Name:

GCSE Further Maths

Solving Index Equations

Corbettmaths

Ensure you have: Pencil, Pen, Calculator

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic

www.corbettmaths.com/gcse-further-maths

1. Solve the equation $3^x = 40$

Give your answer to 3 significant figures.

method 1

$x = \log_3 40$

$\log_3 40 = 3.35776...$

$x = 3.36$

method 2

$\log 3^x = \log 40$

$x \log 3 = \log 40$

$x = \frac{\log 40}{\log 3}$

$x = 3.357...$

3.36

.....

(2)

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2. Solve the equation $8^x = 5$

Give your answer to 3 significant figures.

method 1

$\log_8 5 = x$

$x = 0.77397...$

$x = 0.774$

method 2

$\log 8^x = \log 5$

$x \log 8 = \log 5$

$x = \frac{\log 5}{\log 8}$

$x = 0.77397...$

0.774

.....

(2)

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3. Solve the equation $2^{-x} = 11$

Give your answer to 3 significant figures.

method 1

$\log_2 11 = -x$

$-x = 3.4594...$

$x = -3.4594...$

$x = -3.46$

method 2

$\log 2^{-x} = \log 11$

$-x \log 2 = \log 11$

$-x = \frac{\log 11}{\log 2}$

$-x = 3.459...$

$x = -3.459...$

$x = -3.46$

-3.46

.....

(2)

4. Solve the equation $6^{x+3} = 14$

Give your answer to 3 significant figures.

method 1

$\log_6 14 = x+3$

$x+3 = 1.47288594$

$x = -1.52711..$

$x = -1.53$

method 2

$\log 6^{x+3} = \log 14$

$(x+3)\log 6 = \log 14$

$x+3 = \frac{\log 14}{\log 6}$

$x+3 = 1.47288..$

$x = -1.52711..$

$x = -1.53$

-1.53

.....

(2)

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5. Solve the equation $3^{5x-2} = 80$

Give your answer to 3 significant figures.

method 1

$\log_3 80 = 5x-2$

$5x-2 = 3.98869..$

$5x = 5.98869..$

$x = 1.1977..$

$x = 1.2$

method 2

$\log 3^{5x-2} = \log 80$

$(5x-2)\log 3 = \log 80$

$5x-2 = \frac{\log 80}{\log 3}$

$5x-2 = 3.9886...$

$5x = 5.98869..$

$x = 1.1977..$

$x = 1.2$

1.2

.....

(3)

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6. Solve the equation $7^{(1-2x)} = 3$

Give your answer to 3 significant figures.

method 1

$\log_7 3 = 1-2x$

$0.564... = 1-2x$

$-0.435... = -2x$

$x = 0.2177..$

0.218

method 2

$\log 7^{(1-2x)} = \log 3$

$(1-2x)\log 7 = \log 3$

$1-2x = \frac{\log 3}{\log 7}$

$1-2x = 0.564...$

$-2x = -0.435...$

$x = 0.2172..$

0.218

.....

(3)

7. Solve the equation $14^{(\frac{3}{8}x+1)} = 5$

method 1

$\log_{14} 5 = \frac{3}{8}x+1$

$0.6098... = \frac{3}{8}x+1$

$-0.39014... = \frac{3}{8}x$

$-1.0403... = x$

$x = -1.04$

method 2

$\log 14^{(\frac{3}{8}x+1)} = \log 5$

$(\frac{3}{8}x+1)\log 14 = \log 5$

$\frac{3}{8}x+1 = \frac{\log 5}{\log 14}$

$\frac{3}{8}x+1 = 0.60985...$

$\frac{3}{8}x = -0.3901...$

$x = -1.04...$

-1.04

.....

(3)

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8. Solve the equation $2^{3x+1} = 5^{x+4}$

$\log 2^{3x+1} = \log 5^{x+4}$

$(3x+1)\log 2 = (x+4)\log 5$

$3x\log 2 + \log 2 = x\log 5 + 4\log 5$

$3x\log 2 - x\log 5 = 4\log 5 - \log 2$

$x(3\log 2 - \log 5) = 4\log 5 - \log 2$

$x = \frac{4\log 5 - \log 2}{3\log 2 - \log 5}$

$x = 12.222...$

12.2

.....

(5)

9. Solve the equation $3^{2x+1} = 7^{3x-4}$

$\log 3^{2x+1} = \log 7^{3x-4}$

$(2x+1)\log 3 = (3x-4)\log 7$

$2x\log 3 + \log 3 = 3x\log 7 - 4\log 7$

$2x\log 3 - 3x\log 7 = -4\log 7 - \log 3$

$x(2\log 3 - 3\log 7) = -4\log 7 - \log 3$

$x = \dfrac{-4\log 7 - \log 3}{2\log 3 - 3\log 7}$

$x = 2.43984\ldots$

2.44

(5)

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10. Solve the equation $15^{5-x} = 4^{3x}$

$\log 15^{5-x} = \log 4^{3x}$

$(5-x)\log 15 = 3x\log 4$

$5\log 15 - x\log 15 = 3x\log 4$

$5\log 15 = 3x\log 4 + x\log 15$

$5\log 15 = x(3\log 4 + \log 15)$

$x = \dfrac{5\log 15}{3\log 4 + \log 15}$

$x = 1.9718\ldots$

1.97

(5)

11. (i) Given $8 \times 2^x = 2^y$

show that $y = x + 3$

$2^3 \times 2^x = 2^y$

$2^{x+3} = 2^y$

$\therefore \quad y = x + 3$

(1)

(ii) **Hence** solve the equation $8 \times 2^x = 5^{2x-5}$

From (i) $8 \times 2^x = 2^{x+3}$

$2^{x+3} = 5^{2x-5}$

$\log 2^{x+3} = \log 5^{2x-5}$

$(x+3)\log 2 = (2x-5)\log 5$

$x\log 2 + 3\log 2 = 2x\log 5 - 5\log 5$

$3\log 2 + 5\log 5 = 2x\log 5 - x\log 2$

$3\log 2 + 5\log 5 = x(2\log 5 - \log 2)$

$x = \dfrac{3\log 2 + 5\log 5}{2\log 5 - \log 2}$

$x = 4.00938\ldots$

$= 4.01$

4.01

.....

(5)

11. (i) Given $9 \times 3^{5x} = 3^y$

show that $y = 5x + 2$

$3^2 \times 3^{5x} = 3^y$

$3^{5x+2} = 3^y$

$\therefore \quad y = 5x + 2$

(1)

(ii) **Hence** solve the equation $9 \times 3^{5x} = 8^{x-1}$

From (i) $9 \times 3^{5x} = 3^{5x+2}$

$3^{5x+2} = 8^{x-1}$

$\log 3^{5x+2} = \log 8^{x-1}$

$(5x+2)\log 3 = (x-1)\log 8$

$5x\log 3 + 2\log 3 = x\log 8 - \log 8$

$2\log 3 + \log 8 = x\log 8 - 5x\log 3$

$2\log 3 + \log 8 = x(\log 8 - 5\log 3)$

$x = \dfrac{2\log 3 + \log 8}{\log 8 - 5\log 3}$

$x = -1.2528\ldots$

-1.25

(5)