Name:

Level 2 Further Maths

Geometric Proof

Corbettmaths

Ensure you have: Pencil or pen

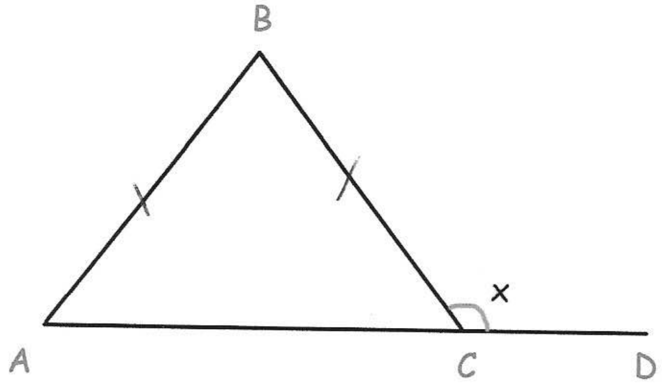
Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic

www.corbettmaths.com/more/further-maths/

1. ABC is an isosceles triangle.
AB = BC
ACD is a straight line.



Angle BCD $= x^{\circ}$

Prove angle ABC $= (2x - 180)^{\circ}$

$\angle BCA = (180 - x)^{\circ}$ as the angles in a straight line add to 180°.

$\angle BAC = (180 - x)^{\circ}$ as two angles in an isosceles triangle are equal.

$\angle ABC = 180 - (180 - x) - (180 - x)$ as the angles in a triangle add to 180°.

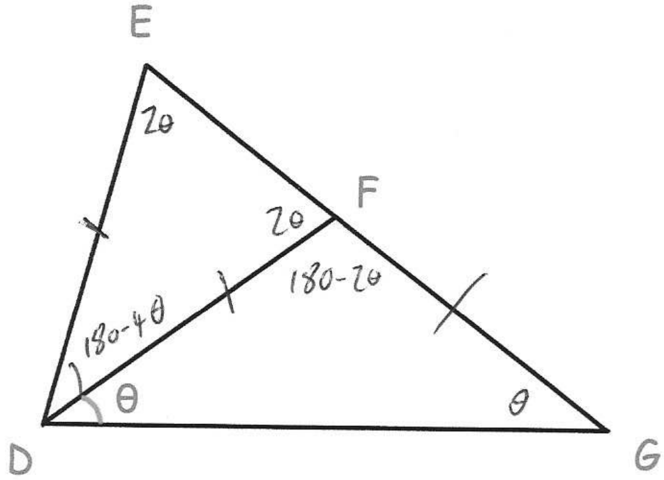
$= -180 + 2x$

$= (2x - 180)^{\circ}$

(3)

QED

2. Shown below is triangle DEG



DE = DF = FG

$\angle FDG = \theta$

Prove that $\angle EDF = 180 - 4\theta$

$\angle DGF = \theta$ two angles in an isosceles triangle are equal.

$\angle DFG = 180 - 2\theta$ angles in a triangle add to 180°.

$\angle EFG = 2\theta$ angles in a straight line add to 180°

$\angle DEF = 2\theta$ two angles in an isosceles triangle are equal.

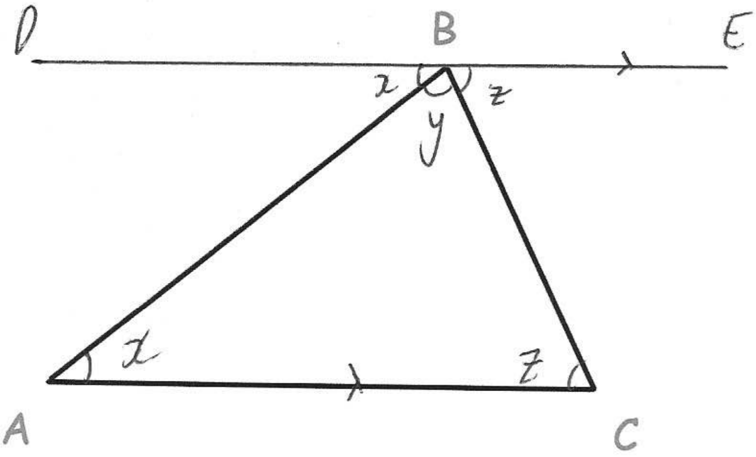
$\angle EDF = 180 - 2\theta - 2\theta$ as the angles in a triangle add to 180°

