

1st July

Higher Plus 5-a-day

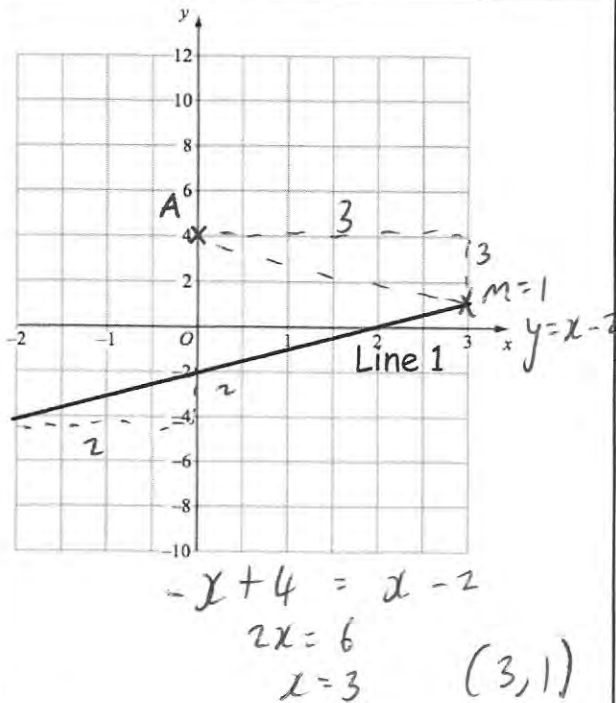


Corbettmaths

Arrange the following in order, smallest first

$25^{-\frac{1}{2}}$        $\left(\frac{2}{3}\right)^{-2}$        $0.1$   
 $\frac{1}{5}$        $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$        $\frac{1}{9}$

$0.1$ ,  $25^{-\frac{1}{2}}$ ,  $\left(\frac{2}{3}\right)^{-2}$



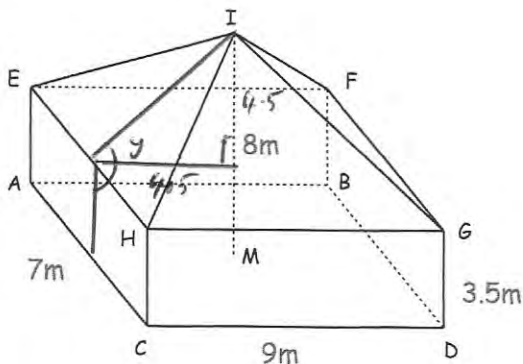
Write down the equation of the line perpendicular to Line 1 and passing through A.

$y = -x + 4$

Find the shortest distance between Line 1 and A.

$y^2 = 3^2 + 3^2$   
 $y^2 = 18$   
 $y = \sqrt{18} = 3\sqrt{2}$

The diagram shows a cuboid and a pyramid. The apex I is directly above the centre M, of ABDC.



Calculate the angle between EHI and ACHE

$\tan y = \frac{4.5}{4.5}$   
 $y = 45^\circ$   
 $90 + 45 = 135^\circ$

2nd July

Higher Plus 5-a-day



Corbettmaths

$(x + 3)(x + a)(bx - 3)$  is expanded to give

$$2x^3 - x^2 - 15x + 18$$

Find a and b.

$$a = -2 \quad b = 2$$

$$3 \times a \times -3 = 18$$

$$-9a = 18$$

$$a = -2$$

$$x \times x \times bx = 2x^3$$

$$b = 2$$

w is proportional to  $\sqrt{x}$

x is decreased by 9.75%

Work out the percentage decrease in w.

$$5\%$$

$$w = k \times \sqrt{x}$$

$$\sqrt{0.9025x}$$

$$= 0.95\sqrt{x}$$

Liquid A has a density of  $0.7\text{g/cm}^3$   
 Liquid B has a density of  $1.5\text{g/cm}^3$   
 Liquid C has a density of  $1.25\text{g/cm}^3$

200g of liquid A, 1kg of liquid B and 500g of liquid C are mixed to make liquid D.

Work out the density of liquid D

(A)  $v = 285.714285\text{cm}^3$

(B)  $v = 666.6\text{cm}^3$

(C)  $v = 400\text{cm}^3$

(D)  $v = 1352.38\text{cm}^3$

$$d = \frac{m}{v}$$

$$\frac{1700}{1352.38} = 1.257\text{g/cm}^3$$

$$v = \frac{m}{d}$$

One solution of a quadratic equation in the form

$$y = ax^2 + bx + c$$

is

$$x = \frac{3 + \sqrt{65}}{4}$$

$$b^2 - 4ac = 65$$

$$9 - 8c = 65$$

$$-8c = 56$$

$$c = -7$$

Find possible values of a, b and c.

$$a = 2$$

$$b = -3$$

$$c = -7$$

$$f(x) = 10 - 5x \quad g(x) = \frac{1}{3}x - 1$$

Solve  $f^{-1}(x) = g^{-1}(x)$

$$y = 10 - 5x$$

$$5x = 10 - y$$

$$x = \frac{10 - y}{5}$$

$$y = \frac{1}{3}x - 1$$

$$3y = x - 3$$

$$x = 3y + 3$$

$$f^{-1}(x) = \frac{10 - x}{5} \quad g^{-1}(x) = 3x + 3$$

$$\frac{10 - x}{5} = 3x + 3$$

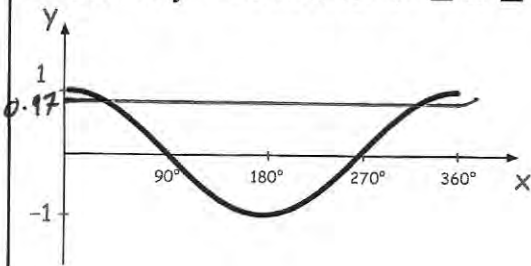
$$10 - x = 15x + 15$$

$$-5 = 16x$$

$$x = -\frac{5}{16}$$



Shown is  $y = \cos x$  for  $0^\circ \leq x \leq 360^\circ$

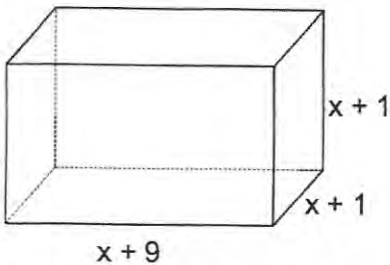


One solution of  $\cos x = 0.97$  is

$$x = 14^\circ$$

Find another solution to  $\cos x = 0.97$

$$360 - 14 = 346^\circ$$



$$(x+1)(x+1)(x+9) = (x^2 + 2x + 1)(x+9)$$

Form an expression for the volume of the cuboid.

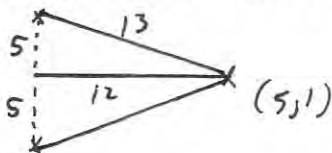
Expand and simplify the expression.

$$x^3 + 9x^2 + 2x^2 + 18x + x + 9$$

$$x^3 + 11x^2 + 19x + 9$$

The distance between  $(-7, a)$  and  $(5, 1)$  is 13 units. *(5, 12, 13 triangle)*

Find two possible values for  $a$ .



$$h^2 + 12^2 = 13^2$$

$$h^2 = 169 - 144$$

$$h^2 = 25$$

$$h = 5$$

$$1 + 5 = 6$$

$$1 - 5 = -4$$

$$a = -4 \text{ or } 6$$

The numbers  $m$  and  $n$  are irrational and are not the same.

$m + n$  is rational

Write down possible values for  $m$  and  $n$

$$m = 8 + \sqrt{2}$$

$$n = 5 - \sqrt{2}$$

$$m + n = 13$$

The ratio of Isaac's age to Max's age is  $x:y$

$$7(x-5) = y-5$$

$$7x - 35 = y - 5$$

Five years ago, the ratio of their ages was 1:7

$$2x - 30 = y \quad \text{--- (1)}$$

In six years time, the ratio of their ages will be 3:10

$$10(x+6) = 3(y+6)$$

$$10x + 60 = 3y + 18$$

Express  $x:y$  in its lowest terms

Substitute (1) into (2)

$$10x + 42 = 3(7x - 30)$$

$$10x + 42 = 21x - 90$$

$$132 = 11x$$

$$x = 12$$

$$y = 54$$

$$12:54$$

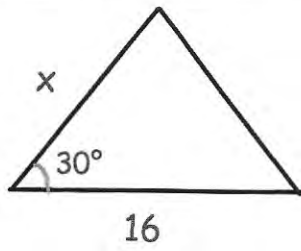
$$2:9$$

4th July

Higher Plus 5-a-day



Corbettmaths



$$\frac{1}{2} ab \sin C$$

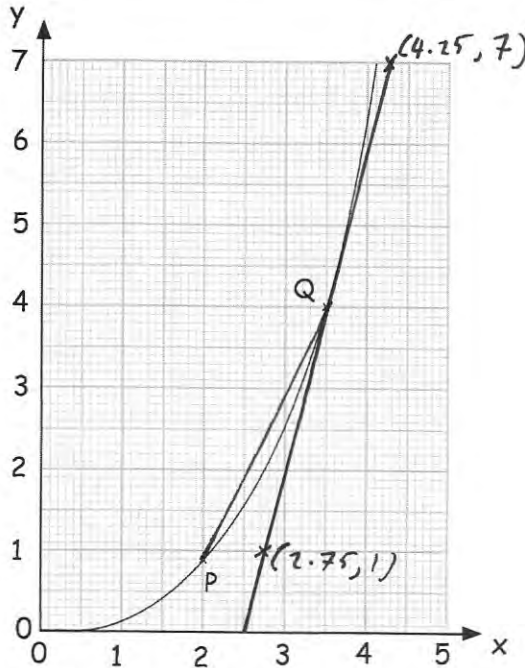
$$\frac{1}{2} x \times 16 \times \sin 30$$

$$\frac{1}{2} x \times 16 \times \frac{1}{2}$$

$$4x$$

Find the area of the triangle in terms of x.

$$4x$$



Work out the average rate of change of y with respect to x between points P and Q.

