Name:

GCSE Further Maths

Simultaneous Equations with Three Unknowns

Corbettmaths

Ensure you have: Pencil, Pen, Calculator

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic

www.corbettmaths.com/gcse-further-maths

1. Solve the simultaneous equations

$$x - y + 3z = 5 \quad -(1)$$
$$x + y + 6z = 12 \quad -(2)$$
$$3x - 2y + 2z = 10 \quad -(3)$$

(1) + (2) → (4)

$2x + 9z = 17 \quad -(4)$

2 × (2) + (3) → (5)

$2x + 2y + 12z = 24$

add $3x - 2y + 2z = 10$

$5x + 14z = 34 \quad -(5)$

5 × (4) − 2 × (5)

$10x + 45z = 85$

$10x + 28z = 68$

$17z = 17$

$z = 1$

Sub $z = 1$ into (5)

$5x + 14 = 34$

$5x = 20$

$x = 4$

Sub $x = 4$ & $z = 1$ into (1)

$4 - y + 3 = 5$

$7 - y = 5$

$y = 2$

check in (2)

$4 + 2 + 6 = 12$ ✓

$x =$ 4 $y =$ 2 $z =$ 1

(5)

2. Solve the simultaneous equations

$$2x + 3y + 5z = 21 \quad -(1)$$

$$3x + 6y + 15z = 51 \quad -(2)$$

$$5x + 4y + 10z = 37 \quad -(3)$$

$2 \times (1) - (2) \rightarrow (4)$

$4x + 6y + 10z = 42$

sub $3x + 6y + 15z = 51$

$x - 5z = -9 \quad -(4)$

$2 \times (2) - 3 \times (3) \rightarrow (5)$

$6x + 12y + 30z = 102$

sub $15x + 12y + 30z = 111$

$-9x = -9 \quad -(5)$

$x = 1$

sub $x = 1$ into (4)

$1 - 5z = -9$

$-5z = -10$

$z = 2$

$x =$ 1 $y =$ 3 $z =$ 2

(5)

sub $x = 1$ & $z = 2$ into (3)

$5 + 4y + 20 = 37$

$4y = 12$

$y = 3$

check in (1)

$2 + 9 + 10 = 21$ ✓

3. Solve the simultaneous equations

$$2x + 4y - z = 15 \quad -(1)$$

$$3x + 8y + z = 44 \quad -(2)$$

$$x + 2y + 2z = 15 \quad -(3)$$

(1) + (2) → (4)

$5x + 12y = 59 \quad -(4)$

2 × (2) − (3) → (5)

$6x + 16y + 2z = 88$

sub $x + 2y + 2z = 15$

$5x + 14y = 73 \quad -(5)$

(5) (4) → (6)

$5x + 14y = 73$

sub $5x + 12y = 59$

$2y = 14$

$y = 7$

$x =$ ..-5.. $y =$ ..7.. $z =$ ..3..

(5)

sub $y = 7$ into (5)

$5x + 98 = 73$

$5x = -25$

$x = -5$

sub $x = -5$ & $y = 7$ into (1)

$-10 + 28 - z = 15$

$18 - z = 15$

$z = 3$

check in (3)

$-15 + 56 + 3 = 44$ ✓

4. Solve the simultaneous equations

$$10x + 60y + 10z = 25 \quad -(1)$$

$$5x + 40y + 20z = 40 \quad -(2)$$

$$20x + 20y + 40z = 30 \quad -(3)$$

$2 \times (1) - (2) \rightarrow (4)$

$$20x + 120y + 20z = 50$$

sub $\quad 5x + 40y + 20z = 40$

$$15x + 80y = 10 \rightarrow (4)$$

$2 \times (3) - 2 \times (2) \rightarrow 5$

$$20x + 20y + 40z = 30$$

sub $\quad 10x + 80y + 40z = 80$

$$10x - 60y = -50 \rightarrow (5)$$

$2 \times (4) - 3 \times (5)$

$$30x + 160y = 20$$

sub $\quad 30x - 180y = -150$

$$340y = 170$$

$$y = \frac{1}{2}$$

sub $y = \frac{1}{2}$ into (4)

$$15x + 40 = 10$$

$$15x = -30$$

$$x = -2$$

sub $x = -2$ & $y = \frac{1}{2}$ into (1)

$$-20 + 30 + 10z = 25$$

$$10 + 10z = 25$$

$$10z = 15$$

$$z = 1.5$$

Check in (2)

$-10 + 20 + 30 = 40$ ✓

$x =$ ..-2.. $y =$ ..0.5.. $z =$ ..1.5..