(3)

$\angle EDF = 180 - 4\theta$

QED

3. ABC is a triangle.



Prove the angles in triangle ABC add up to 180°

straight line DBE is parallel to AC

let $\angle DBA = x°$ $\angle ABC = y°$ $\angle CBE = z°$

$\angle BAC = x°$ as $\angle DBA$ & $\angle BAC$ are alternate angles

$\angle BCA = z°$ as $\angle EBC$ & $\angle BCA$ are alternate angles

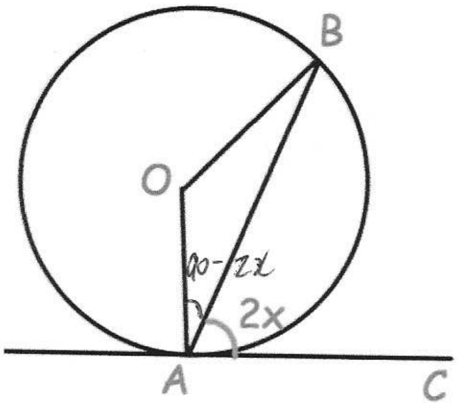
Since DBE is a straight line $x + y + z = 180°$

Since $\angle ABC = y$ & $\angle BCA = z$ & $\angle BAC = x$ **(3)**

the angles in a triangle always add to 180°

QED

4. A and B are points on the circumference of a circle, centre O.



AC is a tangent to the circle.
Angle BAC = $2x$

Prove that angle AOB $= 4x$

Give reasons for each stage of your working.

$\angle OAC = 90°$
$\angle OAB = 90° - 2x$ } radius / tangent meet at 90°

$\angle OAB = \angle OBA = 90 - 2x$ since OA = OB, OAB is an isosceles triangle ∴ two angles are equal

$\angle AOB = 180 - (90-2x) - (90-2x)$ since angles in a triangle add to 180°

$= 4x$

QED

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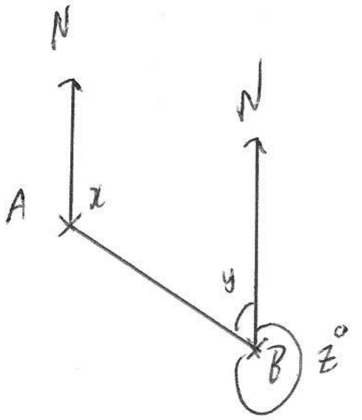
5. The bearing of B from A is x, where x is less than 180°

