Name:

GCSE Further Maths

Sketching Graphs

Corbettmaths

Ensure you have: Pencil, Pen, Calculator

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic

www.corbettmaths.com/gcse-further-maths

1. A curve is defined by the equation $y = x^2 - 4x - 45$

(i) Find the coordinates of the points where the curve meets the x-axis.

$y = c$

$0 = x^2 - 4x - 45$

$0 = (x - 9)(x + 5)$

$x = 9 \quad x = -5$

$(-5, 0)$ and $(9, 0)$

(2)

(ii) Find the coordinates of the turning point of the curve

$\frac{dy}{dx} = 2x - 4$

$0 = 2x - 4$

$2x = 4$

$x = 2$

$y = 2^2 - 4(2) - 45$

$y = 4 - 8 - 45$

$y = -49$

$(2, -49)$

(3)

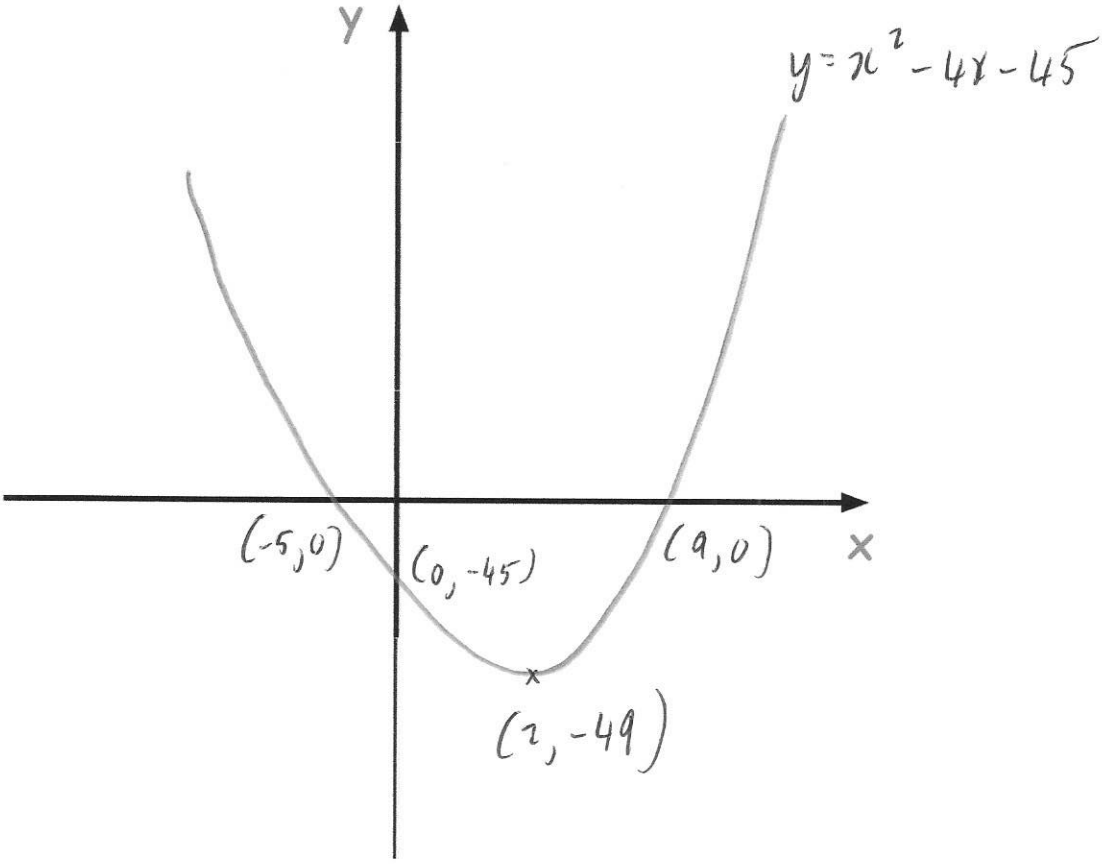
(iii) Using calculus, identify the turning point as either a maximum or minimum point.
You **must** show working to justify your answer.

$\frac{d^2y}{dx^2} = 2$

since $\frac{d^2y}{dx^2} > 0$ when $x = 2$, it is a minimum point.

(2)

(iv) Sketch the curve on the axes below.



(2)

2. A curve is defined by the equation $y = 2x^2 + 5x - 12$

(i) Find the coordinates of the points where the curve meets the x-axis. $\rightarrow y = 0$

$0 = 2x^2 + 5x - 12$

$0 = (2x - 3)(x + 4)$

$x = \frac{3}{2}$ or $x = -4$

$(-4, 0)$ & $(\frac{3}{2}, 0)$

(2)

(ii) Find the coordinates of the turning point of the curve

$\frac{dy}{dx} = 4x + 5$

$y = 2(-\frac{5}{4})^2 + 5(-\frac{5}{4}) - 12$

$y = -15.125$ (or $-\frac{121}{8}$)

$0 = 4x + 5$

$4x = -5$

$x = -\frac{5}{4}$

$(-\frac{5}{4}, -\frac{121}{8})$

(3)