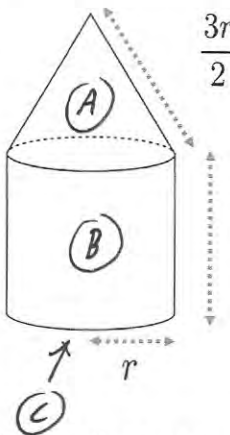
$$\frac{\text{rise}}{\text{run}} = \frac{4 - 0.9}{3.5 - 2} = 2.06$$

Work out the instantaneous rate of change of y with respect to x at point Q.

$$\frac{\text{rise}}{\text{run}} = \frac{7 - 1}{4.25 - 2.75} = 4$$

\* gradients may vary due to tangents.

A cone and cylinder are joined to make a solid



$$(A) \pi r \left( \frac{3r}{2} \right)$$

$$= \frac{3}{2} \pi r^2$$

$$(B) 2\pi r(r+6)$$

$$= 2\pi r^2 + 12\pi r$$

$$(C) \pi r^2$$

$$\frac{3}{2} \pi r^2 + 2\pi r^2 + 12\pi r + \pi r^2$$

$$= \frac{9}{2} \pi r^2 + 12\pi r$$

$$= \frac{3\pi r}{2} (3r + 8)$$

Show the total surface area of the

solid is  $\frac{3\pi r}{2} (3r + 8)$

5th July

Higher Plus 5-a-day



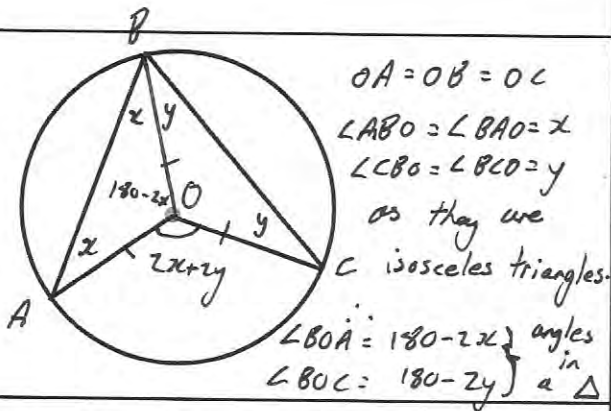
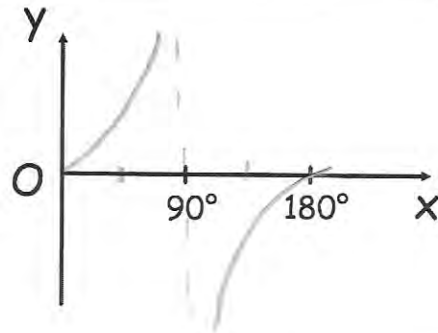
Corbettmaths

Factorise fully

$$98 - 72x^2 \quad 2(49 - 36x^2)$$

$$2(7 - 6x)(7 + 6x)$$

Sketch  $y = \tan x$  for  $0^\circ \leq x \leq 180^\circ$



Prove that the angle at the centre is twice the angle at the circumference.

$$\angle BOA + \angle BOC + \angle AOC = 360 \text{ angles at a point.}$$

$$\therefore \angle AOC = 2x + 2y$$

$$\text{So } \angle AOC = 2 \times \angle ABC$$

A and B are similar cuboids

volume of A: volume of B = 27 : 125

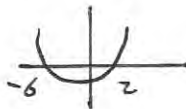
Work out  
surface area of B: surface area of A

$$\sqrt[3]{27} = 3 \quad \sqrt[3]{125} = 5$$

	A	B
sides:	3	5
Area:	9	25

$$\underline{25 : 9}$$

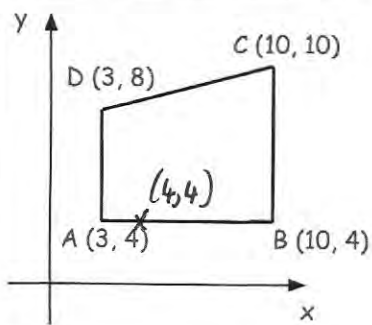
Solve  $x^2 + 4x - 12 > 0$



$$x < -6 \text{ or } x > 2$$

$$(x+6)(x-2)$$

$$x = -6 \text{ or } x = 2$$



ABCD is reflected in the line  $y = x$   
Write down the coordinates of any invariant points.

$$(4, 4) \text{ \& } (10, 10)$$

A garage checks 498 cars for faults with their tyres, brakes and lights.

100 cars had faults with their tyres.  
62 cars had faults with their brakes.  
16 cars had faults with their lights.

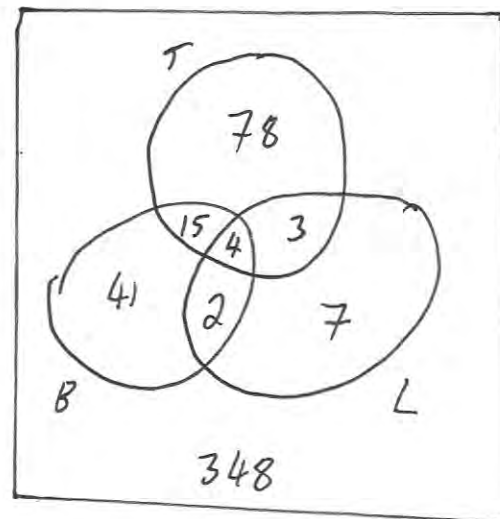
19 cars had faults with both their tyres and brakes.

7 cars had faults with both their tyres and lights.

6 cars had faults with both their brakes and lights.

4 had faults with all three.

Draw a Venn diagram to show this information.



A car that had only one type of fault is picked at random.

Find the probability that the car had a fault with its lights.

$$\frac{7}{41 + 7 + 78} = \frac{1}{18}$$

$$\frac{1}{18}$$

Solve the simultaneous equations

$$x^2 + 3x - xy = 10$$

$$2x - y = 4$$

$$y = 2x - 4$$

$$x^2 + 3x - x(2x - 4) = 10$$

$$x^2 + 3x - 2x^2 + 4x = 10$$

$$0 = x^2 - 7x + 10$$

$$0 = (x - 2)(x - 5)$$

$$x = 2 \text{ or } x = 5$$

$$y = 0 \quad y = 6$$

$$(2, 0) \quad (5, 6)$$



7th July

Higher Plus 5-a-day

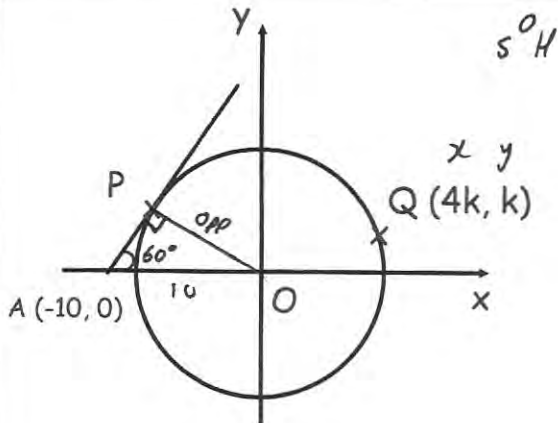


Corbettmaths

A car travelled for 135 minutes, to the nearest 5 minutes.  $132.5 \text{ min}$   
 It travelled for a total distance of 120 km, to the nearest 10km  $125000 \text{ m}$   
 $125 \text{ km}$

Work out the greatest possible average speed, in m/s

$$\begin{aligned} \text{Max } S &= \frac{\text{Max } d}{\text{Min } t} \\ &= \frac{125000(\text{m})}{7950(\text{s})} \\ &= 15.72 \text{ m/s to 2 dp} \end{aligned}$$



AP is a tangent to the circle.  
 Angle OAP =  $60^\circ$

Find the value of k to 1 decimal place.

$$\sin(60) \times 10 = 8.660... \quad (5\sqrt{3})$$

$$x^2 + y^2 = (5\sqrt{3})^2$$

$$x^2 + y^2 = 75$$

$$(4k)^2 + k^2 = 75$$

$$16k^2 + k^2 = 75$$

$$17k^2 = 75$$

$$k^2 = \frac{75}{17}$$

$$k = \pm \sqrt{\frac{75}{17}}$$

$$\boxed{k = 2.1} \text{ or } k = -2.1$$

Make q the subject of

$$\frac{p}{qr} = 2 + \frac{1}{r} \quad \frac{p}{qr} = \frac{2r+1}{r}$$

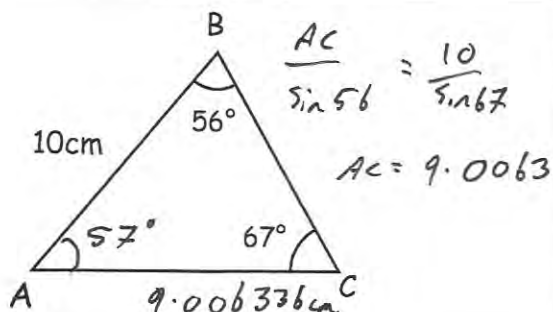
$$\frac{p}{qr} = \frac{2r+1}{r}$$

$$pr = (2r+1)qr$$

$$pr = (2r^2+r)q$$

$$q = \frac{pr}{2r^2+r}$$

$$q = \frac{pr}{r(2r+1)} \quad q = \frac{p}{2r+1}$$



Find the area of ABC.