Prove the bearing of A from B is $(180 + x)^\circ$



Since x & y are co-interior angles,

$y = 180 - y$

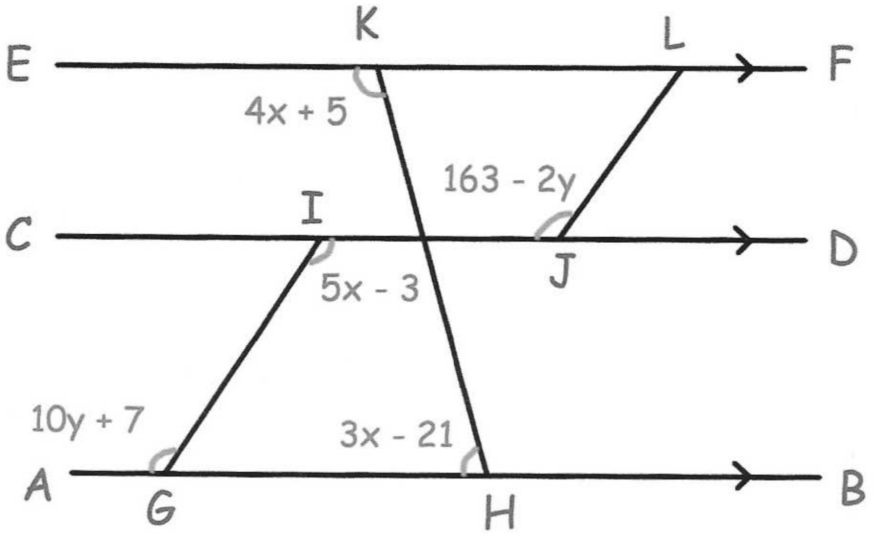
The bearing of A from B is z°

since the angles at a point add up to 360°

$z = 360 - (180 - x)^\circ$ **(3)**

$z = (180 + x)^\circ$

6. The lines AB, CD and EF are parallel.
GI, HK and JL are straight lines.



Show GI and JL are parallel.

$\angle EKH$ & $\angle KHA$ are co-interior, so add to 180°

$(4x+5) + (3x-21) = 180$

$7x - 16 = 180$

$7x = 196$

$x = 28$

$\angle JIG = \angle AGI$ as alternate angle.

$10y + 7 = 5 \times 28 - 3$

$10y + 7 = 137$

$y = 13^\circ$

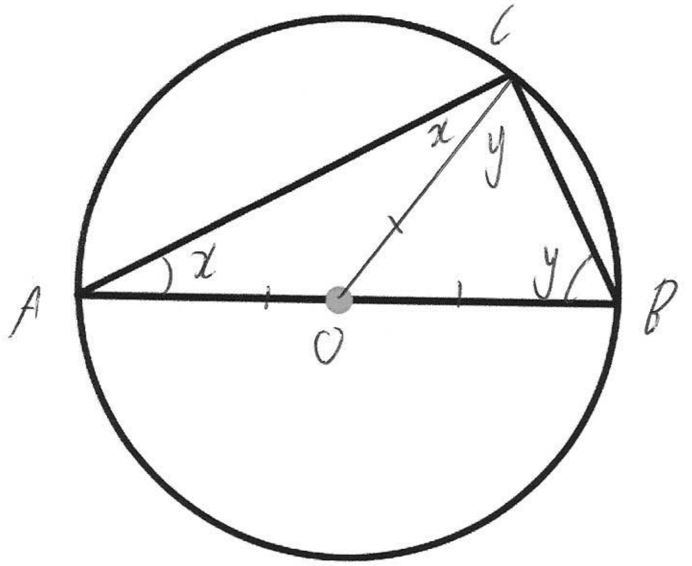
(5)

$\therefore \angle AGI = 137^\circ$

& $\angle IJL = 163 - 2 \times 13 = 137^\circ$

$\therefore$ JL & GI are parallel.

7.



Prove that the angle in a semi-circle is always 90°

Let

$\angle BAC = x^\circ$

$\angle ABC = y^\circ$

$OA = OC = OB$ as all 3 are radii

$\therefore \Delta OAC$ & ΔOBC are isosceles.

$\left.\begin{array}{l}\angle OCA = x \\ \angle OCB = y\end{array}\right\}$ as two angles in an isosceles triangle are equal.

As the angles in a triangle add to 180°

$x + y + (x+y) = 180^\circ$

$2x + 2y = 180$

$x + y = 90$ $\quad \therefore \angle ACB$ is always 90°

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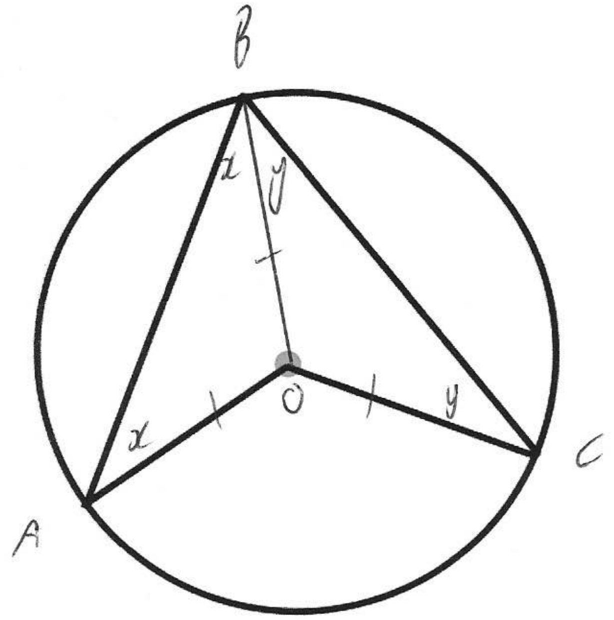
8.