(5)

5. Solve the simultaneous equations

$$x + y + z = 1 \quad -(1)$$

$$4x - 3y + 4z = 32 \quad -(2)$$

$$x - 10y - 2z = 27 \quad -(3)$$

(1) − (3) → (4)

$x + y + z = 1$

sub $x - 10y - 2z = 27$

$11y + 3z = -26 \quad \rightarrow (4)$

4 × (1) − (2) → (5)

$4x + 4y + 4z = 4$

sub $4x - 3y + 4z = 32$

$7y = -28$

$y = -4$

sub $y = -4$ into (4)

$-44 + 3z = -26$

$3z = 18$

$z = 6$

Sub $y = -4$ & $z = 6$ into (1)

$x - 4 + 6 = 1$

$x + 2 = 1$

$x = -1$

$x = \ -1 \ \ y = \ -4 \ \ z = \ 6$

(5)

Check in (2)

$-4 + 12 + 24 = 32$ ✓

6. Solve the simultaneous equations

$$6x + 8y - 2z = 750 \quad -(1)$$

$$18x - 2y + 4z = 1100 \quad -(2)$$

$$4x - 4y + 2z = 100 \quad -(3)$$

$2 \times (1) - (3)$

$$36x - 4y + 8z = 2200$$
$$4x - 4y + 2z = 100$$

$$32x + 6z = 2100 \quad -(4)$$

$2 \times (3) + (1)$

$$8x - 8y + 4z = 200$$
$$6x + 8y - 2z = 750$$

$$14x + 2z = 950 \quad -(5)$$

$3 \times (5) - (4)$

$$42x + 6z = 2850$$
$$32x + 6z = 2100$$

$$10x = 750$$
$$x = 75$$

sub $x = 75$ into (5)

$$1050 + 2z = 950$$
$$2z = -100$$
$$z = -50$$

sub $x = 75$ & $z = -50$ into (1)

$$450 + 8y + 100 = 750$$
$$8y = 200$$
$$y = 25$$

$x = \ 75 \ \ y = \ 25 \ \ z = \ -50$

(5)

7. Solve the simultaneous equations

$$7x + 5y + 4z = 23 \quad -(1)$$

$$21x - 10y + 6z = -4 \quad -(2)$$

$$7x + 15y - 2z = -15 \quad -(3)$$

$3 \times (1) \Rightarrow \quad 21x + 15y + 12z = 69 \quad -(4)$

$3 \times (3) \Rightarrow \quad 21x + 45y - 6z = -45 \quad -(5)$

(4) − (2) ⇒ (6)

$$21x + 15y + 12z = 69$$
$$\text{sub} \quad 21x - 10y + 6z = -4$$
$$25y + 6z = 73 \quad -(6)$$

(5) − (4) ⇒ (7)

$$21x + 45y - 6z = -45$$
$$\text{sub} \quad 21x + 15y + 12z = 69$$
$$30y - 18z = -114 \quad -(7)$$

$3 \times (6) \Rightarrow (8)$

$$75y + 18z = 219 \quad -(8)$$
$$\text{add} \quad 30y - 18z = -114$$
$$105y = 105$$
$$y = 1$$

sub $y = 1$ into (6)

$25 + 6z = 73$

$6z = 48$

$z = 8$

sub $y = 1$ & $z = 8$ into (1)

$7x + 5 + 32 = 23$

$7x = -14$

$x = -2$

check in (2)

$-42 - 10 + 48 = -4$ ✓

$x = -2 \quad y = 1 \quad z = 8$

(5)

8. Solve the simultaneous equations

$$y - x + 2z = 2.1 \quad -(1)$$

$$3x - 2z - y + 2.5 = 0 \quad -(2)$$

$$8z + 10y + 5x = 4.5 \quad -(3)$$

$-x + y + 2z = 2.1$ $\quad 3x - y - 2z = -2.5$ $\quad 5x + 10y + 8z = 4.5$

$(1) + (2) \Rightarrow (4)$

$2x = -0.4$

$x = -0.2$

$4 \times (2) + (3) \Rightarrow (5)$

$12x - 4y - 8z = -10$

add $5x + 10y + 8z = 4.5$

$17x + 6y = -5.5 \quad -(5)$

sub $x = -0.2$ into (5)

$-3.4 + 6y = -5.5$

$6y = -2.1$

$y = -0.35$

sub $x = -0.2$ & $y = -0.35$ into (1)

$0.2 - 0.35 + 2z = 2.1$

$-0.15 + 2z = 2.1$

$z = 1.125$

check in (2)

$x = -0.2$ $\quad y = -0.35$ $\quad z = 1.125$

(5)

$-0.6 + 0.35 - 2.25 = -2.5$

$-2.5 = -2.5$ ✓

9. Roshan sells vehicles for a company.

She receives:

£x commission for each car sold.

£y commission for each motorcycle sold.

£z commission for each van sold.

