

Name:

Level 2 Further Maths

Circle Theorems



Corbettmaths

Ensure you have: Pencil or pen

Guidance

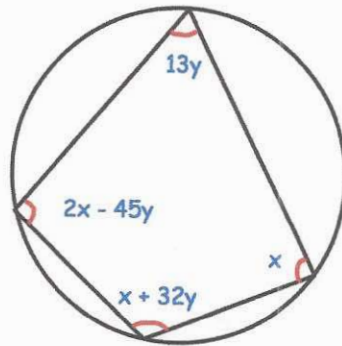
1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic

www.corbettmaths.com/more/further-maths/



1.



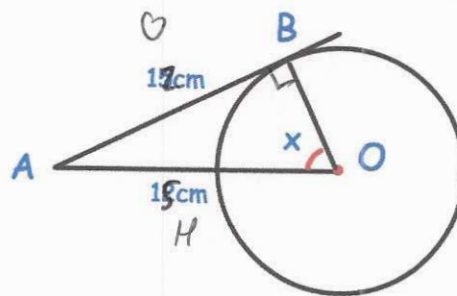
Find the values of x and y

$$\begin{aligned} 3x - 45y &= 180 \\ x + 45y &= 180 \\ \hline 4x &= 360 \\ x &= 90^\circ \end{aligned}$$

$$\begin{aligned} 90 + 45y &= 180 \\ y &= 2 \end{aligned}$$

$$x = \dots 90^\circ \quad y = \dots 2 \dots^\circ \quad (3)$$

2.



AB is a tangent to the circle.

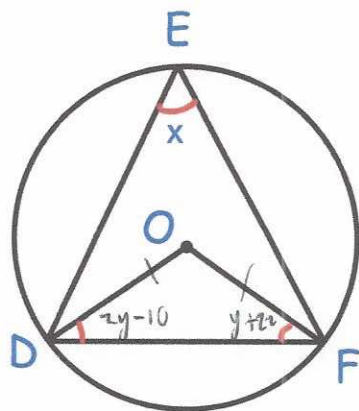
Calculate the size of angle x .

$$\sin x = \frac{12}{15}$$

$$x = 53.13$$

$$x = \dots 53.13^\circ \quad (3)$$

3.



Given $\angle ODF = 2y - 10^\circ$ and $\angle OFD = y + 22^\circ$

Find $\angle DEF$

$$2y - 10 = y + 22$$

$$y = 32^\circ$$

$$\angle OFD = 54^\circ$$

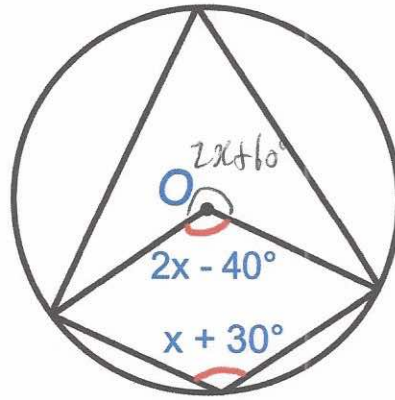
$$54 + 54 = 108$$

$$\angle DOF = 72^\circ$$

$$\angle DEF = 72 \div 2$$

$$\begin{array}{r} 36. \\ \hline (3) \end{array}$$

4.



Work out the size of angle x°

$$2(x + 30) = 2x + 60$$

$$(2x + 60) + (2x - 40) = 360^\circ$$

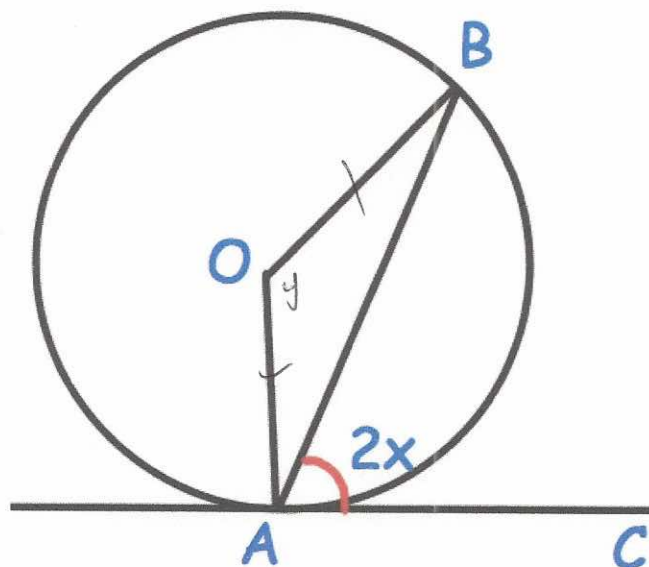
$$4x + 20 = 360$$

$$4x = 340$$

$$x = 85^\circ$$

$$x = \frac{85}{(4)}^\circ$$

5.



