Name:

GCSE Further Maths

Optimising Problems

Corbettmaths

Ensure you have: Pencil, Pen, Calculator

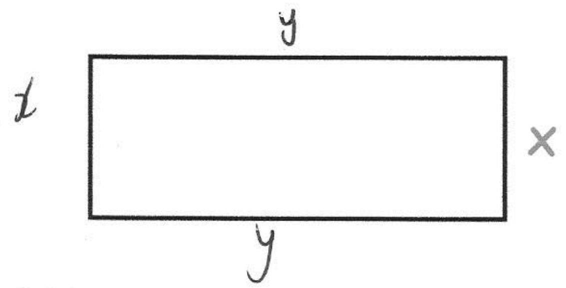
Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic

www.corbettmaths.com/gcse-further-maths

1. A farmer creates a pen for his chickens.



The width of the field is x metres.
The perimeter of the field is 100 metres.

(a) Show that the length of the rectangle is $50 - x$ metres

$x + x + y + y = 100$

$2x + 2y = 100$

$2y = 100 - 2x$

$y = 50 - x$

(1)

(b) Show that the area of the field is $A = 50x - x^2$

$A = xy$

$A = x(50 - x)$

$A = 50x - x^2$

(1)

(c) Find the value of x for which A is a maximum and show it is a maximum.

$\frac{dA}{dx} = 50 - 2x$

$50 - 2x = 0$

$2x = 50$

$x = 25$

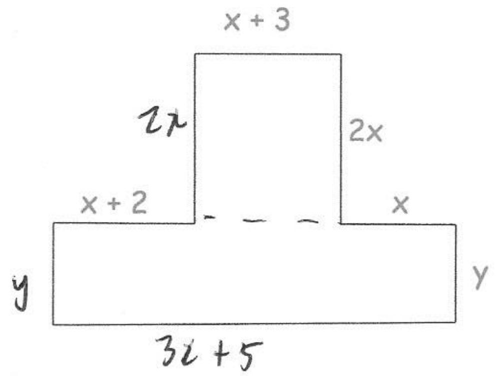
$\frac{d^2A}{dx^2} = -2$

since $\frac{d^2A}{dx^2} < 0$, it is a maximum

$x = 25$

(5)

2. The shape below is made from two rectangle.



The perimeter of the shape is 100cm.

(a) Show that $y = 45 - 5x$

$2y + 3x + 5 + 3x + 5 + 4x = 100$

$2y + 10x + 10 = 100$

$2y + 10x = 90$

$2y = 90 - 10x$

$y = 45 - 5x$

(2)

The area of the shape is $A cm^2$

(b) Show that $A = 225 + 116x - 13x^2$

$A = (45 - 5x)(3x + 5) + 2x(x + 3)$

$= 135x + 225 - 15x^2 - 25x + 2x^2 + 6x$

$= 225 + 116x - 13x^2$

(2)

(c) Find the value of x for which A is a maximum and show it is a maximum.

$\frac{dA}{dx} = 116 - 26x$

$\frac{d^2A}{dx^2} = -26$

Since $\frac{d^2A}{dx^2} < 0$, it is a max

$116 - 26x = 0$

$26x = 116$

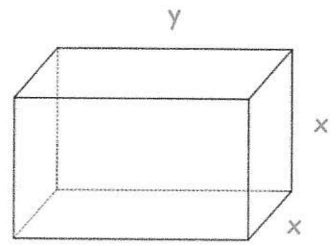
$x = \frac{58}{13}$

$x = 4.461..$

4.461..

(5)

3. Shown below is a metal box in the shape of a cuboid.



The volume of the box is 80cm³

(a) Show that $y = \dfrac{80}{x^2}$

$x^2 y = 80$

$y = \dfrac{80}{x^2}$

(2)

(b) Show that the area of metal to make the box is given by

$$A = 2x^2 + \frac{320}{x}$$

$A = x^2 + x^2 + 4xy$

$A = 2x^2 + 4xy$

$A = 2x^2 + 4x\left(\dfrac{80}{x^2}\right)$

$A = 2x^2 + \dfrac{320}{x}$

or

$A = 2x^2 + 320x^{-1}$

(2)

(c) Find the value of x for which A is a minimum, and show it is a minimum.

$\dfrac{dA}{dx} = 4x - 320x^{-2}$

$\dfrac{dA}{dx} = 4x - \dfrac{320}{x^2}$

$0 = 4x - \dfrac{320}{x^2}$

$\dfrac{320}{x^2} = 4x$

$320 = 4x^3$

$80 = x^3$

$x = \sqrt[3]{80}$

$\dfrac{d^2A}{dx^2} = 4 + 640x^{-3}$

$= 4 + \dfrac{640}{x^3}$

when $x = 4.309..$

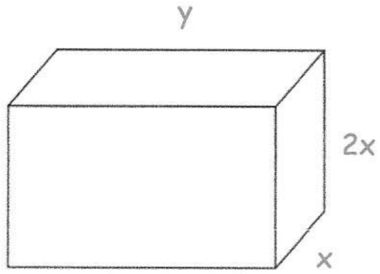
$\dfrac{d^2A}{dx^2} = 48.$

$\dfrac{d^2A}{dx^2} > 0$, $\therefore$ min

4.309

(6)

4. Shown below is a cuboid.



The surface area of the cuboid is 120cm².