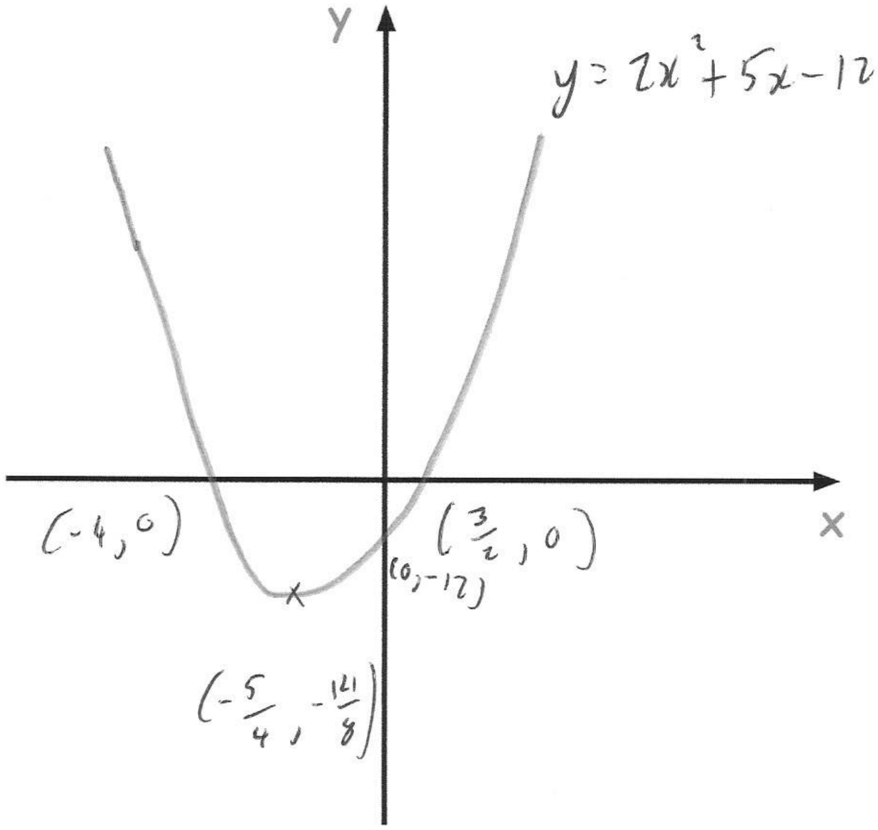
(iii) Using calculus, identify the turning point as either a maximum or minimum point.
You **must** show working to justify your answer.

$\frac{d^2y}{dx^2} = 4$

Since $\frac{d^2y}{dx^2} > 0$, it is a minimum point.

(2)

(iv) Sketch the curve on the axes below.



(2)

3. A curve is defined by the equation $y = (5 - x)(6 - x)$ $y = 30 - 11x + x^2$

(i) Find the coordinates of the points where the curve meets the x-axis.

$y = 0$

$0 = (5-x)(6-x)$

$x = 5$ & $x = 6$

$(5, 0)$ $(6, 0)$

$(5, 0)$ & $(6, 0)$ ....

(2)

(ii) Find the coordinates of the turning point of the curve

$y = 30 - 11x + x^2$

$\frac{dy}{dx} = -11 + 2x$

$0 = -11 + 2x$

$2x = 11$

$x = \frac{11}{2}$

$y = (5 - 5.5)(6 - 5.5)$

$= -0.5 \times 0.5$

$= -0.25$

$\left(\frac{11}{2}, -\frac{1}{4}\right)$ ....

(3)

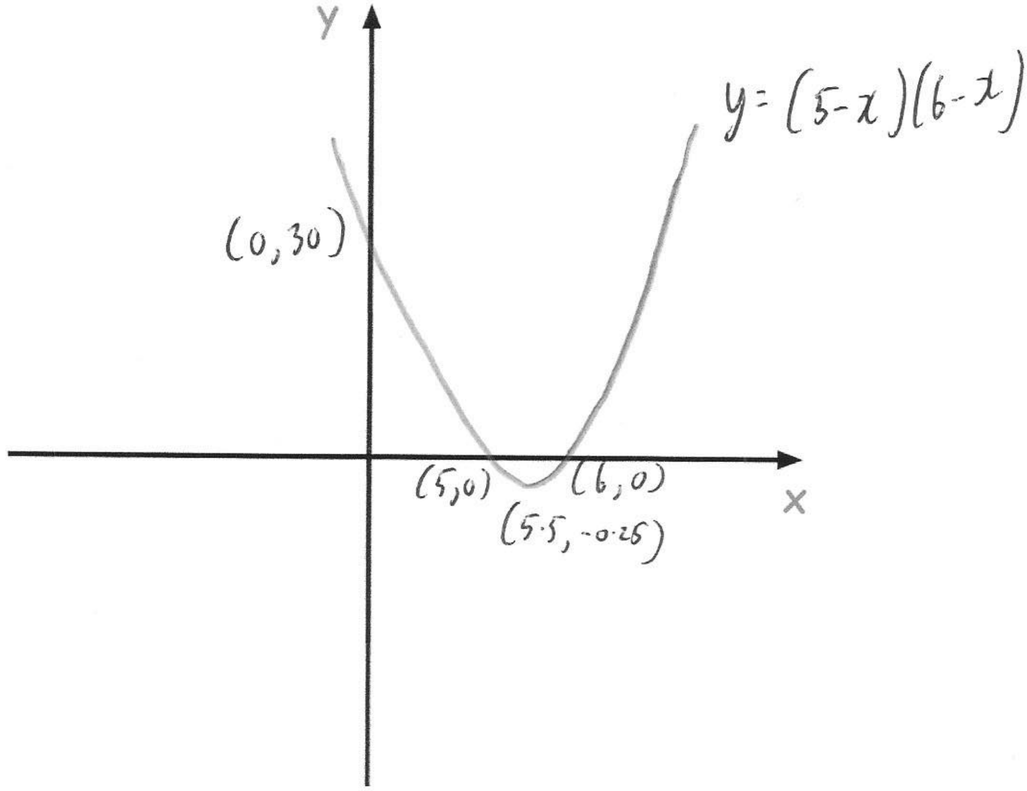
(iii) Using calculus, identify the turning point as either a maximum or minimum point.
You **must** show working to justify your answer.

