

Name: _____

Exam Style Questions

Equation of a Tangent to a Circle



Corbettmaths

Equipment needed: Ruler, calculator, pencil and pen

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Video Tutorial

www.corbettmaths.com/contents

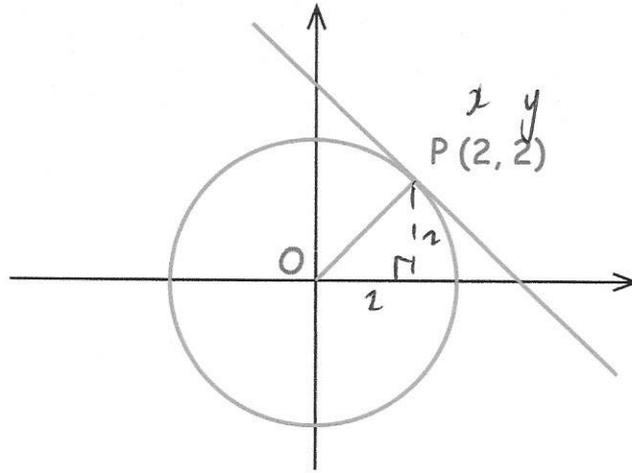
Video 372



Answers and Video Solutions



1. The diagram shows the circle $x^2 + y^2 = 8$ with a tangent at the point $(2, 2)$



- (a) Find the gradient of the line OP.

$$m = \frac{2}{2}$$

$$= 1$$

$$\frac{1}{\dots\dots\dots}$$

(1)

- (b) Find the gradient of the tangent

gradient is "negative reciprocal" as perpendicular to radius.

$$\frac{-1}{\dots\dots\dots}$$

(1)

- (c) Find the equation of the tangent

$$y = -x + c$$

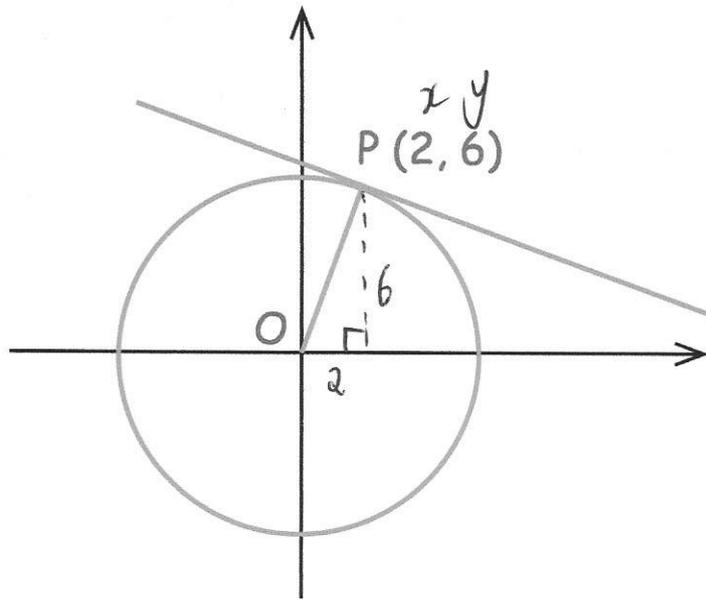
$$2 = -2 + c$$

$$c = 4$$

$$\frac{y = -x + 4}{\dots\dots\dots}$$

(2)

2. The diagram shows the circle $x^2 + y^2 = 40$ with a tangent at the point $(2, 6)$



- (a) Find the gradient of the line OP.

$$m = \frac{6}{2}$$

$$= 3$$

$$\frac{3}{\dots\dots\dots}$$

(1)

- (b) Find the gradient of the tangent

$$\frac{-\frac{1}{3}}{\dots\dots\dots}$$

(1)