Prove that the angle at the centre is twice the angle at the circumference.

OA = OB = OC (radii)

let ∠BAO = x & ∠BCO = y

Since isosceles triangles

∠ABO = x & ∠CBO = y

∴ ∠BOA = 180 − 2x & ∠BOC = 180 − 2y

as the angles in a triangle add to 180°.

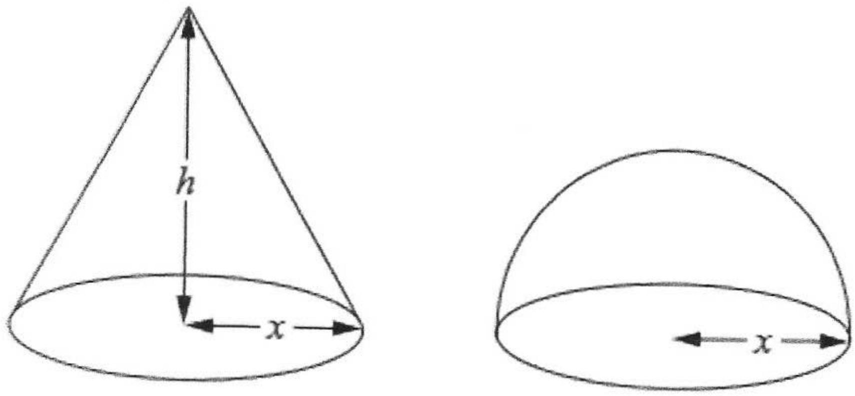
∴ ∠AOC = 2x + 2y as the angles at a point add to 360°.

(4)

As ∠AOC is 2x + 2y = 2(x + y)

∠AOC is twice angle ∠ABC.

9. The diagram shows a cone and a hemisphere.



The hemisphere has base radius x cm.
The cone has base radius x cm and perpendicular height h cm.

The volume of the cone is equal to the volume of the hemisphere.

Show that $h = 2x$

Cone $\frac{1}{3}\pi x^2 h$

hemisphere $\frac{2}{3}\pi x^3$

$\frac{1}{3}\pi x^2 h = \frac{2}{3}\pi x^3$ $(\times 3)$

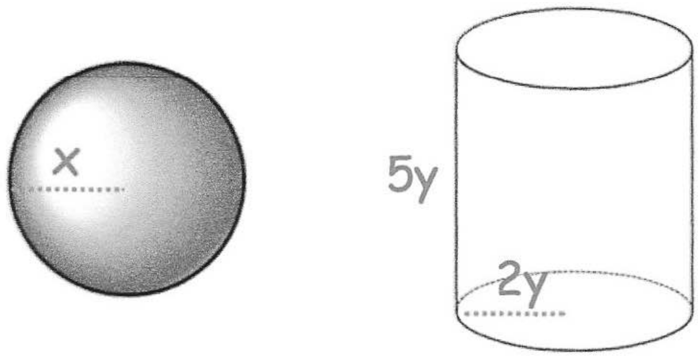
$\pi x^2 h = 2\pi x^3$ $(\div \pi)$

$x^2 h = 2x^3$ $(\div x^2)$

$h = 2x$

(4)

10. A sphere has radius x cm.
A cylinder has radius 2y cm and height 5y cm.



The surface area of both shapes are equal.