$\frac{d^2y}{dx^2} = 2$

$\frac{d^2y}{dx^2} > 0$, therefore it is a minimum.

(2)

(iv) Sketch the curve on the axes below.



(2)

(v) Find the area enclosed by the curve and the x-axis.

$$\int_5^6 (30 - 11x + x^2)\,dx$$

$$= \left[30x - \frac{11}{2}x^2 + \frac{1}{3}x^3\right]_5^6$$

$$= \left(30(6) - \frac{11}{2}(6)^2 + \frac{1}{3}(6)^3\right) - \left(30(5) - \frac{11}{2}(5)^2 + \frac{1}{3}(5)^3\right)$$

$$54 - 54\tfrac{1}{6} = -\tfrac{1}{6}$$

$\frac{1}{6}$ ....

(4)

4. A curve is defined by the equation $y = -3x^2 + 8x - 5$

$y = 0$

(i) Find the coordinates of the points where the curve meets the x-axis.

$0 = -3x^2 + 8x - 5$

$3x^2 - 8x + 5 = 0$

$(3x - 5)(x - 1) = 0$

$x = \frac{5}{3}$ $\quad x = 1$

$(1, 0)$ $\left(\frac{5}{3}, 0\right)$ ....

(2)

(ii) Find the coordinates of the points where the curve meets the y-axis.

$x = 0$

$y = 0 + 0 - 5$

$y = -5$

$(0, -5)$ ....

(1)

(iii) Find the coordinates of the turning point of the curve

$\frac{dy}{dx} = -6x + 8$

$0 = -6x + 8$

$6x = 8$

$x = \frac{8}{6}$

$= \frac{4}{3}$

$y = -3\left(\frac{4}{3}\right)^2 + 8\left(\frac{4}{3}\right) - 5$

$y = \frac{1}{3}$

$\left(\frac{4}{3}, \frac{1}{3}\right)$ ....

(3)

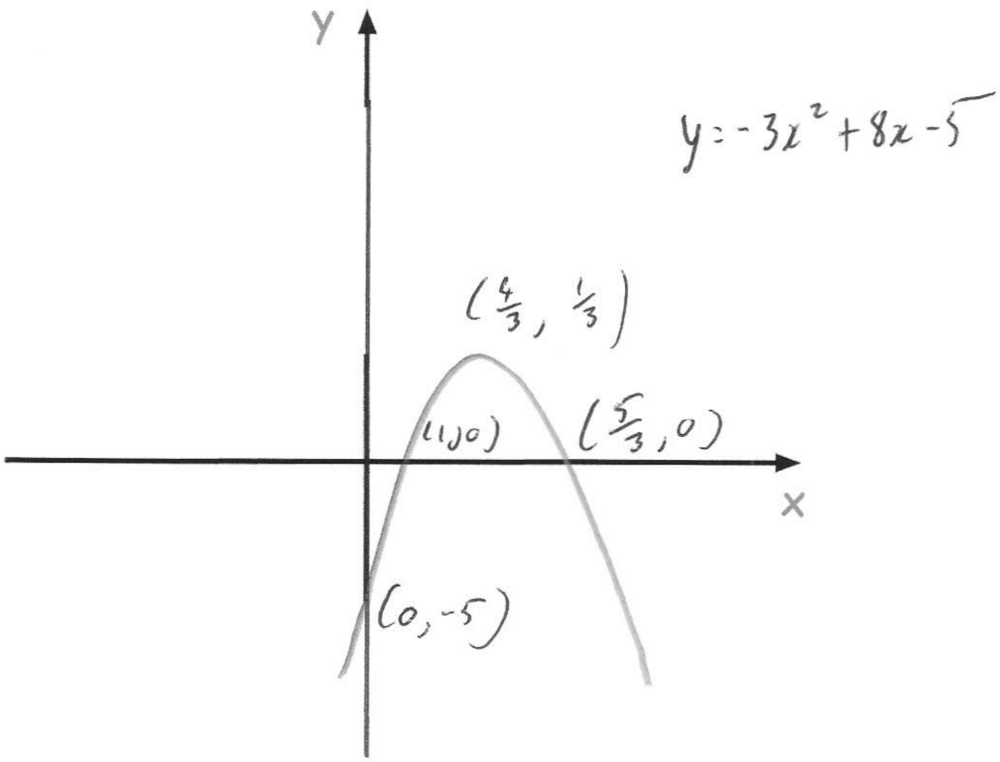
(iv) Using calculus, identify the turning point as either a maximum or minimum point.
You **must** show working to justify your answer.

$$\frac{d^2y}{dx^2} = -6$$

As $\frac{d^2y}{dx^2} < 0$, it is a maximum.