- (c) Find the equation of the tangent

$$y = -\frac{1}{3}x + c$$

$$6 = -\frac{1}{3}(2) + c$$

$$6 = -\frac{2}{3} + c$$

$$6\frac{2}{3} = c$$

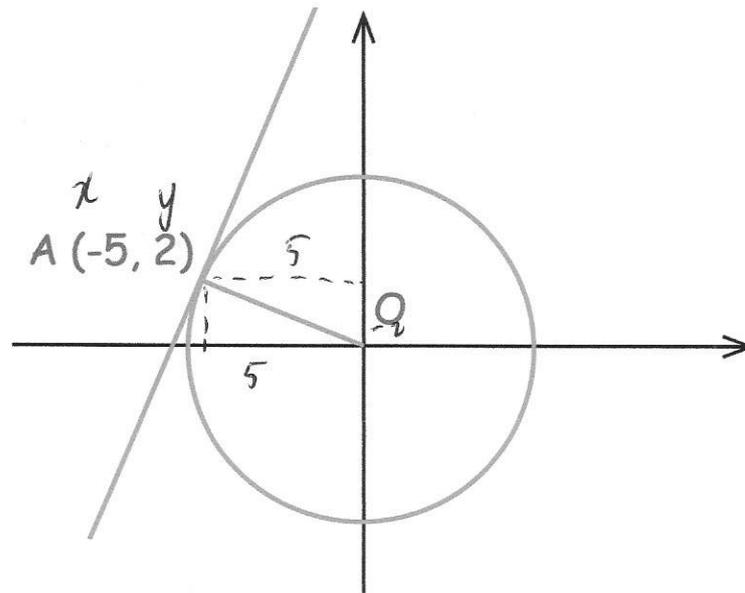
$$y = -\frac{1}{3}x + 6\frac{2}{3}$$

$$\text{or } y = -\frac{1}{3}x + \frac{20}{3}$$

$$\frac{\dots\dots\dots}{\dots\dots\dots}$$

(2)

3. The diagram shows the circle $x^2 + y^2 = 29$ with a tangent at the point $(-5, 2)$



- (a) Find the gradient of the line AO.

$$m = -\frac{2}{5}$$

$$\frac{-\frac{2}{5}}{\dots\dots\dots} \quad (1)$$

- (b) Find the gradient of the tangent

$$\frac{\frac{5}{2}}{\dots\dots\dots} \quad (1)$$

- (c) Find the equation of the tangent

$$y = \frac{5}{2}x + c$$

$$2 = \frac{5}{2}(-5) + c$$

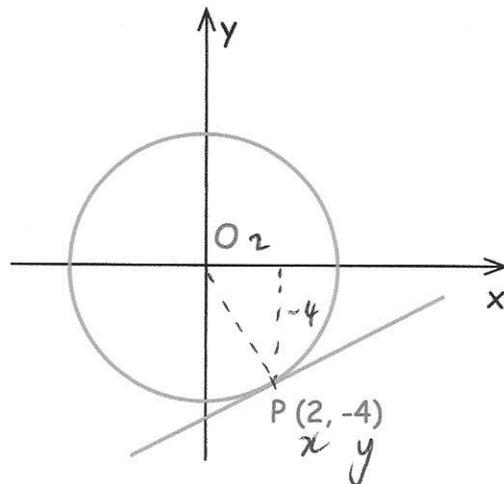
$$2 = -\frac{25}{2} + c$$

$$c = \frac{29}{2}$$

$$\frac{y = \frac{5}{2}x + \frac{29}{2}}{\dots\dots\dots} \quad (2)$$

4.

Here is a circle, centre O, and the tangent to the circle at the point (2, -4).



$$\begin{aligned} \text{gradient of } OP &= \frac{-4}{2} \\ &= -2 \end{aligned}$$

$$\text{gradient of tangent} = \frac{1}{2}$$

Find the equation of the tangent at the point P.

$$y = \frac{1}{2}x + c$$

$$-4 = \frac{1}{2}(2) + c$$

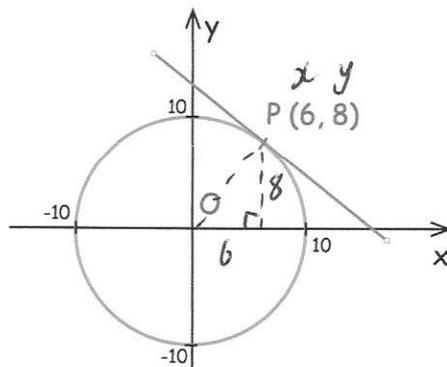
$$-4 = 1 + c$$

$$c = -5$$

$$y = \frac{1}{2}x - 5 \quad (3)$$

5.