A and B are points on the circumference of a circle, centre O.

CA is a tangent to the circle.

Angle $CAB = 2x$

Prove that angle $AOB = 4x$

Give reasons for each stage of your working.

$$\angle OAB = 90 - 2x^\circ \quad (\angle OAC = 90^\circ \text{ as radius \& tangent meet at } 90^\circ)$$

$$\angle OBA = 90 - 2x^\circ \quad (\triangle OAB \text{ is isosceles as } OA = OB = r)$$

As the angles in a triangle add to 180°

$$y + (90 - 2x) + (90 - 2x) = 180^\circ \quad (4)$$

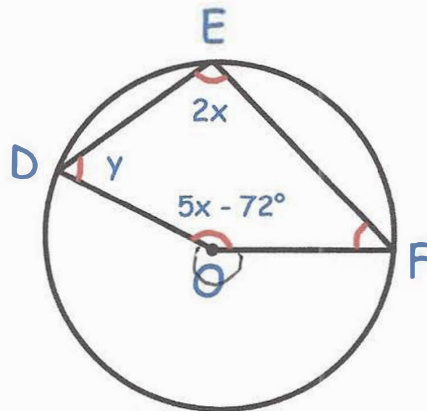
$$y + 180 - 4x = 180$$

$$y - 4x = 0$$

$$y = 4x$$

$$\angle AOB = 4x \quad \text{QED}$$

6.



The points D, E and F are points on a circle, centre O.

$$\text{Angle DEF} = 2x$$

$$\text{Angle DOF} = 5x - 72^\circ$$

$$\text{Angle EDO} = y$$

Angle EFO is 14° smaller than angle EDO.

Work out the value of y

$$(2x) \times 2 = 4x$$

$$4x + 5x - 72 = 360^\circ$$

$$9x = 432^\circ$$

$$x = 48^\circ$$

$$\angle EFO = y - 14^\circ$$

$$7x - 72 + y + y - 14 = 360$$

$$336 - 72 + 2y - 14 = 360$$

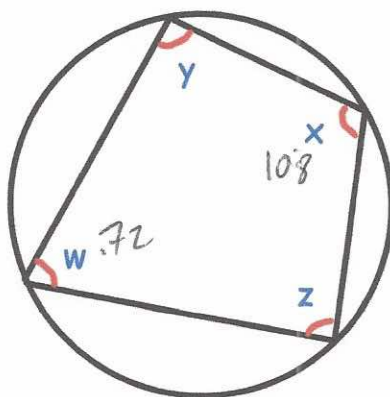
$$250 + 2y = 360$$

$$y = 55^\circ$$

$$y = \dots\dots\dots 55^\circ$$

(5)

7.



$$w : x = 2 : 3$$

$$x : y = 6 : 5 \quad 9 : 8$$

Work out the size of angle z.

