Level 2 Further Maths

Parallel Lines
Perpendicular Lines

Ensure you have: Pencil or pen

Guidance
1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic
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1. Write down the equation of the line that is parallel to \( 4x - 2y = 7 \) and passes through \((9, -3)\).

\[ y = 2x + c \]
\[ -3 = 2(9) + c \]
\[ -3 = 18 + c \]
\[ c = -21 \]

\[ y = 2x - 21 \]

(3)

2. A straight line \( L_1 \) passes through the points \((-6, 1)\) and \((-2, -1)\).

A straight line \( L_2 \) passes through the point \((5, 7)\) and is parallel to \( L_1 \).

Find the equation of the line \( L_2 \)

\[ L_1 \]
\[ m = \frac{1 - (-1)}{-2 - (-6)} \]
\[ m = \frac{2}{4} \]
\[ m = \frac{1}{2} \]

\[ L_2 \]
\[ y = -\frac{1}{2}x + c \]
\[ 7 = -\frac{5}{2} + c \]
\[ 9.5 = c \]
\[ y = -\frac{1}{2}x + 9.5 \]

\[ y = -\frac{1}{2}x + 9\frac{1}{2} \]

(4)

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3. A straight line passes through the point A(1, 4) and B(5, 16)

Find the equation of the line parallel to AB that passes through (−8, 2)

\[
\frac{AB}{m} = \frac{12}{4} = 3
\]

\[y = 3x + c\]
\[2 = -24 + c\]
\[c = 26\]
\[y = 3x + 26\] (4)

4. The line L passes through the points (−5, 3) and (1, −2).
The line N passes through the points (−22, 1) and (−4, −14).

Bryan says that the lines L and N are parallel.
Is Bryan correct? Explain your answer.

\[m_L = \frac{-2 - 3}{1 - (-5)} = -\frac{5}{6}\]

\[m_N = \frac{-14 - 1}{-4 - 22} = \frac{-15}{18} = -\frac{5}{6}\]

\[\text{yes, both lines have the same gradient}\] (4)
5. Line L₁ is parallel to $5x - 7y = 31$ and passes through the point $(4, 16)$

Find the coordinates of the point where L₁ intersects the x-axis.

\[
\begin{align*}
-7y &= -5x + 31 \\
7y &= 5x - 31 \\
y &= \frac{5}{7}x - \frac{31}{7} \\
y &= \frac{5}{7}x + c \\
l &= \frac{20}{7} + c \\
c &= \frac{92}{7} \\
y &= \frac{5}{7}x + \frac{92}{7}
\end{align*}
\]

\[
\begin{align*}
0 &= \frac{5}{7}x + \frac{92}{7} \\
\frac{5}{7}x &= -\frac{92}{7} \\
x &= -\frac{92}{5}
\end{align*}
\]

$(\frac{-184}{5}, 0)$

(5)

6. Write down the equation of the line that is perpendicular to $y = -3x + 4$ and passes through $(9, -3)$

\[
m = \frac{1}{3}
\]

\[
y = \frac{1}{3}x + c
\]

\[
-3 = \frac{1}{3} \cdot 9 + c
\]

\[
c = -6
\]

\[
y = \frac{1}{3}x - 6
\]

(3)
7. A straight line passes through the point A(1, 4) and B(5, 16)

\[ m = \frac{6 - 0}{12 - 3} = \frac{6}{9} = \frac{2}{3} \]

Find the equation of the line perpendicular to AB that passes through C.

\[ y = -\frac{3}{2}x + c \]
\[ 5 = -\frac{3}{2} + c \]
\[ 6\frac{1}{2} = c \]

\[ y = -\frac{3}{2}x + 6\frac{1}{2} \quad (4) \]

8. The line Q passes through the points (−10, −2) and (−8, −8)
The line R passes through the points (1, 2) and (10, a)

The lines Q and R are perpendicular.