(2)

(v) Sketch the curve on the axes below.



(2)

5. A curve is defined by the equation $y = (x-1)(x-2)(x-4)$

(i) Find the coordinates of the points where the curve meets the x-axis.

$y = 0$

$0 = (x-1)(x-2)(x-4)$

$x = 1 \quad x = 2 \quad x = 4$

$(1, 0), (2, 0), (4, 0)$

(2)

(ii) Find the coordinates of the points where the curve meets the y-axis.

$x = 0$

$y = (0-1)(0-2)(0-4)$

$y = -1 \times -2 \times -4$

$y = -8$

$(0, -8)$

(1)

(iii) Expand and simplify $(x-1)(x-2)(x-4)$

$(x^2 - 3x + 2)(x - 4)$

$x^3 - 4x^2 - 3x^2 + 12x + 2x - 8$

$x^3 - 7x^2 + 14x - 8$

$x^3 - 7x^2 + 14x - 8$

(3)

(iv) Find the coordinates of the turning points of the curve

$\frac{dy}{dx} = 3x^2 - 14x + 14$

$0 = 3x^2 - 14x + 14$

$a = 3$
$b = -14$
$c = 14$

$x = \frac{14 \pm \sqrt{196 - (168)}}{6}$

$x = \frac{14 \pm 2\sqrt{7}}{6}$

$x = \frac{7 \pm \sqrt{7}}{3}$

$x = \frac{7 + \sqrt{7}}{3}$ or $x = \frac{7 - \sqrt{7}}{3}$

$x = 3.215...$ $\quad x = 1.451...$

$y = -2.112...$ $\quad y = 0.6311...$

$(1.45, 0.63)$ & $(3.22, -2.11)$

(3)

(v) Using calculus, identify the turning point as either a maximum or minimum point.
You **must** show working to justify your answer.

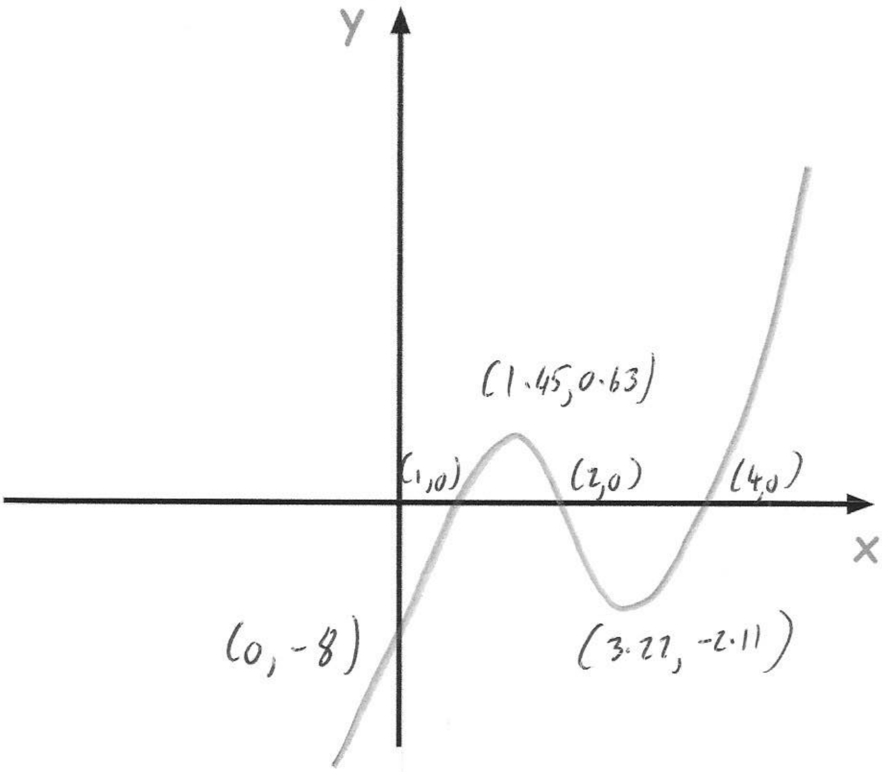
$\frac{d^2y}{dx^2} = 6x - 14$

when $x = 1.45$ $\quad \frac{d^2y}{dx^2} = -5.3$ $\quad$ since $\frac{d^2y}{dx^2} < 0$ $\quad (1.45, 0.63)$ is a max

When $x = 3.22$ $\quad \frac{d^2y}{dx^2} = 5.32$ $\quad$ since $\frac{d^2y}{dx^2} > 0$ $\quad (3.22, -2.11)$ is a min

(2)

(vi) **Hence**, sketch the curve of $y = (x-1)(x-2)(x-4)$



(2)

6. A curve is defined by the equation $y = (x - 1)(2x + 1)^2$

(i) Find the coordinates of the points where the curve meets the x-axis.

$y = 0$

$0 = (x - 1)(2x + 1)^2$

$x = 1$ or $x = -\frac{1}{2}$

$(-\frac{1}{2}, 0)$ & $(1, 0)$

(2)

(ii) Find the coordinates of the points where the curve meets the y-axis.