$$\frac{1}{2} \times 10 \times 9.006... \times \sin 57$$

$$= 37.8 \text{ cm}^2 \text{ to 1 dp.}$$



A sequence of numbers is formed by the iterative process

$$a_{n+1} = (a_n)^2 - 10$$

$$a_1 = 3$$

$$a_2 = 3^2 - 10 = -1$$

Find

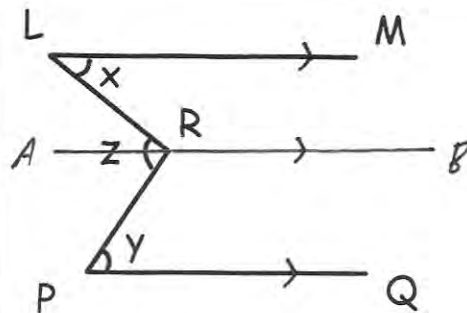
$$a_3 = (-1)^2 - 10 = 1 - 10 = -9$$

LM and PQ are parallel

Prove  $x + y = z$

$$\begin{aligned} \angle MLR &= \angle LRA \quad (\text{alternate angles are equal}) \\ \angle RPQ &= \angle PRA \end{aligned}$$

$$\angle PRL = x + y \quad \therefore z = x + y$$



Ethan has 12 coins.

There are three 10p coins and nine 20p coins.

Ethan chooses 3 coins at random.

Work out the probability that he takes exactly 50p.

$$P(20, 20, 10) = \frac{9}{12} \times \frac{8}{11} \times \frac{3}{10} = \frac{9}{55}$$

$$P(20, 10, 20) = \frac{9}{12} \times \frac{3}{11} \times \frac{8}{10} = \frac{9}{55}$$

$$P(10, 20, 10) = \frac{3}{12} \times \frac{9}{11} \times \frac{8}{10} = \frac{9}{55}$$

$$\boxed{\frac{27}{55}}$$

Solve

$$3^{4x} = 27^{5-x}$$

$$3^{4x} = (3^3)^{5-x}$$

$$4x = 15 - 3x$$

$$7x = 15$$

$$x = \frac{15}{7}$$

Find the nth term for the sequence

0 9 20 33 48  
1 11 13 15  
2 2 2

$$a = 1$$

$$b = 6$$

$$c = -7$$

$$n^2 + 6n - 7$$



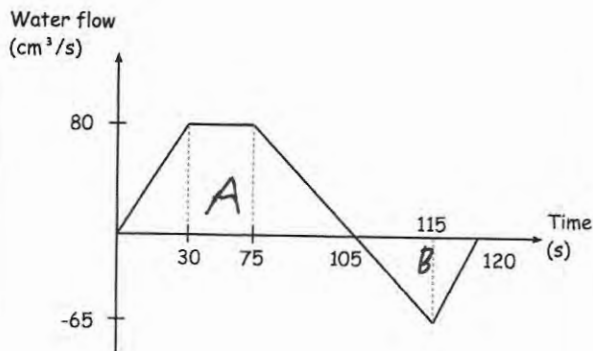
9th July

Higher Plus 5-a-day



Corbettmaths

The graph below shows information on how an empty container is being filled with water.

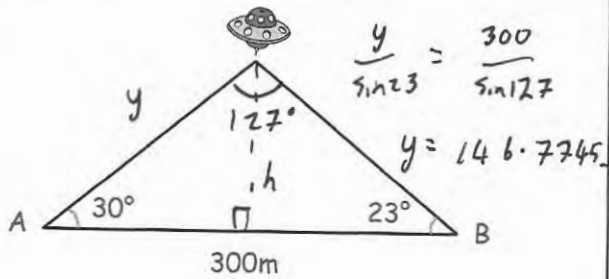


How much water is in the container after 120 seconds?

$$A: \frac{1}{2} (105 + 45) \times 80 = 6000 \text{ cm}^3$$

$$B: \frac{1}{2} \times 15 \times 65 = 487.5 \text{ cm}^3$$

$$6000 - 487.5 = 5512.5 \text{ cm}^3$$



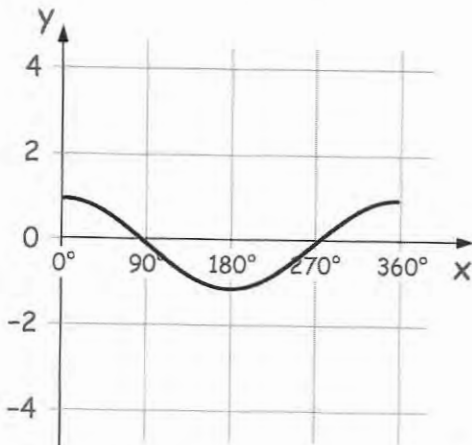
Find the height of the UFO above the ground.

$$5^{\circ} H$$

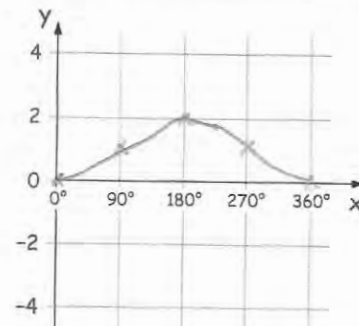
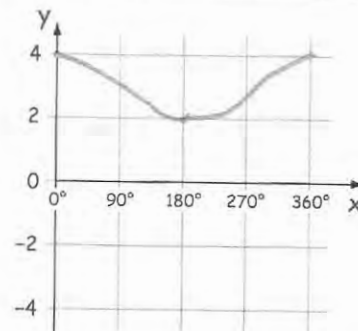
$$\sin(30) \times 146.77$$

$$73.39 \text{ m}$$

Shown is the graph of  $y = \cos x$



Sketch  $y = 3 + \cos x$  and  $y = 1 - \cos x$





Factorise fully

$$7x^2 - 28$$

$$7(x^2 - 4)$$

$$7(x-2)(x+2)$$

Yasmin creates a 6 digit passcode for her phone such that all the digits are prime numbers.

Jack knows that all the digits are prime and he tries to guess the passcode.

What is the probability he guesses correctly?

2, 3, 5, 7

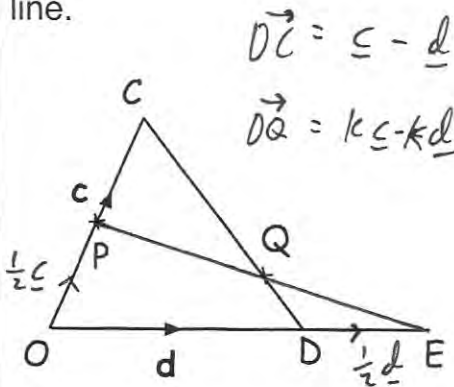
$$4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4096$$

$$\frac{1}{4096}$$

$$\vec{OC} = \underline{c} \quad \vec{OD} = \underline{d}$$

Point P is the midpoint of OC  
ODE is a straight line such that  
OD:OE = 2:3

The points P, Q and E are in a straight line.



$$\vec{DC} = \underline{c} - \underline{d}$$

$$\vec{DQ} = k\underline{c} - k\underline{d}$$

$$\vec{DQ} = k\vec{DC}$$

Find the value of k

$$\vec{PE} = -0.5\underline{c} + 1.5\underline{d}$$

$$\vec{PQ} = -0.5\underline{c} + \underline{d} + k\underline{c} - k\underline{d}$$

$$\vec{PQ} = (-0.5 + k)\underline{c} + (1 - k)\underline{d}$$

$$\frac{-0.5}{-0.5 + k} = \frac{1.5}{1 - k}$$

$$-0.5 + 0.5k = -0.75 + 1.5k$$

$$0.25 = k$$

$$k = \frac{1}{4}$$

Parallel

The first 4 terms of a sequence are:

500, 490, 475, 455 ...

