

Name: \_\_\_\_\_

Level 2 Further Maths

**Matrices**



Corbettmaths

Ensure you have: Pencil or pen

### Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic

[www.corbettmaths.com/more/further-maths/](http://www.corbettmaths.com/more/further-maths/)



1.  $\mathbf{A} = \begin{pmatrix} 4 & -1 \\ 0 & 7 \end{pmatrix}$

Work out the matrix  $3\mathbf{A}$

$$\begin{pmatrix} 12 & -3 \\ 0 & 21 \end{pmatrix}$$

.....  
(1)

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2.  $\mathbf{B} = \begin{pmatrix} -3 & 9 \\ 8 & -1 \end{pmatrix}$

Work out the matrix  $8\mathbf{B}$

$$\begin{pmatrix} -24 & 72 \\ 64 & -8 \end{pmatrix}$$

.....  
(1)

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3. The  $2 \times 2$  matrix  $\mathbf{I}$  is the identity matrix.

Write down the  $2 \times 2$  matrix  $\mathbf{I}$

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

.....  
(1)

4.  $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$        $\mathbf{B} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

Work out the matrix **AB**

$$\begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \times 3 + 1 \times 4 \\ 0 \times 3 + 1 \times 4 \end{pmatrix}$$

$$\begin{pmatrix} 10 \\ 4 \end{pmatrix}$$

(2)

5.  $\mathbf{C} = \begin{pmatrix} -3 & 5 \\ -2 & -8 \end{pmatrix}$        $\mathbf{D} = \begin{pmatrix} 9 \\ -4 \end{pmatrix}$

Work out the matrix **CD**

$$\begin{pmatrix} -3 & 5 \\ -2 & -8 \end{pmatrix} \begin{pmatrix} 9 \\ -4 \end{pmatrix} = \begin{pmatrix} -3 \times 9 + 5 \times -4 \\ -2 \times 9 + -8 \times -4 \end{pmatrix}$$

$$\begin{pmatrix} -47 \\ 14 \end{pmatrix}$$

(2)

6.  $\mathbf{E} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$        $\mathbf{F} = \begin{pmatrix} 0 & 6 \\ -3 & -9 \end{pmatrix}$

Work out the matrix **FE**

$$\begin{pmatrix} 0 & 6 \\ -3 & -9 \end{pmatrix} \begin{pmatrix} 1 \\ -5 \end{pmatrix} = \begin{pmatrix} 0 \times 1 + 6 \times -5 \\ -3 \times 1 + -9 \times -5 \end{pmatrix}$$

$$\begin{pmatrix} -30 \\ 42 \end{pmatrix}$$

(2)

7.  $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 0 & 2 \end{pmatrix}$        $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix}$

Work out the matrix  $\mathbf{AB}$

$$\begin{pmatrix} 4 \times 3 + 1 \times 2 & 4 \times 1 + 1 \times 0 \\ 0 \times 3 + 2 \times 2 & 0 \times 1 + 2 \times 0 \end{pmatrix}$$

$$\begin{pmatrix} 14 & 4 \\ 4 & 0 \end{pmatrix}$$

(3)

8.  $\mathbf{C} = \begin{pmatrix} 4 & -2 \\ -1 & 5 \end{pmatrix}$        $\mathbf{D} = \begin{pmatrix} -7 & 0 \\ 2 & 6 \end{pmatrix}$

Work out the matrix  $\mathbf{CD}$

$$\begin{pmatrix} 4 \times -7 + -2 \times 2 & 4 \times 0 + -2 \times 6 \\ -1 \times -7 + 5 \times 2 & -1 \times 0 + 5 \times 6 \end{pmatrix}$$

$$\begin{pmatrix} -32 & -12 \\ 17 & 30 \end{pmatrix}$$

(3)

9.  $\mathbf{A} = \begin{pmatrix} 0 & 3 \\ 2 & 1 \end{pmatrix}$       $\mathbf{B} = \begin{pmatrix} 4 & -1 \\ -1 & 3 \end{pmatrix}$       $\begin{pmatrix} 4 & -1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 0 & 3 \\ 2 & 1 \end{pmatrix}$

Work out the matrix  $\mathbf{BA}$

$$\begin{pmatrix} (4 \times 0) + (-1 \times 2) & (4 \times 3) + (-1 \times 1) \\ (-1 \times 0) + (3 \times 2) & (-1 \times 3) + (3 \times 1) \end{pmatrix}$$

$$\begin{pmatrix} -2 & 11 \\ 6 & 0 \end{pmatrix}$$

(3)