Show $x : y = \sqrt{7} : 1$

Sphere $4\pi x^2$

Cylinder $20\pi y^2 + \cancel{4y}\, 4\pi y^2 + 4\pi y^2$

$= 28\pi y^2$

$4\pi x^2 = 28\pi y^2$

$x^2 = 7y^2$

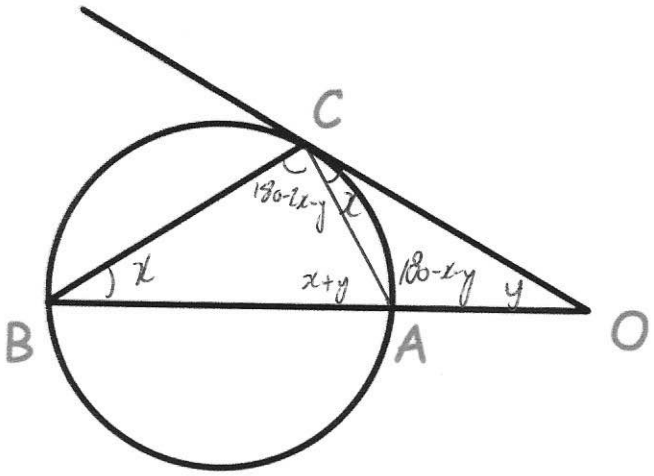
$x = \sqrt{7}\, y$

$\therefore$

$x : y = \sqrt{7} : 1$

(5)

11. OAB is a straight line and OC is a tangent to the circle.



Prove OBC and OAC are similar.

$\angle BOC$ is shared $\therefore = y°$

$\angle OCA = \angle ABC = x°$ (alternate segment theorem)

$\angle OAC = 180 - x - y$ (angles in a triangle add to 180°)

$\angle BAC = x+y$ (angles in a straight line add to 180°)

$\angle ACB = 180 - 2x - y$ (angles in a triangle add to 180°)

$\angle OCB = 180 - 2x - y + x$ (adding $\angle OCA$ & $\angle ACB$) **(4)**

$= 180 - x - y$

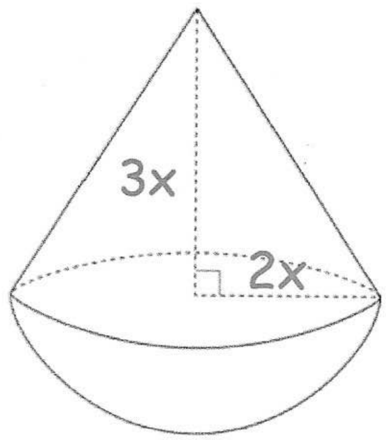
$\therefore$ due to <u>AAA</u> the triangles are similar.

$\angle OBC = \angle OCA = x$

$\angle BOC = y$

$\angle OAC = \angle BCO = 180 - x - y$

12. The diagram shows a solid made up of a cone and a hemisphere.



The radius of the cone is x
The height of the cone is 2x.

Show the volume of the solid is $\frac{28}{3}\pi x^3$

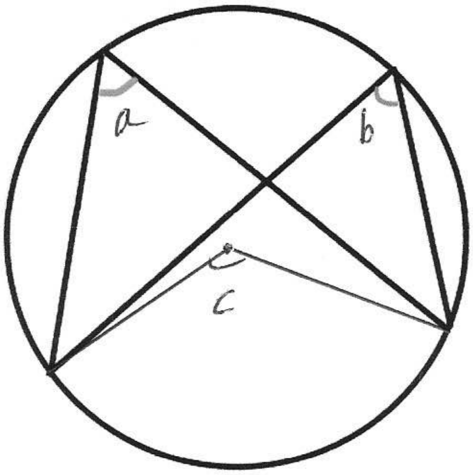
Cone $\frac{1}{3} \times \pi \times (2x)^2 \times 3x = 4\pi x^3$

hemisphere $\frac{2}{3} \times \pi \times (2x)^3 = \frac{16}{3}\pi x^3$

$4\pi x^3 + \frac{16}{3}\pi x^3 = \frac{28}{3}\pi x^3$

(4)

13.



Prove the angles in the same segment are equal.