-10   -15   -20  
-5   -5

$$a = -2.5$$

$$b = -2.5$$

$$c = 505$$

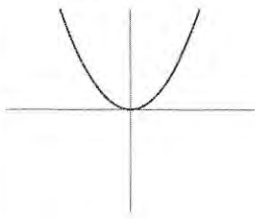
Which term is the first to be negative?

$$-2.5n^2 - 2.5n + 505$$

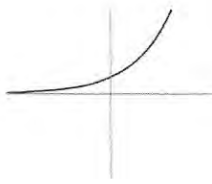
14<sup>th</sup> term is -20



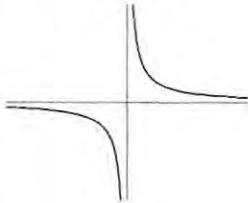
Graph A



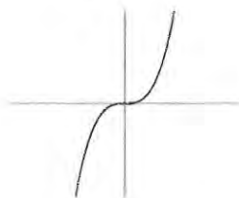
Graph B



Graph C



Graph D



$y = x^2$  is graph A

$y = x^3$  is graph **D**

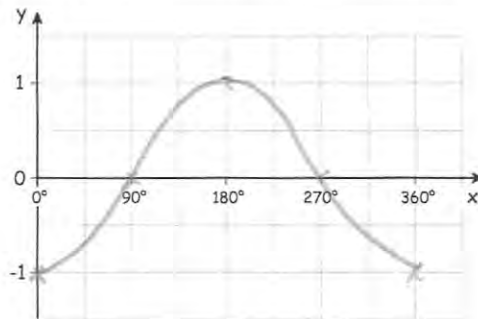
$y = 2^x$  is graph **B**

$y = \frac{1}{x}$  is graph **C**

For all the values of  $x$

$f(x) = x - 180$        $\cos(x - 180)$   
 $g(x) = \cos x$

Draw the graph of the function  $y = gf(x)$  for  $0^\circ \leq x \leq 360^\circ$

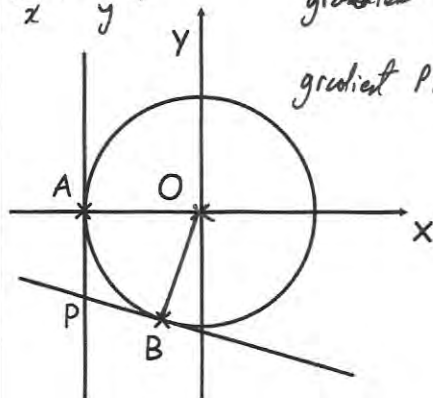


The circle  $x^2 + y^2 = 289$  has  $r = 17$  tangents at points A and B.

The point A has coordinates  $(-17, 0)$

The point B has coordinates

$(-8, -15)$



The tangents meet at the point P.

Work out the equation of the tangent at B.

$y = -\frac{8}{15}x + c$

$-15 = \frac{64}{15} + c$        $c = -\frac{289}{15}$

$y = -\frac{8}{15}x - \frac{289}{15}$

Work out the coordinates of the point P.

$-17 = x$        $x = -17$

$y = -\frac{8}{15}(-17) - \frac{289}{15} = -10.2$

$(-17, -10.2)$

12th July

Higher Plus 5-a-day



Corbettmaths

The curve A with equation  $y = f(x)$  is transformed to curve B with equation  $y = f(-x) + 1$

The point on A with coordinates (4, 5) is mapped to the point P on B

Find the coordinates of P

$$(-4, 6)$$

The straight line L has the equation  $4y = 3x + 5$   $y = \frac{3}{4}x + \frac{5}{4}$   $x$   $y$   
The point A has coordinates (2, -8)

Find an equation of the straight line that is perpendicular to L and passing through A

$$y = -\frac{4}{3}x + c$$

$$-8 = -\frac{8}{3} + c$$

$$c = -\frac{16}{3}$$

$$y = -\frac{4}{3}x - \frac{16}{3}$$

2 2 2 3 4 5 6 7 7 9  
1 2 3 4 5

Tia picks three cards at random, without replacement. She adds the three numbers together to get a score.

EEO  
EOE  
OEE

$$3 \times \frac{5}{36} = \frac{15}{36}$$

Find the probability that the score is an odd number.

$$P(\text{odd}) = \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{1}{12}$$

$$\frac{15}{36} + \frac{1}{12} = \frac{1}{2}$$

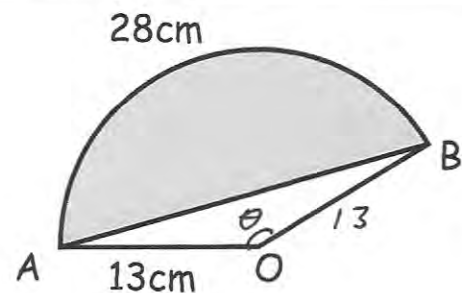
OA = 13cm and the arc AB = 28cm  
Find the area of the shaded segment

$$\frac{\theta}{360} \times \pi \times 26 = 28 \quad \theta = 123.4063$$

$$\text{Sector} : \frac{123.4063}{360} \times \pi \times 13^2 = 182$$

$$\text{Area } \triangle = \frac{1}{2} \times 13 \times 13 \times \sin 123.4 = 70.539...$$

$$\text{Segment} : 111.46 \text{ cm}^2$$



Solve

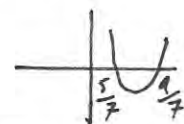
$$(1-x)^2 > \frac{4}{49}$$

$$x^2 - 2x + 1 > \frac{4}{49}$$

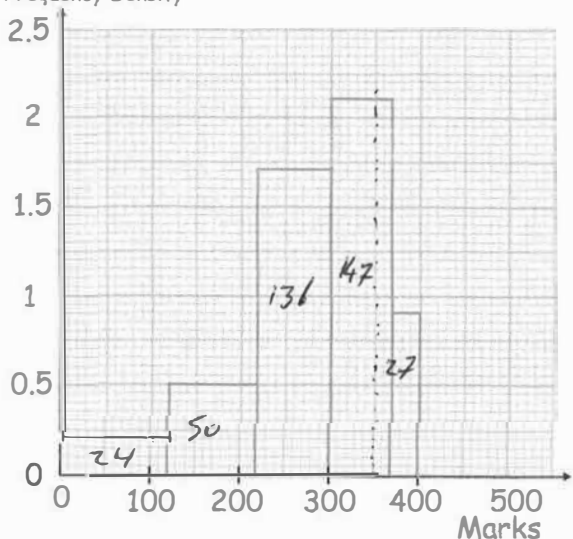
$$x^2 - 2x + \frac{45}{49} > 0$$

$$49x^2 - 98x + 45 > 0$$

$$(7x-5)(7x-9)$$



$$x < \frac{5}{7} \text{ or } x > \frac{9}{7}$$

13th July	
<p>Simplify</p> $\frac{x^3 - x}{x + 2} \div \frac{x(x-1)(x+1)}{x^2 - 5x - 14}$	<p style="text-align: right;">Corbettmaths</p> $\frac{x(x-1)(x+1)}{x+2} \times \frac{(x+2)(x-7)}{x(x-1)}$ $(x+1)(x-7)$
<p>Express <math>\left(\frac{1}{\sqrt{5}}\right)^5</math> in the form <math>\frac{\sqrt{a}}{b}</math></p>	$\frac{\sqrt{5}}{125}$
<p>Given that</p> $x^2 : (x + 6) = 1 : 2$ <p>Find the possible values of x</p>	$2x^2 = x + 6$ $2x^2 - x - 6 = 0$ $(2x + 3)(x - 2) = 0$ $x = -\frac{3}{2} \text{ or } x = 2$
<p>Frequency Density</p>  <p style="text-align: center;">Total 384</p>	<p>Miss Kelly wants to draw a pie chart to represent the grades obtained by the students.</p> <p>If a student scored 350 marks or higher, they obtained a grade 9.</p> <p>What size should the angle of the sector for grade 9 be in her pie chart?</p> $\frac{69}{384} \times 360 = 64.6875^\circ$

14th July



Corbettmaths

$$f(x) = \frac{4x}{9} - 8$$

$$y = \frac{4x}{9} - 8$$

Find

$$f^{-1}(-10)$$

$$y + 8 = \frac{4x}{9}$$

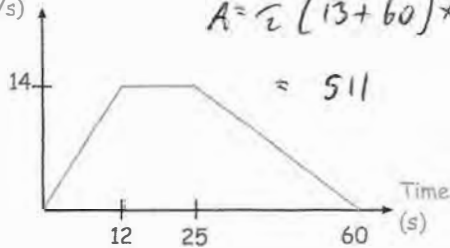
$$4x = 9y + 72$$

$$x = \frac{9y + 72}{4}$$

$$f^{-1}(x) = \frac{9x + 72}{4}$$

$$f^{-1}(-10) = \frac{-90 + 72}{4}$$

$$= -4.5$$

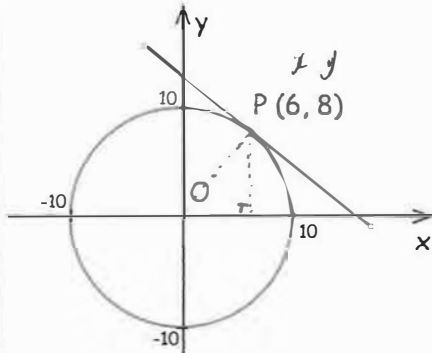
Speed  
(m/s)

$$A = \frac{1}{2} (13 + 60) \times 14$$

$$= 511$$

The graph shows the speed of a bicycle between two houses. Calculate the distance between the houses.