Find a.

\[ Q \]
\[ \frac{-8 - (-2)}{-10 - (-8)} = \frac{-6}{-2} = 3 \]
\[ \frac{-2}{-2} = 1 \]
\[ a - 2 = 3 \]
\[ a = 5 \]

\[ R \]
\[ m = \frac{1}{3} \]
\[ \frac{a - 2}{10 - 1} = \frac{1}{3} \]
\[ \frac{a - 2}{9} = \frac{1}{3} \]

\[ a - 2 = 3 \]
\[ a = 5 \]
9. Point A has coordinates (9, 7)
Point B has coordinates (13, -27)

Find the equation of the line perpendicular to AB that passes through the midpoint of AB

\[ m = -\frac{27 - 7}{13 - 9} = -\frac{20}{4} = -5 \]

\[ -10 = \frac{22}{17} + c \]
\[ c = -\frac{192}{17} \]

\[ y = \frac{2}{17} x - \frac{192}{17} \]

10.

Calculate the shortest distance between Line 1 and the point A.

\[ L_1 \quad m = \frac{2}{-2} = 1 \]
\[ L_2 \quad m = -1 \]

\[ \frac{x_1 + 4}{2} = \frac{x - 2}{2} \]
\[ 6 = 2x \]
\[ x = 3 \]

\[ A(0, 4) \quad P(3, 1) \]

\[ x_1, y_1 \]
\[ x_2, y_2 \]

\[ y = -x + c \]
\[ 4 = c \]

\[ y = -x + 4 \]

\[ \sqrt{(1-4)^2 + (3-0)^2} = 3\sqrt{2} \]

\[ AP \quad \sqrt{18} \]

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11. Two straight lines are shown below.

Line 1 has equation \( y = \frac{3}{2} x - 24 \)

Are Line 1 and Line 2 perpendicular?

\[
L_1: 0 = 1.5x - 24 \\
24 = 1.5x \\
x = 16 \\
\]

\[
L_2: \quad m_2 = \frac{0 - 12}{18 - 0} = \frac{-2}{3} \\
\]

\[
m_1 \times m_2 = \frac{3}{2} \times \frac{-2}{3} = \frac{-6}{6} = -1
\]

Yes, \( L_1 \) and \( L_2 \) are perpendicular.
12. Shown below are the straight lines AB and CD

\[ M(3, 6) \]

M is the midpoint of CD
AB is perpendicular to CD and passes through M.

C is the point (0, 12)
D is the point (6, 0)

Find the coordinates of the point where AB crosses the y-axis.

\[ \text{CD} \quad m = -2 \]

\[ \text{AB} \quad y = \frac{1}{2}x + c \]

\[ 6 = \frac{3}{2} + c \]

\[ 4.5 = c \]

\[ y = \frac{1}{2}x + 4.5 \]
13. The line $L_1$ passes through the points $A(-4, 2)$ and $B(11, 7)$.

The line $L_2$ is parallel to $L_1$ and passes through the point $C(5, 0)$.
The line $L_3$ is perpendicular to $L_2$ and passes through the point $D(-3, 8)$.
The lines $L_2$ and $L_3$ intersect at the point $E$.

Find the area of triangle $OCE$.

$L_1$ \[\frac{7 - 2}{11 - (-4)} = \frac{5}{15} = \frac{1}{3}\]

$L_2$ \[m = \frac{1}{3}\]
\[y = \frac{1}{3}x + c\]
\[0 = \frac{5}{3} + c\]
\[c = -\frac{5}{3}\]
\[y = \frac{1}{3}x - \frac{5}{3}\]

$L_3$ \[m = -3\]
\[y = -3x + c\]
\[8 = 9 + c\]
\[c = -1\]
\[y = -3x - 1\]

\[\frac{1}{3} x - \frac{5}{3} = -3x + 1\]
\[3\frac{1}{3}x = \frac{2}{3}\]
\[x = \frac{1}{6}\]
\[y = -\frac{3}{5} - 1\]
\[y = -\frac{8}{5}\]
\[
\begin{pmatrix}
\frac{1}{6} \\
-\frac{8}{5}
\end{pmatrix}
\]

\[A = \frac{1}{2} \times 5 \times \frac{8}{5} = 4\]

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