$x = 0$

$y = (0 - 1)(2(0) + 1)^2$

$y = -1 \times 1^2$

$y = -1$

$(0, -1)$

(1)

(iii) Expand and simplify $(x - 1)(2x + 1)^2$

$(x - 1)(2x + 1)(2x + 1)$

$= (x - 1)(4x^2 + 4x + 1)$

$= 4x^3 + 4x^2 + x - 4x^2 - 4x - 1$

$= 4x^3 - 3x - 1$

$4x^3 - 3x - 1$

(3)

(iv) Find the coordinates of the turning points of the curve

$\frac{dy}{dx} = 12x^2 - 3$

$0 = 12x^2 - 3$

$0 = 4x^2 - 1$

$0 = (2x+1)(2x-1)$

$x = -\frac{1}{2}$ or $x = \frac{1}{2}$

$y = (x-1)(2x+1)^2$

$y = (-\frac{1}{2} - 1)(2(-\frac{1}{2}) + 1)^2$

$y = 0$

$y = (\frac{1}{2} - 1)(2(\frac{1}{2}) + 1)^2$

$y = (-\frac{1}{2})(4)$

$y = -2$

$(-\frac{1}{2}, 0)$ $(\frac{1}{2}, -2)$

(3)

(v) Using calculus, identify the turning point as either a maximum or minimum point.
You **must** show working to justify your answer.

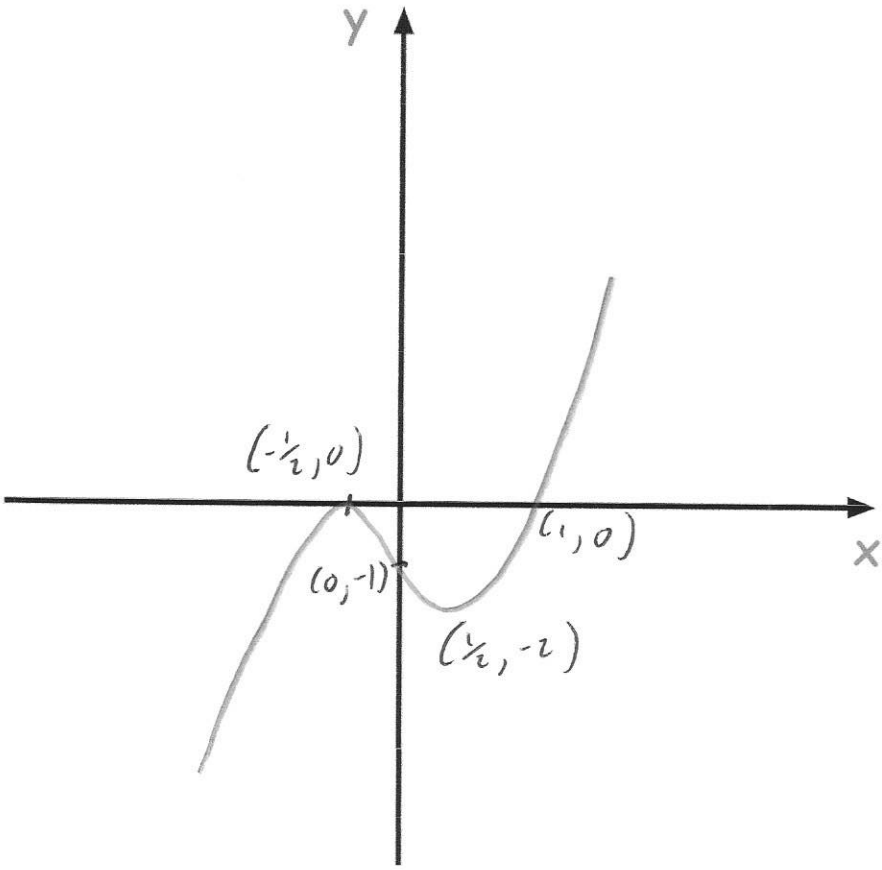
$\frac{d^2y}{dx^2} = 24x$

when $x = -\frac{1}{2}$ $\frac{d^2y}{dx^2} = -12$ $\therefore (-\frac{1}{2}, 0)$ is a max

when $x = \frac{1}{2}$ $\frac{d^2y}{dx^2} = 12$ $\therefore (\frac{1}{2}, -2)$ is a min

(2)

(vi) **Hence**, sketch the curve of $y = (x - 1)(2x + 1)^2$



(2)

7. A curve is defined by the equation $y = x(x-12)^2$

(i) Find the coordinates of the points where the curve meets the x-axis.

$y = x(x-12)^2$ $\quad y=0$

$0 = x(x-12)^2$

$x = 0$ or $x = 12$

$(0,0)$ & $(12,0)$

(3)

(ii) Find the coordinates of the turning points of the curve

$y = x(x^2 - 24x + 144) \rightarrow y = x^3 - 24x^2 + 144x$

$dy/dx = 3x^2 - 48x + 144$

$0 = 3x^2 - 48x + 144$

$0 = x^2 - 16x + 48$

$0 = (x-4)(x-12)$

$x = 4 \quad x = 12$

$y = 4(4-\cancel{12})^2$ $\quad y = 12(0)^2$

$y = 4(-8)^2$ $\quad y = 0$

$y = 256$ $\quad (12,0)$

$(4, 256)$

$(4, 256)$ & $(12, 0)$

(6)

(iii) Using calculus, identify the turning point as either a maximum or minimum point.
You **must** show working to justify your answer.