511m



Write down the equation of the circle

$$x^2 + y^2 = 10^2$$

or

$$x^2 + y^2 = 100$$

Here is a circle, centre O, and the tangent to the circle at the point (6, 8).

$$\text{gradient of } OP = \frac{4}{3}$$

$$\text{gradient of tangent} = -\frac{3}{4}$$

Find the equation of the tangent at the point P.

$$y = -\frac{3}{4}x + c$$

$$8 = -\frac{9}{2} + c$$

$$c = 12.5$$

$$y = -\frac{3}{4}x + 12.5$$

Work out

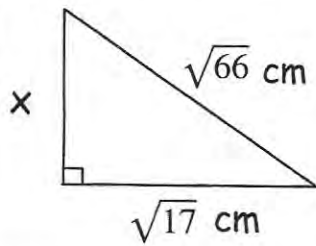
$$27^{-\frac{2}{3}} \div 0.25$$

$$\frac{1}{9} \div \frac{25}{100} = \frac{11}{25}$$





Find x



$$x^2 + (\sqrt{17})^2 = (\sqrt{66})^2$$

$$x^2 + 17 = 66$$

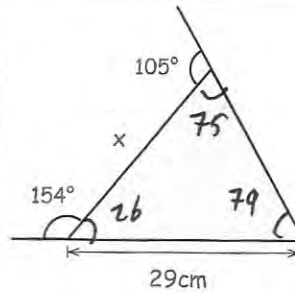
$$x^2 = 49$$

$$x = 7 \text{ cm}$$

Find the length of the side, x.

$$\frac{x}{\sin 79} = \frac{29}{\sin 75}$$

$$x = 29 \cdot 471 \text{ cm}$$



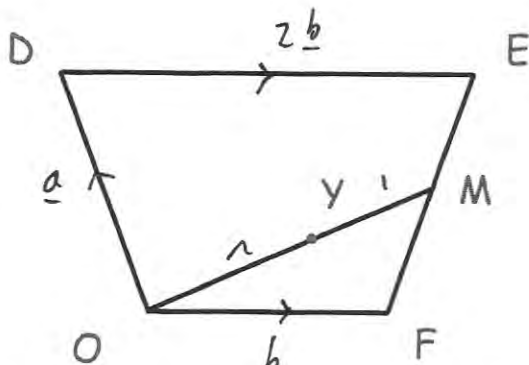
Factorise

$$2x^2 + 11xy + 15y^2$$

$$(2x + 5y)(x + 3y)$$

ODEF is a quadrilateral

$$\vec{OD} = \mathbf{a} \quad \vec{OF} = \mathbf{b} \quad \vec{DE} = 2\mathbf{b}$$



$$\vec{EF} = -\mathbf{a} - \mathbf{b} \quad \vec{EM} = -\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$$

$$\vec{DF} = -\mathbf{a} + \mathbf{b} \quad \vec{OM} = \frac{1}{2}\mathbf{a} + \frac{3}{2}\mathbf{b}$$

M is the midpoint of EF

Y is a point on OM such that

$$OY:YM = n : 1$$

DYF is a straight line.

Work out the value of n

$$\vec{DY} = -\mathbf{a} + \frac{n}{n+1} \left( \frac{1}{2}\mathbf{a} + \frac{3}{2}\mathbf{b} \right)$$

$$\vec{DY} = \frac{-n-2}{2n+2} \mathbf{a} + \frac{3n}{2n+2} \mathbf{b}$$

$$\text{Since } \vec{DF} = -\mathbf{a} + \mathbf{b}$$

$$\frac{-n-2}{2n+2} = \frac{3n}{2n+2}$$

$$-n-2 = 3n$$

$$n = 1$$

$$\vec{OY} = \frac{1}{n+1} \left( \frac{1}{2}\mathbf{a} + \frac{3}{2}\mathbf{b} \right)$$



Write as a fraction

$$0.2\dot{8}$$

$$x = 0.2888\dots$$

$$10x = 2.888\dots$$

$$100x = 28.888\dots$$

$$90x = 26$$

$$x = \frac{26}{90}$$

$$x = \frac{13}{45}$$

$$f(x) = \frac{ax + 3}{4}$$

Given

$$f(7) = 6$$

Find a

$$\frac{7a+3}{4} = 6$$

$$7a+3 = 24$$

$$7a = 21$$

$$a = 3$$

$$3^x = 9\sqrt{3} \quad \text{and} \quad 3^y = \frac{1}{\sqrt{3}}$$

Work out  $3^{x-y}$ 

$$3^3 = 27$$

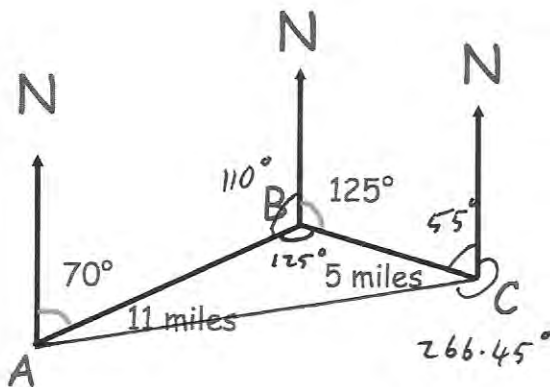
$$3^x = 3^2 \times 3^{\frac{1}{2}} = 3^{2\frac{1}{2}}$$

$$3^y = \frac{1}{3^{\frac{1}{2}}} = 3^{-\frac{1}{2}}$$

$$x = 2\frac{1}{2} \quad y = -\frac{1}{2}$$

$$x - y = 3$$

Shown are three towns, Antrim, Ballyclare and Carrickfergus.



Find the bearing of Antrim from Carrickfergus.

$$AC^2 = 5^2 + 11^2 - 2 \times 5 \times 11 \times \cos 125$$

$$AC = 14.46006 \text{ miles}$$

$$\frac{\sin 125}{14.46} = \frac{\sin x}{11}$$

$$x = 38.546^\circ$$

$$266.45^\circ$$



Find the nth term for the sequence

0 6 16 30 48  
 6 10 14 18  
 4 4 4

$a=2 \quad b=0 \quad c=-2$

$2n^2 - 2$

Height (x cm)	Frequency
$0 < x \leq 10$	3
$10 < x \leq 20$	7
$20 < x \leq 30$	12 *
$30 < x \leq 40$	31
$40 < x \leq 50$	27 *

80

The table shows the heights of some plants in a greenhouse

$LQ : 20^m$

$UQ : 60^m$

Work out the interquartile range

LQ

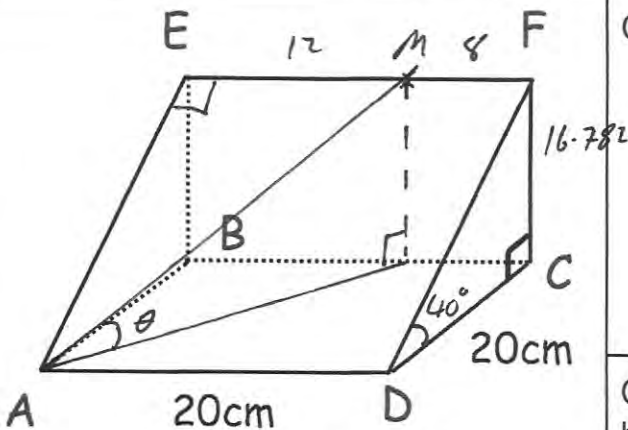
$20 + \frac{10}{12} \times 10 = 28.3$

UQ

$40 + \frac{7}{27} \times 10 = 42.592$

$42.592 - 28.3 = 14.259..$

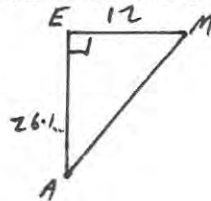
$IQR : 14.26 \text{ cm}$



Angle CDF =  $40^\circ$   $CF = \tan(40) \times 20 = 16.782$

M is a point on EF such that  $EM : MF = 3 : 2$   $OF = \frac{20}{2+3} = 26.10815$

Calculate the distance AM



$AM^2 = 12^2 + 26.10815^2$

$AM = 28.73 \text{ cm}$

Calculate the size of the angle between AM and the base of the prism.

$\sin \theta = \frac{16.78199262}{28.73386965}$

$\theta = 35.731^\circ$



$$g(x) = \frac{2x - 9}{5}$$

$$y = \frac{2x - 9}{5}$$

Find

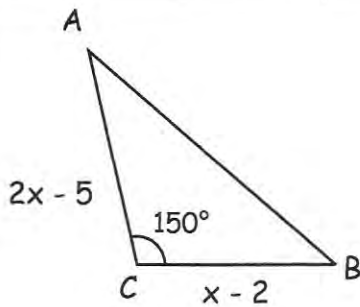
$$5y = 2x - 9$$

$g^{-1}(x)$

$$5y + 9 = 2x$$

$$x = \frac{5y + 9}{2}$$

$$g^{-1}(x) = \frac{5x + 9}{2}$$



Write an expression for the area of the triangle.

$$\frac{1}{2} (x-2)(2x-5) \sin 150$$

$$\sin 150 = \frac{1}{2}$$

$$\frac{1}{2} (x-2)(2x-5) \times \frac{1}{2}$$

$$\frac{1}{4} (x-2)(2x-5)$$

Given the area of the triangle is greater than  $16.5\text{cm}^2$ , show that

$$2x^2 - 9x - 56 > 0$$

$$\frac{1}{4} (x-2)(2x-5) > 16.5 \quad \times 4$$

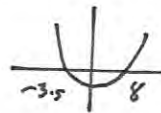
$$(x-2)(2x-5) > 66$$

$$2x^2 - 9x + 10 > 66$$

$$2x^2 - 9x - 56 > 0$$

Find the possible range of x.

$$(2x + 7)(x - 8)$$



$$x < -3.5 \quad x > 8$$

Shown is a sketch of the circle with equation  $x^2 + y^2 = 25$

$$r = 5$$

The circle is translated 3 squares downwards.

Sketch the circle and label the coordinates where the circle crosses both the x-axis and y-axis.

