Name:

Level 2 Further Maths

Simultaneous Equations (non-linear)

Corbettmaths

Ensure you have: Pencil or pen

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic

www.corbettmaths.com/more/further-maths/

1. Solve the simultaneous equations

$$y = x^2 + x - 14$$

$$y = x - 5$$

$x^2 + x - 14 = x - 5$

$x^2 - 9 = 0$

$x = \pm 3$

$x = 3 \quad y = -2$

$x = -3 \quad y = -8$

$x = 3 \quad \text{or} \quad x = -3$
$y = -2 \qquad\qquad y = -8$

$(3, -2)$ e $(-3, -8)$

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2. Solve the simultaneous equations

$$y = x + 3$$

$$x^2 + y^2 = 149$$

$x^2 + (x+3)^2 = 149$

$x^2 + x^2 + 6x + 9 = 149$

$2x^2 + 6x - 140 = 0$

$x^2 + 3x - 70 = 0$

$(x + 10)(x - 7) = 0$

$x = -10 \quad \text{or} \quad x = 7$

$y = -7 \quad \text{or} \quad y = 10$

$x = -10 \quad \text{or} \quad x = 7$
$y = -7 \qquad\qquad y = 10$

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3. Solve the simultaneous equations

$x - 4y = 2$

$x^2 - 8y^2 = 68$

$x = 2 + 4y$

$(2 + 4y)(2 + 4y) - 8y^2 = 68$

$4 + 8y + 8y + 16y^2 - 8y^2 = 68$

$4 + 16y + 16y^2 - 8y^2 = 68$

$4 + 16y + 8y^2 = 68$

$8y^2 + 16y - 64 = 0$

$y^2 + 2y - 8 = 0$

$(y + 4)(y - 2) = 0$

$y = -4$ or $y = 2$

$x = -14$ $x = 10$

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4. Solve the simultaneous equations

$y = 2x^2 + x + 1$

$y = x^2 - 5x - 7$

$2x^2 + x + 1 = x^2 - 5x - 7$

$x^2 + 6x + 8 = 0$

$(x + 2)(x + 4) = 0$

$x = -2$ or $x = -4$

$y = 7$ $y = 29$

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5. Solve the simultaneous equations

$$x^2 + 3x - xy = 10$$

$$2x - y = 4$$

$$y = 2x - 4$$

$$x^2 + 3x - x(2x - 4) = 10$$

$$x^2 + 3x - 2x^2 + 4x = 10$$

$$-x^2 + 7x = 10$$

$$0 = x^2 - 7x + 10$$

$$(x - 2)(x - 5) = 0$$

$$x = 2 \qquad or \qquad x = 5$$
$$y = 0 \qquad\qquad y = 6$$

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6. Solve the simultaneous equations

$$y = 9x^2 + 11x + 3$$

$$5x - y + 2 = 0$$

$$5x - (9x^2 + 11x + 3) + 2 = 0$$

$$5x - 9x^2 - 11x - 3 + 2 = 0$$

$$-9x^2 - 6x - 1 = 0$$

$$0 = 9x^2 + 6x + 1$$

$$0 = (3x + 1)(3x + 1)$$

$$x = -\frac{1}{3}$$

$$y = \frac{1}{3}$$

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7. Solve the simultaneous equations

$2x + y = 7$

$x^2 - y^2 = 8$

$y = 7 - 2x$

$x^2 - (7 - 2x)^2 = 8$

$x^2 - (49 - 28x + 4x^2) = 8$

$-3x^2 + 28x - 49 = 8$

$3x^2 - 28x + 57 = 0$

$(3x - 19)(x - 3) = 0$

$x = \frac{19}{3}$ or $x = 3$

$x = 3$ or $x = \frac{19}{3}$
$y = 1$ $y = -\frac{17}{3}$

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8. Solve the simultaneous equations

$y = x^2 - 9x - 3$

$y = x$

$x = x^2 - 9x - 3$

$0 = x^2 - 10x - 3$

$a = 1$ $b = -10$ $c = -3$

$x = 5 + 2\sqrt{7}$ or $x = 5 - 2\sqrt{7}$
$y = 5 + 2\sqrt{7}$ $y = 5 - 2\sqrt{7}$