$$w + x = 180 \quad (\text{opposite angles in a cyclic quadrilateral})$$

$$2 + 3 = 5$$

$$180 \div 5 = 36$$

$$36 \times 2 = 72 \quad \underline{w}$$

$$36 \times 3 = 108 \quad \underline{x}$$

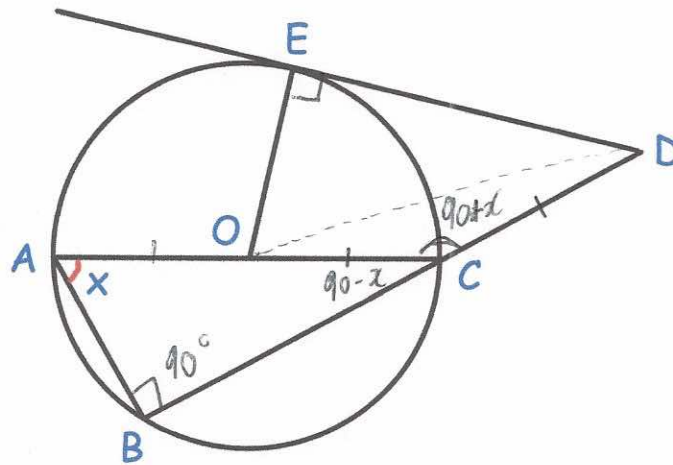
$$108 \div 9 = 12$$

$$12 \times 8 = 96$$

$$180 - 96 = 84$$

$$z = \frac{84}{5}^\circ$$

8.



AC is the diameter of a circle, centre O.
DE is the tangent to the circle.
BCD is a straight line.

Angle BAC = x

Express angle COD in terms of x .

$$\begin{aligned} \angle ABC &= 90^\circ \quad (\text{angle in semi-circle}) \\ \angle ACB &= 90^\circ - x \quad (\text{angles in } \triangle ABC \text{ add to } 180^\circ) \\ \angle BCD &= 90^\circ + x \quad (\text{BCD is a straight line}) \\ \angle OED &= 90^\circ \quad (\text{radius / tangent meet at } 90^\circ) \end{aligned}$$

$$180 - (90 + x) = 90 - x$$

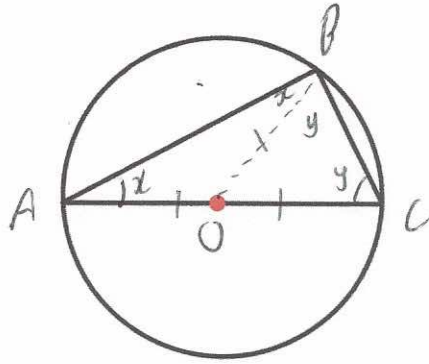
$$\frac{90 - x}{2}$$

$$= 45 - \frac{1}{2}x$$

$$\dots\dots\dots 45 - \frac{1}{2}x$$

(5)

9.



Prove that the angle in a semi-circle is always 90°

$$OA = OB = OC = \text{radius}$$

$$\left. \begin{array}{l} \angle OAB = \angle OBA = x^\circ \\ \angle OCB = \angle OBC = y^\circ \end{array} \right\} \begin{array}{l} \text{Two angles in an isosceles triangle} \\ \text{are equal.} \end{array}$$

Angles in a triangle add to 180°

$$\therefore x + (x+y) + y = 180$$

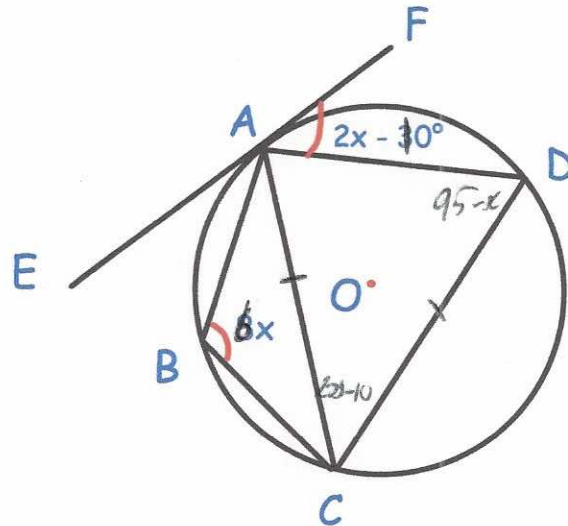
$$2x + 2y = 180$$

$$x + y = 90^\circ$$

As $\angle ABC = x + y$
it equals 90°

(3)

10.



EF is a tangent to a circle, centre O.

$$\angle DAF = 2x - 30^\circ$$

$$\angle ABC = 8x$$

Find the size of angle $\angle DAF$

$$\angle ACD = 2x - 10 \quad (\text{alternate segment theorem})$$

$$\angle ADC = \frac{180 - (2x - 10)}{2} = \frac{190 - 2x}{2} = 95 - x$$

$$6x + (95 - x) = 180$$

$$5x + 95 = 180$$

$$5x = 85$$

$$x = 17$$

$$2 \times 17 - 10 = 24^\circ$$

24°

(3)