4 + 4 = 8 \\
\therefore \text{it is not on } C
\end{align*}
\]

(2)
9. Find the equation of the circle.

\[
(x-3)^2 + (y-4)^2 = 3^2
\]

(2)

10. Find the equation of the circle.

\[
(x+2)^2 + (y-1)^2 = 4^2
\]

(2)
11. A circle C has centre P

The points A (0, 6) and B (8, 6) lie on the diameter of C.

(a) Find the coordinates of the centre P.

(b) Write down the equation of the circle.

\[
(x - 4)^2 + (y - 6)^2 = 4^2
\]
12. AB is a diameter of a circle C. 
   Q is the centre of the circle 
   A has coordinates (-2, 12) and B has coordinates (8, 2). 

   (a) Find the centre of the circle, Q.

   \( (\ldots, \ldots, 7) \)

   \((1)\)

   (b) Find the equation of C

   \((3, 7)\)

   \[ (x - 3)^2 + (y - 7)^2 \]

   \((2)\)

   (c) Show the point D, (10, 8) lies on C.

   \[ (10 - 3)^2 + (8 - 7)^2 \]

   \[ 7^2 + 1^2 = 50 \]

   \[ \therefore 0 \text{ is on the circle} \]

   \((2)\)

   (d) Find the gradient of OD.

   \((10, 8)\)

   \[(3, 7)\]

   \((2)\)

   (e) Find the equation of the tangent to C at the point D.

   \[ y = -7x + C \]

   \[ 8 = -70 + C \]

   \[ C = 78 \]

   \[ y = -7x + 78 \]

   \((3)\)

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13. A circle has equation \((x - 5)^2 + (y + 2)^2 = 20\)

(a) Find the centre of the circle.

\((5, -2)\)  

(b) State, with a reason, whether this circle intersects the y-axis.

\(\text{No, centre is } (5, -2) \text{ but radius is } \sqrt{20}\)  

(c) Find the equation of a line parallel to \(y = 2x\) that passes through the centre of the circle.

\[y = 2x + c\]
\[-2 = 10 + c\]
\[c = -12\]

\(y = 2x + 12\)  

(d) Show \(y = 2x - 2\) is a tangent to the circle

\[(x - 5)^2 + (2x - 2 + 2)^2 = 20\]
\[(x - 5)^2 + (2x)^2 = 20\]
\[x^2 - 10x + 25 + 4x^2 = 20\]
\[5x^2 - 10x + 5 = 0\]
\[x^2 - 2x + 1 = 0\]

\[(x - 1)(x + 1) = 0\]
\[x = 1\]
\[y = 0\]

\((1, 0)\)  

Since only 1 point of intersection, it must be a tangent.

(e) Find the coordinates of the point of contact.
14. A circle has centre C and equation \( x^2 + y^2 - 6x + 14y + 49 = 0 \)

(a) Find the centre of the circle.

\[
\begin{align*}
2^2 - 6x + y^2 + 14y + 49 &= 0 \\
(x - 3)^2 - 9 + (y + 7)^2 - 49 + 49 &= 0 \\
(x - 3)^2 + (y + 7)^2 &= 9
\end{align*}
\]

Centre: \((-3, -7)\)

(b) Find the radius of the circle

Radius: \(3\)
15. Circle 1 has an equation of \((x - 4)^2 + (y - 1)^2 = 36\)

Circle 2 has an equation of \((x + 7)^2 + (y - 8)^2 = 100\)

Calculate the distance between the centres of Circle 1 and Circle 2

\[ \sqrt{7^2 + 11^2} = \sqrt{170} \]
16. Shown below is the circle, centre A, with equation \((x + 9)^2 + y^2 = 225\)

Find the equation of the line passing through A and B.

\[
x = 0
\]
\[
x^2 + y^2 = 225
\]
\[
y^2 = 144
\]
\[
y = \pm 12
\]

\[
y = \frac{12}{9} x + c
\]
\[
y = \frac{4}{3} x + c
\]
\[
0 = \frac{4}{3} (-9) + c
\]
\[
0 = -12 + c
\]
\[
c = 12
\]

(\text{what that was obvious!})

\[
y = \frac{4}{3} x + 12
\]

(4)
17. The line \( y = 2x - 1 \) intersects the circle \((x - 2)^2 + (y + 5)^2 = 144\) at the points A and B.

Find the coordinates of A and B.

\[
(x - 2)^2 + (2x - 1 + 5)^2 = 144
\]
\[
x^2 - 4x + 4 + (2x + 4)^2 = 144
\]
\[
x^2 - 4x + 4 + 4x^2 + 16x + 16 = 144
\]
\[
x^2 + 12x - 124 = 0
\]
\[
a = 5 \quad b = 12 \quad c = -124
\]
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm 4\sqrt{541}}{5}
\]
\[
y = -17 + 8\sqrt{541} \quad \text{or} \quad y = -17 - 8\sqrt{541}
\]

\[
\left(\frac{-b + 4\sqrt{541}}{5}, \frac{-17 + 8\sqrt{541}}{5}\right) \quad \left(\frac{-b - 4\sqrt{541}}{5}, \frac{-17 - 8\sqrt{541}}{5}\right)
\]

.................

(6)
18. A circle has equation \((x + 7)^2 + (y - 6)^2 = 49\)

Is the point \((-4, 12)\) inside or outside the circle?
You must show your workings.

\[ r = 7 \]

Centre \((-7, 6)\)

Point \((-4, 12)\)

\[ \sqrt{45} < 7 \]

\[ \sqrt{x^2 + b^2} = \sqrt{45} \]

As the distance between the centre \((-7, 6)\) and the point \((-4, 12)\) is less than the radius of 7 then the point is inside the circle.

(6)