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9. Solve the simultaneous equations

$$y = x^2 + x - 7$$

$$4x + 2y + 1 = 0$$

$4x + 2(x^2 + x - 7) + 1 = 0$

$4x + 2x^2 + 2x - 14 + 1 = 0$

$2x^2 + 6x - 13 = 0$

$x = -\dfrac{3 + \sqrt{35}}{2}$ $x = -\dfrac{3 - \sqrt{35}}{2}$

$y = \dfrac{5}{2} - \sqrt{35}$ $y = \dfrac{5}{2} + \sqrt{35}$

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10. Solve the simultaneous equations

$$y = x - 2$$

$$2x^2 - xy = 11$$

$2x^2 - x(x-2) = 11$

$2x^2 - x^2 + 2x = 11$

$x^2 + 2x - 11 = 0$

$a = 1 \quad b = 2 \quad c = -11$

$x = -1 + 2\sqrt{3}$ $x = -1 - 2\sqrt{3}$

$y = -3 + 2\sqrt{3}$ $y = -3 - 2\sqrt{3}$

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11. Find the coordinates where the line $y = x + 8$ and the curve
$y = x^2 + 19x + 80$ intersect

$x + 8 = x^2 + 19x + 80$

$0 = x^2 + 18x + 72$

$(x + 6)(x + 12) = 0$

$x = -6$ or $x = -12$

$y = 2$ $y = -4$

$(-6, 2)$ and $(-12, -4)$

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12. Find the coordinates where the line $x + y = 3$ and the curve
$x^2 + 3y = 27$ intersect

$x = 3 - y$

$(3 - y)(3 - y) + 3y = 27$

$9 - 6y + y^2 + 3y = 27$

$y^2 - 3y - 18 = 0$

$(y + 3)(y - 6) = 0$

$y = -3$ $y = 6$

$x = 6$ $x = -3$

$(6, -3)$ & $(-3, 6)$

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13. How many points of intersection does the circle $x^2 + y^2 = 8$ have with the line $x + y = 4$?

$$x = 4 - y$$

$$(4 - y)(4 - y) + y^2 = 8$$

$$16 - 4y - 4y + y^2 + y^2 = 8$$

$$2y^2 - 8y + 8 = 0$$

$$y^2 - 4y + 4 = 0$$

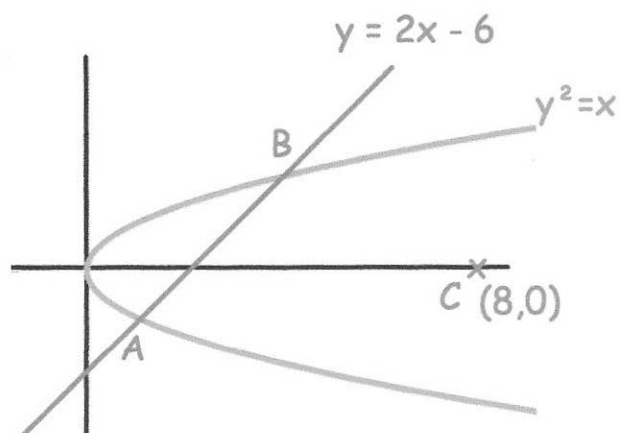$$(y - 2)(y - 2) = 0$$

$$y = 2$$

$$x = 2$$

$$(2, 2)$$

1

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14. Shown is the curve $y^2 = x$ and the line $y = 2x - 6$

y = 2x - 6

$y^2 = x$

B

C (8,0)

A

The curve and the line meet at the points A and B.
The point C is (8, 0)

Show ABC is a right angled triangle.

$$y = 2y^2 - 6$$

$$2y^2 - y - 6 = 0$$

$$(2y + 3)(y + 2)$$

$$y = -\frac{3}{2} \quad \text{or} \quad y = -2$$

$$x = 2.25 \qquad y = 4$$

$$A(\overset{x_1}{2.25}, \overset{y_1}{-1.5}) \quad B(\overset{x_2}{4}, \overset{y_2}{2}) \qquad C(\overset{x_3}{8}, \overset{y_3}{0})$$

$$\text{gradient of } AB = \frac{2 - (-1.5)}{4 - 2.25} = 2$$

$$\text{gradient of } BC = \frac{0 - 2}{8 - 4} = \frac{-2}{4} = -\frac{1}{2}$$

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$$\therefore AB \And BC \text{ are perpendicular } (90°)$$
$$\therefore ABC \text{ is a right angled triangle}$$