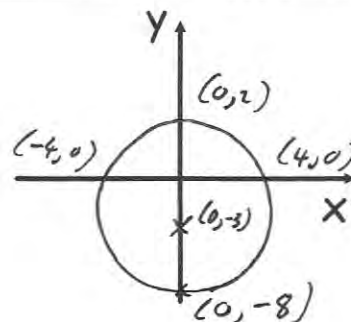
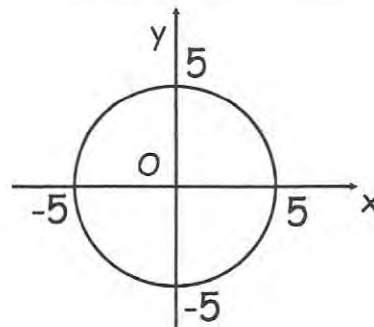
$$x^2 + 3^2 = 25$$

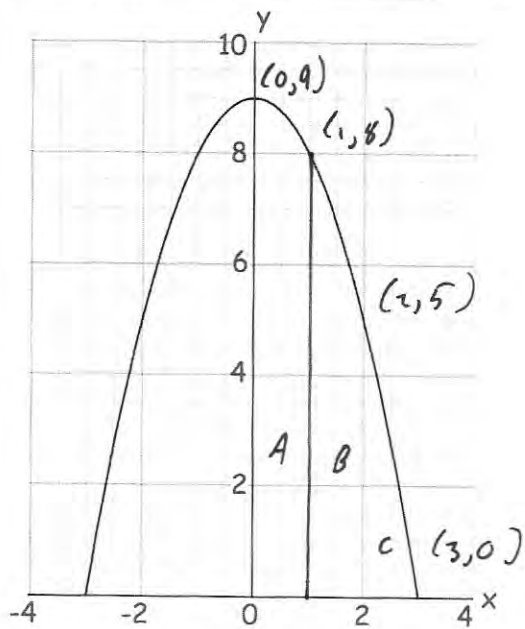
$$x^2 + 9 = 25$$

$$x^2 = 16$$

$$x = \pm 4$$

$$(4, 0) \quad (-4, 0)$$





Here is a sketch of  $y = 9 - x^2$   
 The graph is used to model the cross section of a tunnel.  
 Calculate an estimate of the area under the graph.

A)  $\frac{1}{2}(9+8) \times 1 = 8.5$

B)  $\frac{1}{2}(8+5) \times 1 = 6.5$

C)  $\frac{1}{2} \times 1 \times 5 = 2.5$

$\begin{array}{r} + \\ 17.5 \end{array}$

$\begin{array}{r} 17.5 \times 2 = 35 \\ \hline \end{array}$

Find the nth term of

-12 -9 -4 3 ...  
 $\begin{array}{cccc} & 3 & & 5 & & 7 \\ & & 2 & & 2 & \end{array}$

$2a = 2$

$a = 1$

$3a + b = 3$

$3 + b = 3$

$b = 0$

$a + b + c = -12$

$1 + 0 + c = -12$

$c = -13$

$n^2 - 13$

Solve the simultaneous equations

$y = 9x^2 + 11x + 3$

$5x - y + 2 = 0$

$y = 5x + 2$

$5x + 2 = 9x^2 + 11x + 3$

$0 = 9x^2 + 6x + 1$

$0 = (3x + 1)(3x + 1)$

$x = -\frac{1}{3}$

$y = \frac{1}{3}$

Simplify fully

$\frac{3x^2 + 20x - 7}{16x^2 - 1} \div \frac{x + 7}{4x + 1}$

$\frac{(3x-1)(x+7)}{(4x-1)(4x+1)} \times \frac{4x+1}{x+7}$

$\frac{3x-1}{4x-1}$

20th July

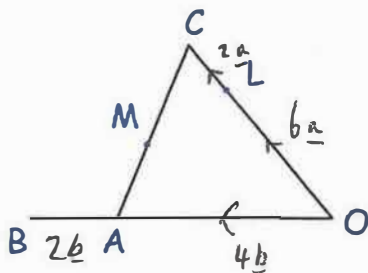


Corbettmaths

Convert the following recurring decimal to a fraction

$$\begin{aligned} \dots & & x &= 1.646464\dots \\ 1.64 & & 100x &= 164.6464\dots \\ & & 99x &= 163 \end{aligned}$$

$$x = \frac{163}{99}$$



$$\begin{aligned} \vec{OC} &= 8a & \vec{CA} &= -8a + 4b \\ \vec{OA} &= 4b & \vec{CM} &= -4a + 2b \\ \vec{AB} &= 2b & & \\ \vec{OL} &= 6a & & \end{aligned}$$

M is the midpoint of AC

Work out the vector

$$\begin{aligned} \vec{LM} &= \vec{LC} + \vec{CM} \\ &= 2a + (-4a + 2b) \\ &= -2a + 2b \end{aligned}$$

Show that L, M and B lie on a straight line.

$$\begin{aligned} \vec{MB} &= -4a + 2b + 2b = -4a + 4b \\ \vec{MB} &= 2\vec{LM} \quad \therefore \text{parallel} \\ &\text{as both vectors pass through M,} \\ &\text{they are co-linear.} \end{aligned}$$

Express as a single fraction

$$\begin{aligned} \frac{b}{a} - \frac{a-1}{b+1} &= \frac{b^2+b}{a(b+1)} - \frac{a^2-a}{a(b+1)} \\ &= \frac{b^2+b-a^2+a}{a(b+1)} \end{aligned}$$

Write down the coordinates of the minimum point on the curve

$$y = x^2 - 6x - 20$$

$$\begin{aligned} (x-3)^2 - 9 - 20 \\ (x-3)^2 - 29 \end{aligned}$$

$$(3, -29)$$



21st July

Higher Plus 5-a-day



Corbettmaths

Write 128 in the form  $4^n$

$$(2^2)^n = 2^7$$

$$2^{2n} = 2^7$$

$$2n = 7$$

$$n = \frac{7}{2}$$

$$4^{\frac{7}{2}}$$

The line AB has equation  $4x + 3y = 9$

Find an equation of the line perpendicular to the line AB that passes through the point

$(-3, -1)$

$x$   $y$

$$3y = -4x + 9$$

$$y = -\frac{4}{3}x + 3$$

$$y = \frac{3}{4}x + c$$

$$-1 = -\frac{9}{4} + c$$

$$c = \frac{5}{4}$$

$$y = \frac{3}{4}x + \frac{5}{4}$$

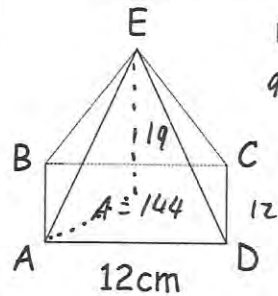
Shown is a square based pyramid. E is directly over the centre of ABCD. The volume of the pyramid is  $912\text{cm}^3$

Find the length of AE.

$$AC^2 = 12^2 + 12^2$$

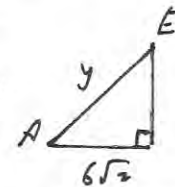
$$AC = 12\sqrt{2}$$

$$\boxed{20.81\text{cm}}$$



$$V = \frac{1}{3}(144) \times h$$

$$912 = 48h \quad h = 19\text{cm}$$



$$y^2 = 19^2 + (12\sqrt{2})^2$$

$$y = 20.80865$$

The equation  $x^3 - 2x^2 + 19 = 0$  has a root in the interval  $(-3, -2)$

Use an appropriate iteration formula to find an approximate to 1 decimal place for the root of

$$x^3 - 2x^2 + 19 = 0$$

in the interval  $(-3, -2)$

$$x^3 = 2x^2 - 19$$

$$x = \sqrt[3]{2x^2 - 19}$$

$$x_{n+1} = \sqrt[3]{2(x_n)^2 - 19}$$

$$x_0 = -2$$

$$x_1 = -2.223980091$$

$$x_2 = -2.08835773$$

$$x_3 = -2.174183353$$

$$x_4 = -2.121313821$$

$$x_5 = -2.154438665$$

$$x_6 = -2.133900886$$

$$x_7 = -2.146718196$$

$$x_8 = -2.1387515613$$

$$x_9 = -2.143715813$$

$$x_{10} = -2.140629311$$

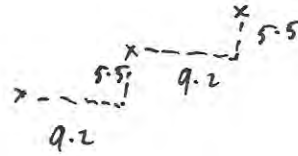
$$\boxed{x = -2.1}$$



AB is a straight line

The coordinates of A are  $(-1, -7)$   
The midpoint of AB is  $(8.2, -1.5)$

Work out the coordinates of B



$(17.4, 4)$

The curve A with equation  $y = f(x)$  is transformed to curve B with equation  $y = -f(x + 2)$

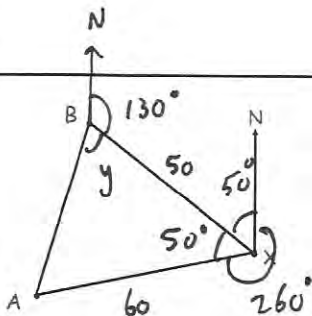
The point on A with coordinates  $(2, 1)$  is mapped to the point P on B

Find the coordinates of P

$(0, -1)$

Write down the exact value of  $\sin 60^\circ$

$$\frac{\sqrt{3}}{2}$$



Ship A is 60km from X on a bearing of  $260^\circ$

Ship B is 50km from X on a bearing of  $310^\circ$

Calculate the distance between A and B.

$$AB^2 = 50^2 + 60^2 - 2 \times 50 \times 60 \times \cos 50^\circ$$

$$AB = 47.36 \text{ km}$$

Calculate the bearing of A from B.