During January, Roshan sold: 12 cars, 6 motorcycles and 3 vans; she received a total of £1590 commission.

(i) Show that x, y and z satisfy the equation

$$4x + 2y + z = 530$$

$12x + 6y + 3z = 1590 \quad \div 3$

$4x + 2y + z = 530$ ✓

(1)

During February, Roshan sold: 10 cars, 5 motorcycles and 5 vans; she received a total of £1600 commission.

(ii) Show that x, y and z satisfy the equation

$$2x + y + z = 320$$

$10x + 5y + 5z = 1600 \quad \div 5$

$2x + y + z = 320$ ✓

(1)

During March, Roshan sold: 18 cars, 4 motorcycles and 6 vans; she received a total of £1600 commission.

(iii) Show that x, y and z satisfy the equation

$$9x + 2y + 3z = 1175$$

$18x + 4y + 6z = 2350 \quad \div 2$

$9x + 2y + 3z = 1175$ ✓

(1)

(iv) Solve the equations

$$4x + 2y + z = 530 \quad -(1)$$

$$2x + y + z = 320 \quad -(2)$$

$$9x + 2y + 3z = 1175 \quad -(3)$$

to find the commission Roshan receives for selling each type of vehicle.

Show clearly each stage of your solution.

(1) − (2) = (4)

$4x + 2y + z = 530$

sub $2x + y + z = 320$

$2x + y = 210$ −(4)

(3) − 3 × (2) = (5)

$9x + 2y + 3z = 1175$

sub $6x + 3y + 3z = 960$

$3x - y = 215$ −(5)

(4) + (5) = (6)

$2x + y = 210$

add $3x - y = 215$

$5x = 425$ (6)

$x = 85$

sub $x = 85$ into (4)

$170 + y = 210$

$y = 40$

sub $x = 85$ & $y = 40$ into (1)

$340 + 80 + z = 530$

$420 + z = 530$

$z = 110$

check $x = 85$, $y = 40$ & $z = 110$ in (3)

$765 + 80 + 330 = 1175$ ✓

Car £ 85

Motorcycle £ 40

Van £ 110

(8)

10. A football team sells three types of season ticket: adult, child and pensioner.

This year, the McManus family bought:

4 adult season tickets at £x each.

6 child season tickets at £y each.

2 pensioner tickets at £z each.

The total cost of the tickets is £5650

(i) Show that x, y and z satisfy the equation

$$2x + 3y + z = 2825$$

$4x + 6y + 2z = 5650 \quad \div 2$

$2x + 3y + z = 2825$ ✓

(1)

The cost of an two adult season ticket is £100 more than the total cost of two child season tickets **and** two pensioner season tickets.

(ii) Show that x, y and z satisfy the equation

$$x - y - z = 50$$

$2x - 2y - 2z = 100 \quad \div 2$

$x - y - z = 50$

(1)

Last year, all season tickets cost £25 less than the current prices.

Last year, the Wilcox family bought 4 adult season tickets, 8 child season tickets and 4 pensioner tickets.

The total price was £6600

(iii) Show that x, y and z satisfy the equation

$$x + 2y + z = 1750$$

$4(x-25) + 8(y-25) + 4(z-25) = 6600$

$4x - 100 + 8y - 200 + 4z - 100 = 6600$

$4x + 8y + 4z = 7000$

$x + 2y + z = 1750$

(2)

(iv) Solve the equations

$$2x + 3y + z = 2825 \quad -(1)$$

$$x - y - z = 50 \quad -(2)$$

$$x + 2y + z = 1750 \quad -(3)$$

to find the cost of the current season tickets.

Show clearly each stage of your solution.

(1) + (2) = (4)

$2x + 3y + z = 2825$

Add $x - y - z = 50$

$3x + 2y = 2875 \quad -(4)$

(2) + (3) = (5)

$x - y - z = 50$

Add $x + 2y + z = 1750$

$2x + y = 1800 \quad -(5)$

2 × (5) − (4)

$4x + 2y = 3600$

sub $3x + 2y = 2875$

$x = 725$

sub $x = 725$ into (5)

$1450 + y = 1800$

$y = 350$

sub $y = 350$ & $x = 725$ into (2)

$725 - 350 - z = 50$

$375 - z = 50$

$z = 325$

Check in (3)

$725 + 700 + 325 = 1750$ ✓

Adult £725

Child £350

Pensioner £325

(8)

11. Oisín is training for a race and as part of his training, Oisín completes three types of run: short, medium and long.

Let x, y and z represent the length, in kilometres, of a short, medium and long run respectively.

In September, Oisín completed 24 short runs, 18 medium runs and 6 long runs. The total distance ran during September was 450km

(i) Show that x, y and z satisfy the equation

$$4x + 2y + z = 75$$

$24x + 18y + 6z = 450$ $\div 6$

$4x + 3y + z = 75$ ✓

(1)

In October, Oisín completed 5 short runs, 15 medium runs and 20 long runs. The total distance ran during October was 615km.