10.  $\mathbf{M} = \begin{pmatrix} -3 & 1 \\ -2 & -4 \end{pmatrix}$

Work out the matrix  $\mathbf{M}^2$

$$\mathbf{M}^2 = \begin{pmatrix} -3 & 1 \\ -2 & -4 \end{pmatrix} \begin{pmatrix} -3 & 1 \\ -2 & -4 \end{pmatrix}$$

$$= \begin{pmatrix} (-3 \times -3) + (1 \times -2) & (-3 \times 1) + (1 \times -4) \\ (-2 \times -3) + (-4 \times -2) & (-2 \times 1) + (-4 \times -4) \end{pmatrix}$$

$$\begin{pmatrix} 7 & -7 \\ 14 & 14 \end{pmatrix}$$

(3)

11.  $3 \begin{pmatrix} a \\ 4 - 2a \end{pmatrix} = \begin{pmatrix} 15 \\ b \end{pmatrix}$

Work out the values of  $a$  and  $b$

$$3a = 15$$

$$a = 5$$

$$12 - 6a = b$$

$$12 - 30 = b$$

$$-18 = b$$

$$a = \dots 5 \dots$$

$$b = \dots -18 \dots$$

(3)

12.  $4 \begin{pmatrix} 6 - c \\ d \end{pmatrix} = c \begin{pmatrix} -2 \\ 9 \end{pmatrix}$

Work out the values of  $c$  and  $d$

$$24 - 4c = -2c$$

$$24 = 2c$$

$$c = 12$$

$$4d = 9c$$

$$4d = 108$$

$$d = 27$$

$$c = \dots 12 \dots$$

$$d = \dots 27 \dots$$

(3)



13.  $\mathbf{A} = \begin{pmatrix} 6 & -2 \\ 1 & -1 \end{pmatrix}$        $\mathbf{B} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

(a) Work out the matrix  $\mathbf{A}^2$

$$\begin{pmatrix} 6 & -2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 6 & -2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} (6 \times 6) + (-2 \times 1) & (6 \times -2) + (-2 \times -1) \\ (1 \times 6) + (-1 \times 1) & (1 \times -2) + (-1 \times -1) \end{pmatrix}$$

$$\begin{pmatrix} 34 & -10 \\ 5 & -1 \end{pmatrix}$$


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(3)

(b) Give a reason why it is not possible to work out  $\mathbf{B}^2$

The number of columns that B has does not equal the number of rows that B has, therefore it cannot be multiplied.

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(1)

(c)  $k\mathbf{B} = \begin{pmatrix} 2k + 15 \\ k + 15 \end{pmatrix}$  where  $k$  is a constant.

Work out the value of  $k$

$$k \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 2k + 15 \\ k + 15 \end{pmatrix}$$

$$4k = 2k + 15$$

$$2k = 15$$

$$k = 7.5$$

$$7.5$$


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(2)

14.  $\mathbf{A} = \begin{pmatrix} 3 & 9 \\ -2 & 8 \end{pmatrix}$       $\mathbf{B} = \begin{pmatrix} p \\ -1 \end{pmatrix}$       $\mathbf{C} = \begin{pmatrix} 9 \\ q \end{pmatrix}$

$p$  and  $q$  are constants.

Given  $\mathbf{AB} = \mathbf{C}$

Work out the values of  $p$  and  $q$

$$\begin{pmatrix} 3 & 9 \\ -2 & 8 \end{pmatrix} \begin{pmatrix} p \\ -1 \end{pmatrix} = \begin{pmatrix} 9 \\ q \end{pmatrix}$$

$$3p + (-9) = 9$$

$$-2p - 8 = q$$

$$3p - 9 = 9$$

$$-12 - 8 = q$$

$$3p = 18$$

$$q = -20$$

$$p = 6$$

$$p = \dots 6 \dots$$

$$q = \dots -20 \dots$$

(3)

15.  $\begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \mathbf{I}$  where  $\mathbf{I}$  is the identity matrix

Work out the values of  $a$ ,  $b$  and  $c$

$$\begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$2a = 1$$

$$2b - 1 = 0$$

$$a = \frac{1}{2}$$

$$2b = 1$$

$$2b - c = 0$$

$$b = \frac{1}{2}$$

$$a = \dots \frac{1}{2} \dots$$

$$b = \dots \frac{1}{2} \dots$$

$$c = 1$$

$$c = \dots 1 \dots$$

(3)



$$16. \begin{pmatrix} -6 & 2 \\ 5 & -4 \end{pmatrix} \begin{pmatrix} -4 & -2 \\ -5 & m \end{pmatrix} = 14 \mathbf{I}$$