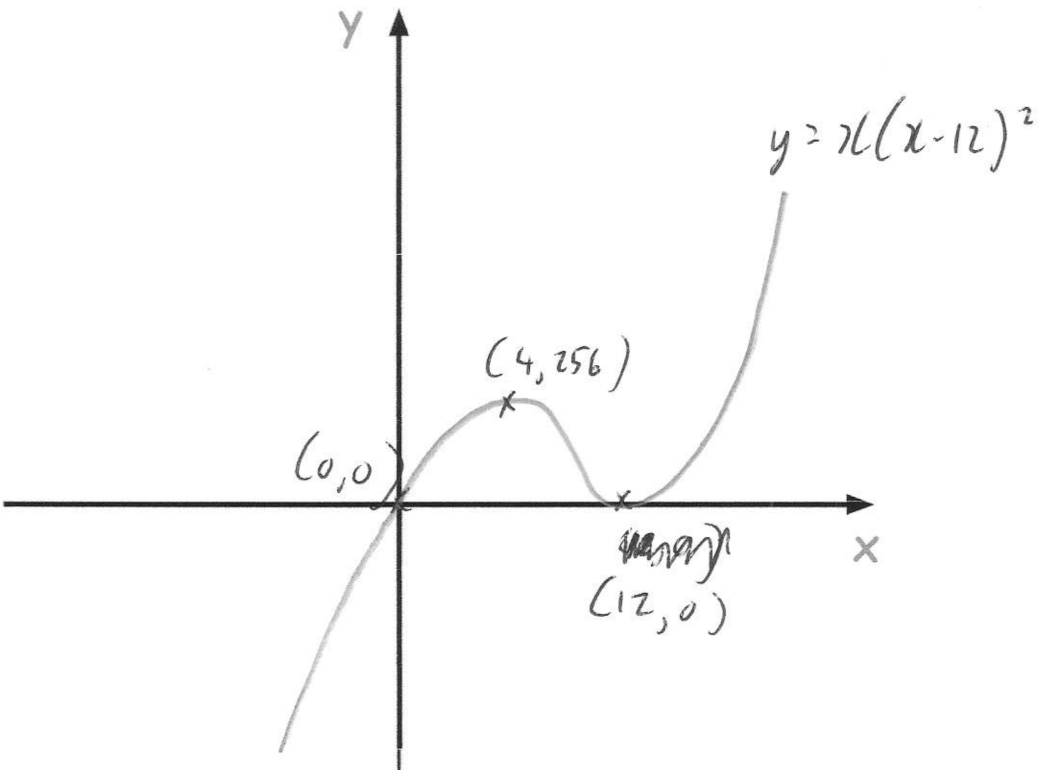
$\frac{d^2y}{dx^2} = 6x - 48$

$x = 4 \quad \frac{d^2y}{dx^2} = -24 \quad \therefore (4, 256)$ is a max as $\frac{d^2y}{dx^2} < 0$

$x = 12 \quad \frac{d^2y}{dx^2} = 24 \quad \therefore (12, 0)$ is a min as $\frac{d^2y}{dx^2} > 0$

(3)

(iv) sketch the curve on the axes below.



(2)

8. A curve is defined by the equation $y = x^3 + 4x^2 + 4x$

(i) Find the coordinates of the points where the curve meets the x-axis.

$y=0$

$0 = x^3 + 4x^2 + 4x$

$0 = x(x^2 + 4x + 4)$

$0 = x(x+2)(x+2)$

$x = 0 \qquad x = -2$

$(0,0)$ & $(-2,0)$

(3)

(iv) Find the coordinates of the turning points of the curve

$\frac{dy}{dx} = 3x^2 + 8x + 4$

$0 = 3x^2 + 8x + 4$

$0 = (3x+2)(x+2)$

$x = \frac{-2}{3} \qquad x = -2$

$\downarrow \qquad \downarrow$

$y = -\frac{32}{27} \qquad y = 0$

$(-2,0)$ & $\left(-\frac{2}{3}, -\frac{32}{27}\right)$

(6)

(iii) Using calculus, identify the turning point as either a maximum or minimum point.
You **must** show working to justify your answer.