(ii) Show that x, y and z satisfy the equation

$$x + 3y + 4z = 123$$

$5x + 15y + 20z = 615$ $\div 5$

$x + 3y + 4z = 123$ ✓

(1)

In November, Oisin replaced the long run with an ultra run.
The ultra run is 40% longer than the long run. ultra $= 1.4 \times z$ or $1.4z$

That month he ran 14 short runs, 4 medium runs and 10 ultra runs, covering a total distance of 420km.

(iii) Show that x, y and z satisfy the equation

$$7x + 2y + 7z = 210$$

$14x + 4y + 10(1.4z) = 420$

$14x + 4y + 14z = 420$ $\div 2$

$7x + 2y + 7z = 210$ ✓

(1)

(iv) Solve the equations

$$4x + 3y + z = 75 \quad - (1)$$

$$x + 3y + 4z = 123 \quad - (2)$$

$$7x + 2y + 7z = 210 \quad - (3)$$

to find the length of a short, medium and long run.

Show clearly each stage of your solution.

(1) - (2) = (4)

$4x + 3y + z = 75$

sub $x + 3y + 4z = 123$

$3x - 3z = -48 \quad - (4)$

3 × (3) − 2 × (2) = (5)

$21x + 6y + 21z = 630$

sub $2x + 6y + 8z = 246$

$19x + 13z = 384 \quad - (5)$

13 × (4) + 3 × (5)

$39x - 39z = -624$

Add $57x + 39z = 1152$

$96x = 528$

$x = 5.5$

sub $x = 5.5$ into (4)

$16.5 - 3z = -48$

$-3z = -64.5$

$z = 21.5$

sub $x = 5.5$ & $z = 21.5$ into (1)

$22 + 3y + 21.5 = 75$

$3y = 31.5$

$y = 10.5$

check in (2)

$5.5 + 31.5 + 86 = 123$

$123 = 123$ ✓

Short 5.5 km

Medium 10.5 km

Long 21.5 km

(8)

12. A concert venue sells three types of tickets: standing, seated and VIP.

For each concert

the venue sells a maximum of x standing tickets.
the venue sells a maximum of y seated tickets.
the venue sells a maximum of z VIP tickets.

The venue can sell a total of 18000 tickets for each concert.

Therefore, x, y and z satisfy the equation

$$x + y + z = 18000$$

For a recent pop concert, 75% of the standing tickets, 90% of the seated tickets and 45% of the VIP tickets **were sold**.
A total of 15600 tickets were sold for the pop concert.

(i) Show that x, y and z satisfy the equation

$$5x + 6y + 3z = 104000$$

$0.75x + 0.9y + 0.45z = 15600$ $\times 100$

$75x + 90y + 45z = 1560000$ $\div 15$

$5x + 6y + 3z = 104000$ ✓

(2)

For a recent rock concert, 4% of the standing tickets, $\frac{1}{5}$ of the seated tickets and 52% of the VIP tickets **were not sold.**
A total of 14640 tickets were sold for the rock concert.

(ii) Show that x, y and z satisfy the equation

$$6x + 5y + 3z = 91500$$

$0.96x + 0.8y + 0.48z = 14640$ $\times 100$

$96x + 80y + 48z = 1464000$ $\div 16$

$6x + 5y + 3z = 91500$ ✓

(2)

(iv) Solve the equations

$$x + y + z = 18000 \quad -(1)$$

$$5x + 6y + 3z = 104000 \quad -(2)$$

$$6x + 5y + 3z = 91500 \quad -(3)$$

to find the maximum number of standing, seated and VIP tickets available for each concert.

Show clearly each stage of your solution.

(1) × 3 = (4) $3x + 3y + 3z = 54000 \quad -(4)$

(2) − (4) = (5)

$5x + 6y + 3z = 104000$

sub $3x + 3y + 3z = 54000$

$2x + 3y = 50000 \quad -(5)$

(3) − (4) = (6)

$6x + 5y + 3z = 91500$

sub $3x + 3y + 3z = 54000$

$3x + 2y = 37500 \quad -(6)$

3 × (6) − 2 × (5)

$9x + 6y = 112500$

sub $4x + 6y = 100000$

$5x = 12500$

$x = 2500$

Sub $x = 2500$ into (5)

$5000 + 3y = 50000$

$3y = 45000$

$y = 15000$

sub $x = 2500$ & $y = 15000$ into (1)

$2500 + 15000 + z = 18000$

$z = 500$

check in (3) $15000 + 7500 + 1500 = 91500$ ✓

Standing 2500

Seated 15000

VIP 500

(8)