

19th July



Corbettmaths

Use factor theorem to show that  $(x - 1)$  is a factor of

$$x^3 - 3x^2 - 13x + 15 = f(x)$$

$$f(1) = 1 - 3 - 13 + 15 = 0$$

$$\Rightarrow \underline{x-1 \text{ factor}}$$

Expand and simplify  $(2\sqrt{3} - 1)^3$

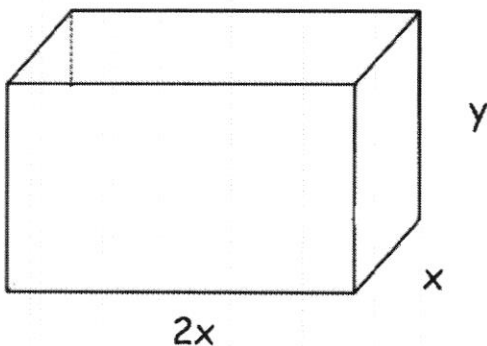
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(2\sqrt{3}-1)^3 = (2\sqrt{3})^3 + 3(2\sqrt{3})^2(-1) + 3(2\sqrt{3})(-1)^2 + (-1)^3$$

$$= 24\sqrt{3} - 36 + 6\sqrt{3} - 1$$

$$= \underline{-37 + 30\sqrt{3}}$$

An open-topped tank in the shape of a cuboid is shown below.



The surface area of the cuboid is  $300\text{cm}^2$

The volume of the cuboid is  $V$

Show that  $y = \frac{50}{x} - \frac{x}{3}$

$$SA = 2(2xy) + 2(xy) + x(2x)$$

$$= 6xy + 2x^2 = 300$$

$$\Rightarrow 6xy = 300 - 2x^2$$

$$\Rightarrow \underline{y = \frac{50}{x} - \frac{x}{3}}$$

Show that the volume of the tank is

$$V = 100x - \frac{2}{3}x^3$$

$$V = 2x^2y = 2x^2\left(\frac{50}{x} - \frac{x}{3}\right)$$

$$= \underline{100x - \frac{2}{3}x^3}$$

Use differentiation to find the value of  $x$  for which  $V$  is a maximum

$$\frac{dV}{dx} = 100 - 2x^2 \quad \left[ \frac{d^2V}{dx^2} = -4x < 0 \Rightarrow \text{max} \right]$$

$$\text{At max } 100 - 2x^2 = 0$$

$$\Rightarrow x^2 = 50$$

$$\Rightarrow \underline{x = \sqrt{50} \text{ (7.07 cm)}}$$