$$\frac{\sin y}{60} = \frac{\sin 50^\circ}{47.36}$$

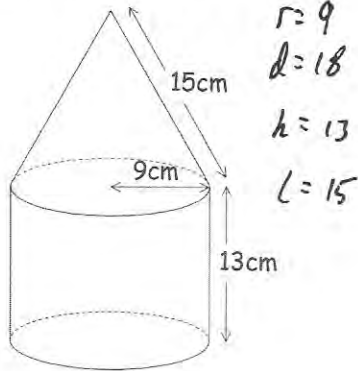
$$y = 76.032^\circ$$

$206^\circ$



A cylinder and a cone are joined together to make a solid.  
The cylinder has a radius of 9cm and height of 13cm.  
The cone has a slant height of 15cm.

$$\pi r l + \pi d h + \pi r^2$$



Find the total surface area of the solid.

$$(\pi \times 9 \times 15) + (\pi \times 18 \times 13) + (\pi \times 9^2) = 1413.716694..$$

$$1413.72 \text{ cm}^2$$

Express  $3x^2 + 24x - 1$  in the form  $a(x + b)^2 + c$

$$3(x^2 + 8x) - 1$$

$$3[(x+4)^2 - 4^2] - 1$$

$$3[(x+4)^2 - 16] - 1$$

$$3(x+4)^2 - 48 - 1$$

$$3(x+4)^2 - 49$$

A circle has equation

$$x^2 + y^2 = 0.25$$

Write down the length of its diameter

$$r = \sqrt{0.25} = \frac{1}{2}$$

$$d = 2 \times \frac{1}{2} = 1$$

A clock has two hands.  
A minute hand which is 7cm long and  
an hour hand which is 5cm long.

Find the distance between the tips of  
the two hands at 7:20am  $100^\circ$

$$d^2 = 5^2 + 7^2 - 2 \times 5 \times 7 \times \cos 100$$

$$d = 9.282 \text{ cm}$$



Make  $y$  the subject of

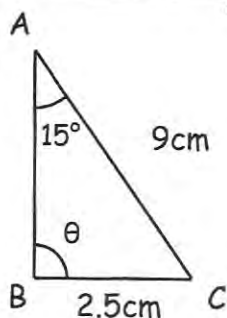
$$\frac{x-3y}{y+x} = p \quad \frac{x-3y}{y+x} = p$$

$$x-3y = p(y+x)$$

$$x-3y = py+px$$

$$x-px = y(p+3)$$

$$y = \frac{x-px}{p+3}$$



$$\frac{\sin \theta}{9} = \frac{\sin 15}{2.5}$$

$$\sin \theta = 0.9317\dots$$

Find the two possible values of  $\theta$

$$\theta = 68.71^\circ$$

or

$$\theta = 111.29^\circ$$

Prove that when any odd integer is squared, the result is always one more than a multiple of 8.

$$(2n+1)^2 = 4n^2 + 4n + 1$$

$$4n(n+1) + 1$$

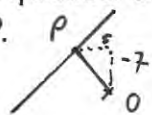
$n(n+1)$  is even as it is the product of two consecutive numbers.

$4n(n+1)$  is a multiple of 8.  
even

$\therefore 4n(n+1) + 1$  is one more than a multiple of 8.

The point  $P(-5, 7)$  is a point on the circle  $x^2 + y^2 = 74$

Find the equation of the tangent to the circle at P.



gradient of  $OP$   
 $= -\frac{7}{5}$

$$y = \frac{5}{7}x + c$$

$$7 = -\frac{25}{7} + c$$

$$c = \frac{74}{7}$$

$$y = \frac{5}{7}x + \frac{74}{7}$$

Find the coordinates of the point of intersection of this tangent and the line  $y = x$

$$x = \frac{5}{7}x + \frac{74}{7}$$

$$\frac{2}{7}x = \frac{74}{7}$$

$$x = 37$$

$$y = 37$$

$$(37, 37)$$



Make  $f$  the subject of

$$x = \frac{2f-3}{f-1}$$

$$x(f-1) = 2f-3$$

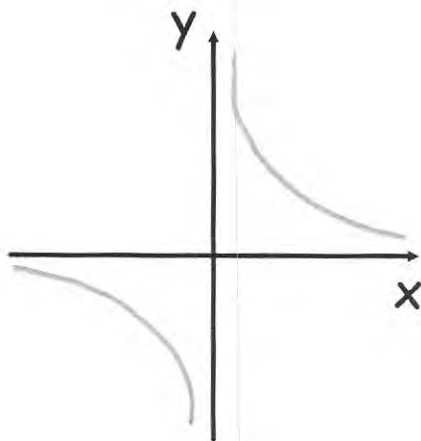
$$fx - x = 2f-3$$

$$fx - 2f = x-3$$

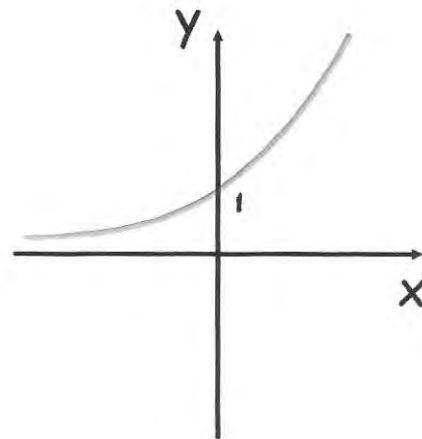
$$f(x-2) = x-3$$

$$f = \frac{x-3}{x-2}$$

Sketch  $y = \frac{1}{x}$



Sketch  $y = 4^x$



The histogram shows the speeds of some cars while they travelled along a road.

156 cars were travelling less than 10mph.

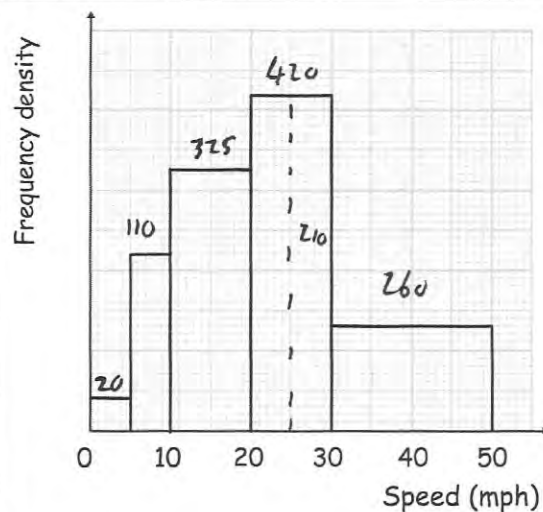
Estimate how many cars were travelling at a speed greater than 25mph.

$$110 + 20 = 130 \text{ squares}$$

$$156 \div 130 = 1.2 \text{ cars per square}$$

$$210 + 260 = 470$$

$$470 \times 1.2 = 564 \text{ cars}$$





Make  $x$  the subject of  $y = \sqrt[3]{x^5}$

$$y^3 = x^5$$

$$x = \sqrt[5]{y^3}$$

Simplify

$$\sqrt{48} + \sqrt{300}$$

$$4\sqrt{3} + 10\sqrt{3} = 14\sqrt{3}$$

The curve  $y = x^2 - 3x - 4$  is reflected in the  $x$ -axis.

Find the equation of the new curve.

$$y = -x^2 + 3x + 4$$

Solve the simultaneous equations

$$2x = 6 - y \quad y = 6 - 2x$$

$$x^2 + y^2 = 8 \quad x^2 + (6 - 2x)^2 = 8$$

$$x^2 + 36 - 24x + 4x^2 = 8$$

$$5x^2 - 24x + 28 = 0$$

$$(5x - 14)(x - 2) = 0$$

$$x = \frac{14}{5} \quad x = 2$$

$$y = \frac{2}{5} \quad y = 2$$

The  $n$ th term of a sequence is  $n^2 - 4n + 5$

By using completing the square, show that every term is positive.

$$(n - 2)^2 - 4 + 5$$

$$(n - 2)^2 + 1$$

$$(n - 2)^2 \geq 0$$

$$\therefore (n - 2)^2 + 1 > 0$$





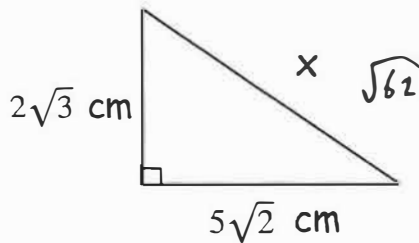
The square of  $w$  is 6

Write down the value of  $w^3$

$$w^2 = 6$$

$$w = \pm\sqrt{6}$$

$$w^3 = 6\sqrt{6} \text{ or } -6\sqrt{6}$$



Find  $x$

$$x^2 = (2\sqrt{3})^2 + (5\sqrt{2})^2$$

$$x^2 = 62$$

$$x = \sqrt{62} \text{ cm}$$

Scott has drawn  $y = x^2 - 4x - 8$  and  $y = 3x + 6$

Find the quadratic equation whose solutions are the  $x$ -coordinates of the points of intersection of  $y = 3x + 6$  and  $y = x^2 - 4x - 8$

$$x^2 - 4x - 8 = 3x + 6$$

$$x^2 - 7x - 14 = 0$$

Solve

$$\frac{11}{(x-1)(x+4)} + \frac{5}{x-1} = 1$$

$$\frac{11}{(x-1)(x+4)} + \frac{5(x+4)}{(x-1)(x+4)} = 1$$

$$5x+31 = (x-1)(x+4)$$

$$5x+31 = x^2+3x-4$$

$$0 = x^2 - 2x - 35$$

$$(x-7)(x+5) = 0$$

$$x = 7 \text{ or } x = -5$$

A triangle has side lengths of 9cm, 10cm and 5cm.