(a) Show that $y = \dfrac{20}{x} - \dfrac{2x}{3}$

$2x^2 + 2x^2 + xy + xy + 2xy + 2xy = 120$

$4x^2 + 6xy = 120$

$6xy = 120 - 4x^2$

$y = \dfrac{120}{6x} - \dfrac{4x^2}{6x}$

$y = \dfrac{20}{x} - \dfrac{2x}{3}$

(3)

(b) Show that the volume of the cuboid is given by

$$V = 40x - \frac{4}{3}x^3$$

$V = 2x^2 y$

$V = 2x^2\left(\dfrac{20}{x} - \dfrac{2x}{3}\right)$

$V = \dfrac{40x^2}{x} - \dfrac{4x^3}{3}$

$V = 40x - \dfrac{4}{3}x^3$

(2)

(c) Find the value of x for which V is a maximum, and show it is maximum.

$\dfrac{dV}{dx} = 40 - 4x^2$

$0 = 40 - 4x^2$

$4x^2 = 40$

$x^2 = 10$

$x = \sqrt{10}$

$\dfrac{d^2V}{dx^2} = -8x$

when $x = \sqrt{10}$ $\dfrac{d^2V}{dx^2} = -8\sqrt{10}$

since $\dfrac{d^2V}{dx^2} < 0$, it is a max

$\sqrt{10}$ ....

or

3.162

(5)

(d) Use your answer to (c) to find the maximum volume of the cuboid

$\sqrt{10} \times 2\sqrt{10} \times \frac{4\sqrt{10}}{3}$

$= \frac{80\sqrt{10}}{3}$

84.33 cm²
to 2dp **(2)**

5. The volume of a container with a height of x, is given by

$$V = x(x-1)(9-x) \quad \text{where } 1 < x < 9$$

$v = x(9x - x^2 - 9 + x)$

(a) Find $\frac{dV}{dx}$

$v = x(10x - x^2 - 9)$

$v = 10x^2 - x^3 - 9x$

$\frac{dv}{dx} = 20x - 3x^2 - 9$

$20x - 3x^2 - 9$

(3)

(b) Hence find the value of x for which the volume is a maximum.
Give your answer to 1 decimal place.

$\frac{dv}{dx} = 20x - 3x^2 - 9$

$20x - 3x^2 - 9 = 0$

$a = -3$

$b = 20$

$c = -9$

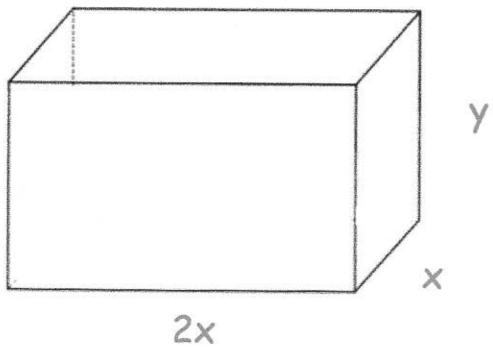
$x = \frac{10 \pm \sqrt{73}}{3}$

$x = 0.5$ or $x = 6.2$

6.2

(3)

6. An open-topped tank in the shape of a cuboid is shown below.



The surface area of the cuboid is 300cm²

(a) Show that $y = \dfrac{50}{x} - \dfrac{x}{3}$

$xy + xy + 2x^2 + 2xy + 2xy = 300$

$2xy + 2x^2 + 4xy = 300$

$2x^2 + 6xy = 300$

$6xy = 300 - 2x^2$

$y = \frac{50}{x} - \frac{x}{3}$

(3)

(b) Show that the volume of the tank is $V = 100x - \dfrac{2}{3}x^3$

$V = 2x^2\left(\frac{50}{x} - \frac{x}{3}\right)$

$= 100x - \frac{2x^3}{3}$

$= 100x - \frac{2}{3}x^3$

(3)

(c) Find the value of x for which V is a maximum

$\frac{dV}{dx} = 100 - 2x^2$

$0 = 100 - 2x^2$

$2x^2 = 100$

$x^2 = 50$

$x = \sqrt{50}$ or $5\sqrt{2}$ ....

(3)

(d) Show the answer is (c) is a maximum.

$\frac{d^2v}{dx^2} = -4x$ $\qquad \therefore \frac{d^2v}{dx^2} < 0$

when $x = \sqrt{50}$ $\quad \frac{d^2v}{dx^2} = -4\sqrt{50}$

so it is a max

(2)

(e) Find the maximum volume of the tank

$2\sqrt{50} \times \sqrt{50} \times \frac{10\sqrt{2}}{3}$

471.4 cm^3

(2)