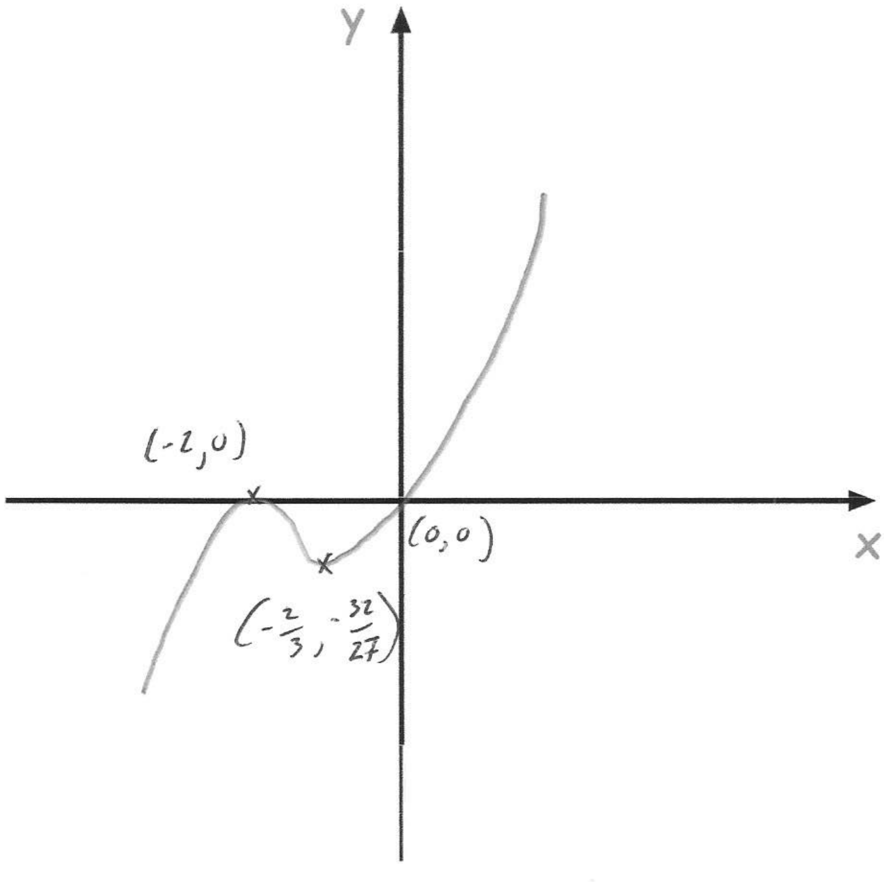
$\frac{d^2y}{dx^2} = 6x + 8$

when $x = -2$ $\frac{d^2y}{dx^2} = -4$ $\therefore$ $(-2, 0)$ is a max as $\frac{d^2y}{dx^2} < 0$

when $x = -\frac{2}{3}$ $\frac{d^2y}{dx^2} = 4$ $\therefore$ $\left(-\frac{2}{3}, -\frac{32}{27}\right)$ is a min as $\frac{d^2y}{dx^2} > 0$

(3)

(iv) sketch the curve on the axes below.



(2)

9. A curve is defined by the equation $y = 2x^3 + 6x^2 + 4x$

(i) Find the coordinates of the points where the curve meets the x-axis.

$y = x(2x^2 + 6x + 4)$

$0 = x(x^2 + 3x + 2)$

$0 = x(x+1)(x+2)$

$x = 0 \quad x = -1 \quad x = -2$

$(-2, 0) \; (-1, 0) \; (0, 0)$

(3)

(iv) Find the coordinates of the turning points of the curve

$\frac{dy}{dx} = 6x^2 + 12x + 4$

$0 = 6x^2 + 12x + 4$

$0 = 3x^2 + 6x + 2$

$a = 3$
$b = 6$
$c = 2$

$x = \frac{-6 \pm \sqrt{36 - 24}}{6}$

$x = \frac{-6 \pm 2\sqrt{3}}{6}$

$x = -0.4226$ or $x = -1.5774$

$y = -0.7698 \qquad y = 0.7698$

$(-1.5774, 0.7698)$

&

(6)

$(-0.4226, -0.7698)$

(iii) Using calculus, identify the turning point as either a maximum or minimum point.
You **must** show working to justify your answer.