Find the size of the largest angle.

$$86.18^\circ$$

$$\cos A = \frac{5^2 + 9^2 - 10^2}{2 \times 5 \times 9}$$



Here are the first 5 terms of a quadratic sequence

24 30 38 48 60

Find an expression, in terms of  $n$ , for the  $n$ th term of this quadratic sequence

24 30 38 48

6 8 10

2 2

$$n^2 + 3n + 20$$

$$a = 1$$

$$b = 3$$

$$c = 20$$

A circle has equation  $x^2 + y^2 = 196$

Work out the length of the diameter.

$$r = 14$$

$$d = 2 \times 14 = 28$$

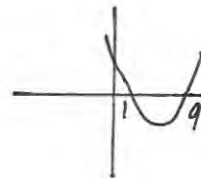
Solve

$$x^2 - 10x + 9 > 0$$

$$x^2 + 9 > 10x$$

$$(x-9)(x-1)$$

$$x=9 \quad x=1$$



$$x < 1 \quad \text{or} \quad x > 9$$

Clive has a cone of base diameter 20cm.

He removes a cone diameter 12cm from the top of his cone to leave a frustum.

The height of the frustum is 6cm.

Find the volume of the frustum

$$\frac{1}{3}(\pi \times 10^2) \times 15 - \frac{1}{3}(\pi \times 6^2) \times 9$$

$$= 1231.5 \text{ cm}^3$$

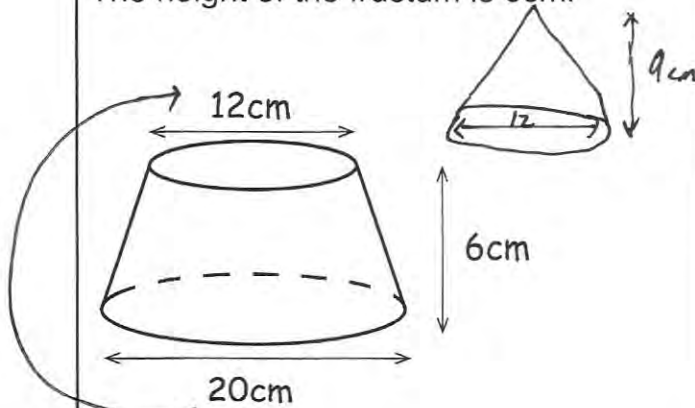
The frustum has the same volume as a sphere.

Find the radius of the sphere.

$$\frac{4}{3}\pi r^3 = 1231.50432$$

$$r^3 = 294$$

$$r = 6.6494 \text{ cm}$$





Expand and simplify

$$(2x - 1)(2x - 3)(x + 5)$$

$$(4x^2 - 6x - 3x + 3)(x + 5)$$

$$(4x^2 - 9x + 3)(x + 5)$$

$$4x^3 - 8x^2 + 3x + 20x^2 - 40x + 15$$

$$4x^3 + 12x^2 - 37x + 15$$

Point A has coordinates (9, 7)  
 Point B has coordinates (14, -8)  
 (11.5, -0.5)  
 Find the equation of the line perpendicular to AB, that passes through the midpoint of AB.

$$\text{gradient of } AB = -3$$

$$y = \frac{1}{3}x + c$$

$$-0.5 = \frac{23}{6} + c$$

$$c = -\frac{13}{3}$$

$$y = \frac{1}{3}x - \frac{13}{3}$$

A group of scientists want to estimate the number of eels in a lake. They catch and ring 400 eels. They return the 400 eels to the lake. They then catch 700 eels. Of these, 16 are ringed.

Estimate the number of eels in the lake.

$$\frac{400}{N} = \frac{16}{700}$$

$$280000 = 16N$$

$$N = 17500$$

There are only yellow and blue counters in a box. A counter is to be taken at random from the box. The probability that the counter is blue is  $\frac{2}{5}$ .

The counter is returned to the box. 4 more yellow counters and 1 blue counter is added to the box.

The probability of a yellow counter is now  $\frac{8}{13}$ .

Find the number of yellow counters and blue counters that were in the bag originally.

$$m = \text{yellow} \quad n = \text{blue} \quad \text{total} = m + n$$

$$\frac{n}{m+n} = \frac{2}{5}$$

$$\frac{m+4}{m+n+5} = \frac{8}{13}$$

$$5n = 2m + 2n$$

$$3n = 2m$$

$$13m + 52 = 8m + 8n + 40$$

$$5m + 12 = 8n$$

$$2.5(2m) + 12 = 8n$$

$$2.5(3n) + 12 = 8n$$

$$7.5n + 12 = 8n$$

$$0.5n = 12$$

$$n = 24$$

$$\text{blue} = 24$$

$$\text{yellow} = 36$$



$$W = \frac{a^3}{4c}$$

max a  
min c

$a = 15.4$  correct to 1 decimal place  
 $c = 20$  correct to 2 significant figures.

Find the upper bound for W

$$W = \frac{15.45^3}{4 \times 19.5}$$

$$= 47.28145 \dots$$

Write as a single fraction

$$\frac{1-x}{x+7} - \frac{4}{x-2}$$

$$\frac{-x^2 - x - 30}{(x+7)(x-2)}$$

$$= \frac{(1-x)(x-2) - 4(x+7)}{(x+7)(x-2)}$$

$$= \frac{x-2-2^2+2x-4x-28}{(x+7)(x-2)}$$

$$= \frac{-x^2 - x - 30}{(x+7)(x-2)}$$

Given

$$x^2 : (10x + 48) = 1 : 3$$

Find the possible values of x

$$3x^2 = 10x + 48$$

$$3x^2 - 10x - 48 = 0$$

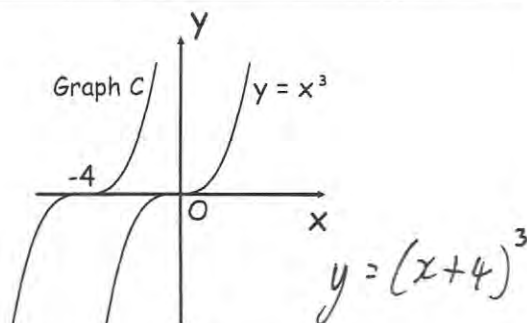
$$(3x+8)(x-6) = 0$$

$$x = -\frac{8}{3} \quad \text{or} \quad x = 6$$

Shown is the graph of  $y = x^3$  and of graph C.

Write down the equation of Graph C

$$y = (x+4)^3$$



$(3, -4)$  is a point on the graph with equation  $y = (x+7)^2 + a$

$$-4 = 100 + a$$

Find the coordinates of the turning point.

$$(-7, -104)$$

$$a = -104$$

31st July

Higher Plus 5-a-day



Corbettmaths

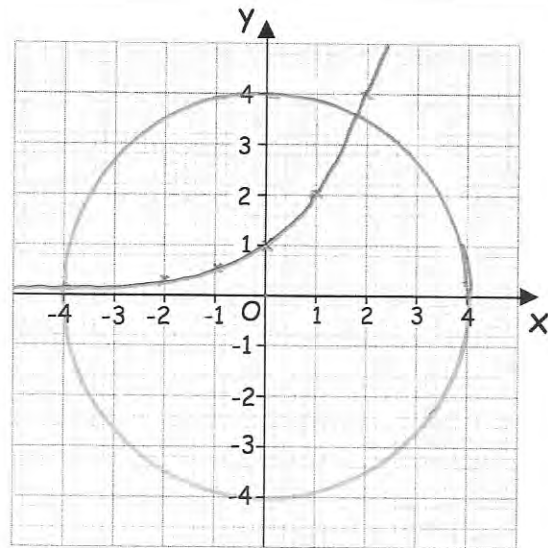
A cylinder has a height of 18cm and volume of 1715cm<sup>3</sup>.

Work out the surface area of the cylinder.

$$\begin{aligned} \text{Area of circle} &= \frac{1715}{18} \\ &= 95.27 \end{aligned}$$

$$\begin{aligned} r &= 5.507073506 \text{ cm} \\ \pi \times d \times h &= 622.83534 \dots \text{ cm}^2 \\ 622.835 \dots + 95.27 + 95.27 \\ &= 813.39 \text{ cm}^2 \end{aligned}$$

Draw  $x^2 + y^2 = 16$



By sketching  $y = 2^x$ , show that the graphs of  $x^2 + y^2 = 16$  and  $y = 2^x$  have two points of intersection.

Solve

$$\frac{x+1}{x-3} + \frac{2}{x-4} = 2$$

Give your solutions to 3 significant figures

$$\frac{(x+1)(x-4) + 2(x-3)}{(x-3)(x-4)} = 2$$

$$\frac{x^2 - 3x - 4 + 2x - 6}{x^2 - 7x + 12} = 2$$

$$x^2 - x - 10 = 2x^2 - 14x + 24$$

$$0 = x^2 - 13x + 34$$

$$a=1 \quad b=-13 \quad c=34$$

$$x = \frac{13 \pm \sqrt{23}}{2}$$

$$x = 9.37 \quad \text{or} \quad x = 3.63$$