As the angle at the centre is twice the angle at the circumference

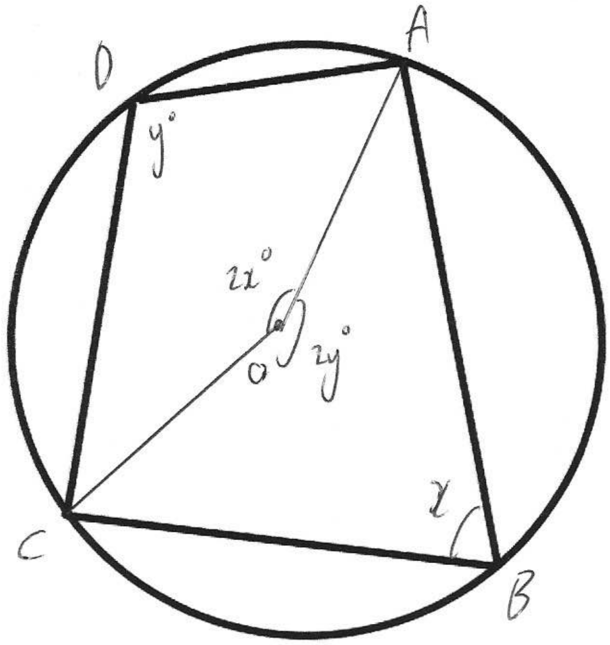
$c = 2a$ & $c = 2b$

$\therefore$ $2a = 2b$

$a = b$

(4)

14.



Prove the opposite angles in a cyclic quadrilateral add to 180°

Let $\angle ABC = x^\circ$ & $\angle ADC = y^\circ$

As the angle at the centre is twice the angle at the circumference

$\angle AOC^{\text{(minor)}} = 2x$ & $\angle AOC^{\text{(major)}} = 2y$

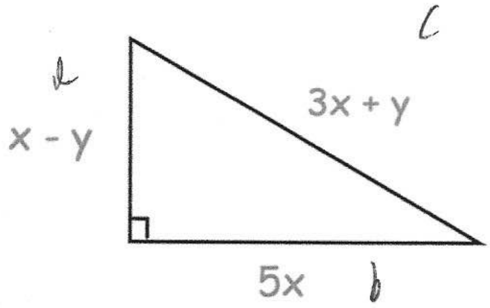
As the angles at a point add up to 360°

$2x + 2y = 360$

$x + y = 180^\circ$

(4)

15.



Prove x : y = 8 : 17

$$a^2 + b^2 = c^2$$

$$(x-y)^2 + (5x)^2 = (3x+y)^2$$

$$x^2 - 2xy + \cancel{y^2} + 25x^2 = 9x^2 + 6xy + \cancel{y^2}$$

$$26x^2 - 2xy = 9x^2 + 6xy$$

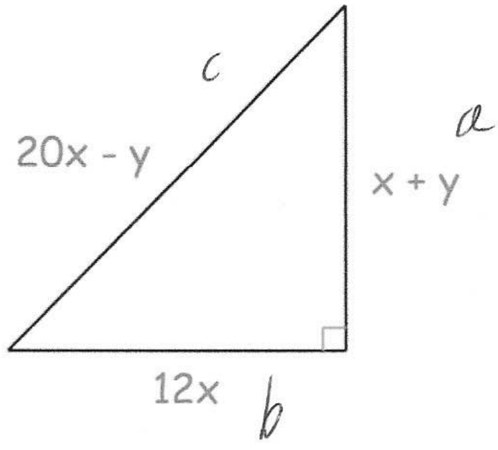
$$17x^2 = 8xy$$

$$17x = 8y$$

$$x : y = 8 : 17$$

(4)