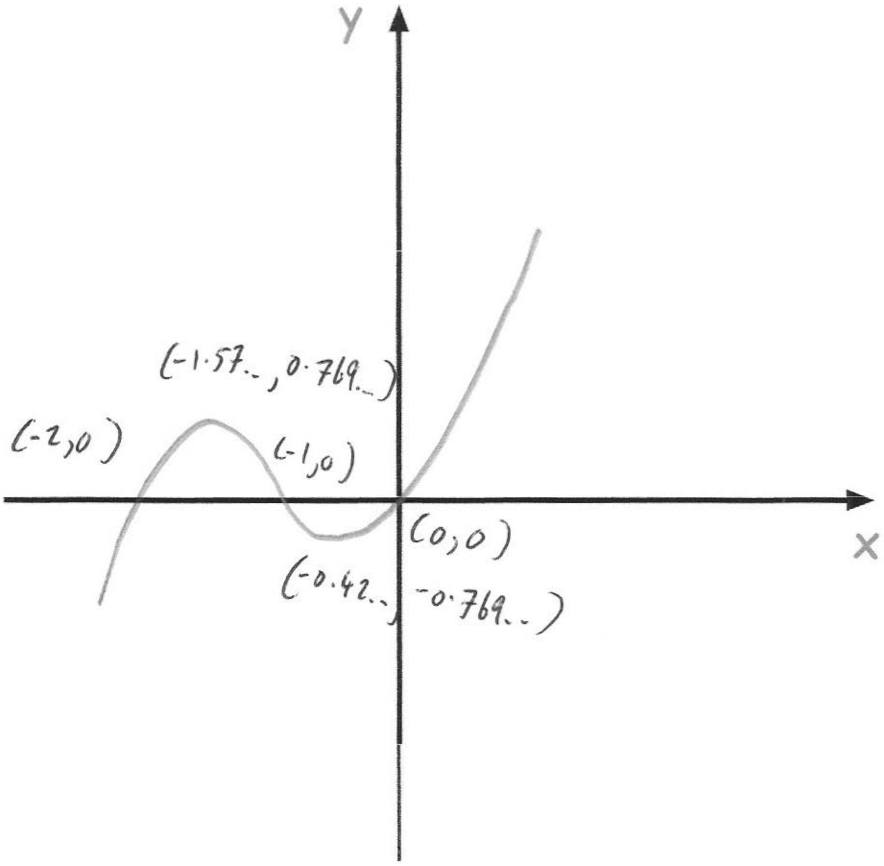
$$\frac{d^2y}{dx^2} = 12x + 12$$

when $x = -1.5774$, $\frac{d^2y}{dx^2} = -6.928...$ $\therefore (-1.5774, 0.7698)$ is a max as $\frac{d^2y}{dx^2} < 0$

when $x = -0.4226$, $\frac{d^2y}{dx^2} = 6.928...$ $\therefore (-0.4226, -0.7698)$ is a min as $\frac{d^2y}{dx^2} > 0$

(3)

(iv) sketch the curve on the axes below.



(2)

10. A curve is defined by $y = (x-1)(x-4)^2$

(i) Expand and simplify $(x-1)(x-4)^2$

$(x-1)(x^2-8x+16)$

$x^3-8x^2+16x-x^2+8x-16$

$x^3-9x^2+24x-16$

$x^3-9x^2+24x-16$

(3)

(ii) Find the coordinates of the turning points of the curve, and, using calculus, identify each turning point as either a maximum or a minimum point.

$\frac{dy}{dx} = 3x^2-18x+24$

$0 = 3x^2-18x+24$

$0 = x^2-6x+8$

$0 = (x-2)(x-4)$

$x = 2$ & $x = 4$

$(2, 4)$ $(4, 0)$

$y = (2-1)(2-4)^2$

$= 1 \times 4$

$= 4$

$y = (4-1)(4-4)^2$

$y = 0$

$\frac{d^2y}{dx^2} = 6x-18$

when $x = 2$ $\frac{d^2y}{dx^2} = -6$

$\therefore$ max

as $\frac{d^2y}{dx^2} < 0$

when $x = 4$ $\frac{d^2y}{dx^2} = 6$

$\therefore$ min

as $\frac{d^2y}{dx^2} > 0$

$(4, 0)$ & $(2, 4)$

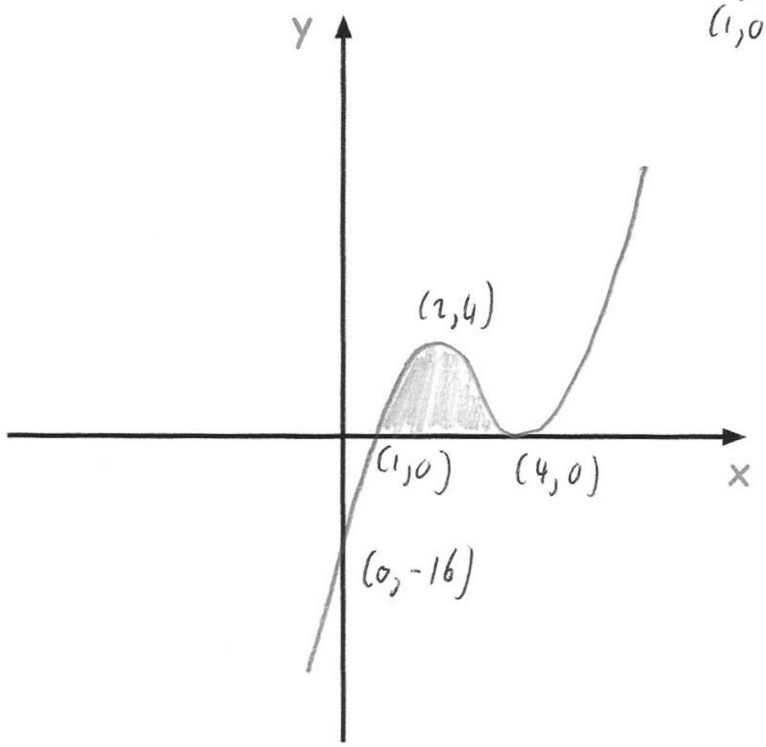
min max

(6)

(iii) sketch the curve on the axes below.

$y = (x-1)(x-4)^2$

$(1,0)$ $(4,0)$



(2)

(iv) Find the area enclosed by the curve and the x-axis.

$$\int_1^4 (x^3 - 9x^2 + 24x - 16)\,dx$$

$$= \left[\tfrac{1}{4}x^4 - 3x^3 + 12x^2 - 16x\right]_1^4$$

$$= \left(\tfrac{1}{4}(4)^4 - 3(4)^3 + 12(4)^2 - 16(4)\right) - \left(\tfrac{1}{4}(1)^4 - 3(1)^3 + 12(1)^2 - 16(1)\right)$$

$$= (64 - 192 + 192 - 64) - \left(\tfrac{1}{4} - 3 + 12 - 16\right)$$

$$= 6.75$$

6.75 ....

(4)