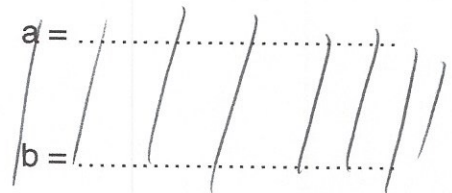
Work out the value of  $m$

$$\begin{pmatrix} 14 & 12 + 2m \\ 0 & -10 - 4m \end{pmatrix} = \begin{pmatrix} 14 & 0 \\ 0 & 14 \end{pmatrix}$$

$$12 + 2m = 0$$

$$2m = -12$$

$$m = -6$$



$$m = -6$$

(3)

$$17. \text{ Given that } \begin{pmatrix} 5 & 1 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} d + 6 \\ -4c - 4d \end{pmatrix}$$

Work out the values of  $c$  and  $d$

$$5c + d = d + 6$$

$$5c = 6$$

$$c = 1.2$$

$$-3c + 2d = -4c - 4d$$

$$-3 \cdot 6 + 6d = -4 \cdot 8$$

$$6d = -1.2$$

$$d = -0.2$$

$$c = 1.2$$

$$d = -0.2$$

(4)

18. Given that  $\begin{pmatrix} -2 & 3 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} b-8 \\ a+2 \end{pmatrix}$

Work out the values of  $a$  and  $b$

$$\begin{aligned} -2a + 3b &= b-8 \\ -2a + 2b &= -8 \end{aligned}$$

$$\begin{aligned} 4a + b &= a+2 \\ 3a + b &= 2 \end{aligned}$$

Subtract

$$\begin{array}{r} -2a + 2b = -8 \\ 6a + 2b = 4 \\ \hline -8a = -12 \\ a = 1.5 \end{array}$$

$$\begin{aligned} 4.5 + b &= 2 \\ b &= -2.5 \end{aligned}$$

$a = 1.5$

$b = -2.5$

(5)

18.  $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$        $B = \begin{pmatrix} p & q \\ 4 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

$AB = BA$

Work out the values of  $p$  and  $q$

$$AB = \begin{pmatrix} p & q \\ p+4 & q+3 \end{pmatrix} \quad BA = \begin{pmatrix} p+q & q \\ 7 & 3 \end{pmatrix}$$

$$p = p+q$$

$$p+4 = 7$$

$$q = 0$$

$$p = 3$$

$p = 3$

$q = 0$

(5)

19.  $\begin{pmatrix} 3 & -1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 23 \\ 8 \end{pmatrix}$

Work out the values of  $x$  and  $y$

$$\begin{aligned} 3x - y &= 23 \\ 2x + 3y &= 8 \end{aligned}$$

$$\begin{array}{r} 9x - 3y = 69 \\ \text{add } 2x + 3y = 8 \\ \hline 11x = 77 \\ x = 7 \end{array}$$

$$\begin{aligned} 14 + 3y &= 8 \\ 3y &= -6 \\ y &= -2 \end{aligned}$$

$$x = \overset{7}{\dots\dots\dots}$$

$$y = \overset{-2}{\dots\dots\dots}$$

(4)

20.  $\begin{pmatrix} 2 & y \\ x & -12 \end{pmatrix} \begin{pmatrix} x \\ 2 \end{pmatrix} = \begin{pmatrix} 7 \\ 5x \end{pmatrix}$

Work out the possible values for x and y

$$2x + 2y = 7$$

$$x^2 - 24 = 5x$$

$$x^2 - 5x - 24 = 0$$

$$(x - 8)(x + 3) = 0$$

$$x = 8 \text{ or } x = -3$$

When  $x = 8$

$$16 + 2y = 7$$

$$2y = -9$$

$$y = -4.5$$

When  $x = -3$

$$-6 + 2y = 7$$

$$2y = 13$$

$$y = 6.5$$

$$x = 8, y = -4.5 \text{ or } x = -3, y = 6.5$$

(5)

21.  $\begin{pmatrix} 1 & 2 \\ x & 3x \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 10 \end{pmatrix}$

Work out the possible values for x and y

$$x + 2y = 3$$

$$x = 3 - 2y$$

$$x^2 + 3xy = 10$$

$$(3 - 2y)(3 - 2y) + 3y(3 - 2y) = 10$$

$$9 - 12y + 4y^2 + 9y - 6y^2 = 10$$

$$9 - 3y - 2y^2 = 10$$

$$0 = 2y^2 + 3y + 1$$

$$(2y + 1)(y + 1) = 0$$

$$y = -\frac{1}{2} \text{ or } y = -1$$

$$x = 4 \text{ or } x = 5$$

(6)