16. Below is a right angled triangle.



Prove $x : y = 14 : 85$

$$(x+y)^2 + (12x)^2 = (20x-y)^2$$

$$x^2 + 2xy + \cancel{y^2} + 144x^2 = 400x^2 - 40xy + \cancel{y^2}$$

$$145x^2 + 2xy = 400x^2 - 40xy$$

$$42xy = 255x^2$$

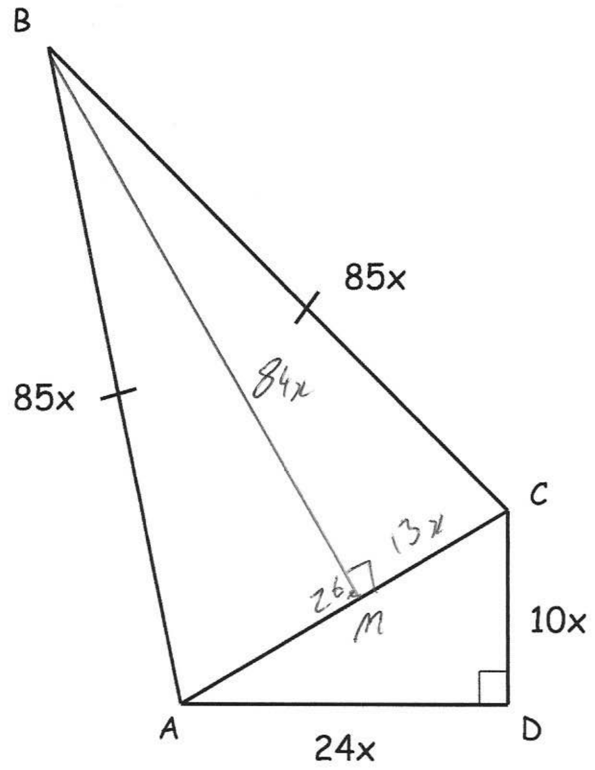
$$14xy = 85x^2$$

$$14y = 85x$$

(4)

$$x : y = 14 : 85$$

17. Shown below is quadrilateral ABCD.
ABC is an isosceles triangle.
ACD is a right angled triangle.



Show that the area of quadrilateral ABCD is $1212x^2$

$AC^2 = (10x)^2 + (24x)^2$

$AC^2 = 100x^2 + 576x^2$

$AC^2 = 676x^2$

$AC = 26x$

$CM = 13x$

$BM^2 = BC^2 - CM^2$

$BM^2 = (85x)^2 - (13x)^2$

$= 7225x^2 - 169x^2$

$= 7056x^2$

$BM = 84x$

Area $\triangle ACD$

$= \frac{1}{2} \times 24x \times 10x$

$= 120x^2$

Area $\triangle ACB$ =

$= \frac{1}{2} \times 26x \times 84x =$

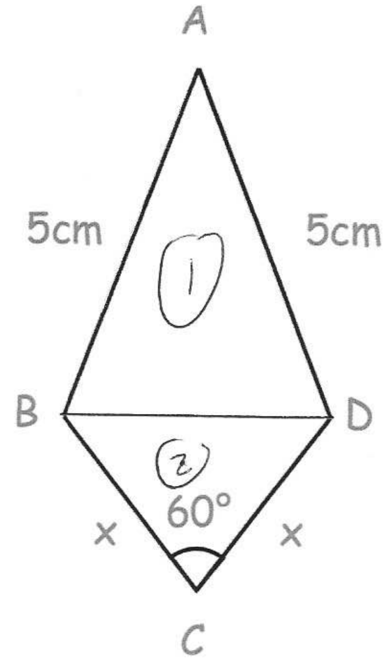
$= 1092x^2$

$120x^2 + 1092x^2 =$

$1212x^2$

(6)

18. Shown below is a kite, ABCD.



Prove $CosBAD = 1 - \frac{x^2}{50}$

① $BD^2 = 5^2 + 5^2 - 2 \times 5 \times 5 \times \cos BAD$

$BD^2 = 50 - 50 \cos BAD$

② $BD^2 = x^2 + x^2 - 2 \times x \times x \times \cos 60$ 0.5

$BD^2 = 2x^2 - 2x^2 (\frac{1}{2})$

$= 2x^2 - x^2 = x^2$

$\therefore \quad x^2 = 50 - 50 \cos BAD$

$50 \cos BAD = 50 - x^2$

$\cos BAD = \frac{50 - x^2}{50}$

$= 1 - \frac{x^2}{50}$ QED.

(6)