Here is a circle, centre O, and the tangent to the circle at the point (6, 8).



$$\begin{aligned} \text{gradient of } OP &= \frac{8}{6} \\ &= \frac{4}{3} \end{aligned}$$

$$\text{gradient of tangent} = -\frac{3}{4}$$

$$y = -\frac{3}{4}x + c$$

$$8 = -\frac{3}{4}(6) + c$$

$$8 = -\frac{9}{2} + c$$

$$c = \frac{25}{2}$$

Find the equation of the tangent at the point P.

$$y = -\frac{3}{4}x + \frac{25}{2}$$

$$y = -\frac{3}{4}x + \frac{25}{2} \quad (3)$$

6. The line l is a tangent to the circle $x^2 + y^2 = 68$ at the point P.
 P is the point $(2, 8)$

Work out the equation of the line l

$$\text{Gradient of } OP = \frac{8}{2} = 4$$

$$y = -\frac{1}{4}x + c$$

$$8 = -\frac{1}{4}(2) + c$$

$$8 = -\frac{1}{2} + c$$

$$c = 8\frac{1}{2}$$

$$c = \frac{17}{2}$$

$$y = -\frac{1}{4}x + \frac{17}{2}$$

(3)

7. The diagram shows the circle $x^2 + y^2 = 17$

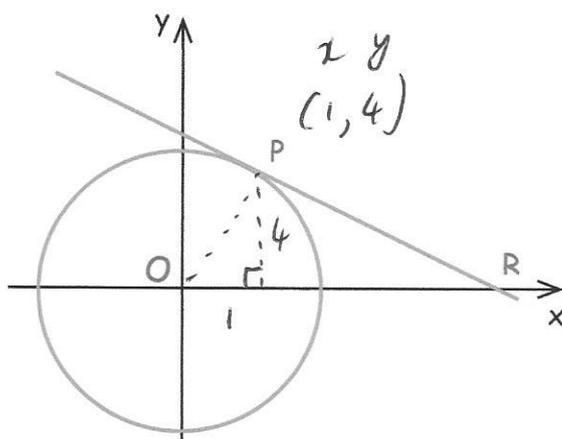


$$1 + y^2 = 17$$

$$y^2 = 16$$

$$y = \pm 4$$

$$(1, 4)$$



$$\text{gradient of } OP = \frac{4}{1} = 4$$

P lies on the circle and has x-coordinate 1.
 The tangent at P intersects the x-axis at R.

Work out the coordinates of R

$$y = -\frac{1}{4}x + c$$

$$4 = -\frac{1}{4} + c$$

$$c = 4\frac{1}{4}$$

$$c = \frac{17}{4}$$

$$y = -\frac{1}{4}x + \frac{17}{4}$$

Cross x-axis when $y=0$

$$0 = -\frac{1}{4}x + \frac{17}{4}$$

$$\frac{1}{4}x = \frac{17}{4}$$

$$x = 17$$

$$(17, 0)$$

(5)

8.

The points D $(-12, -5)$ and E $(12, 5)$ lie on the circle C.
DE is a diameter of C.

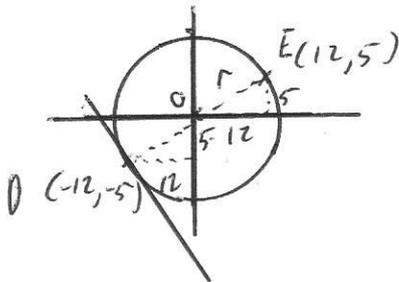
(a) Write down the coordinates of the centre of C.

midpoint of $(-12, -5)$ & $(12, 5)$

$$\left(\frac{-12+12}{2}, \frac{-5+5}{2} \right) = \left(\frac{0}{2}, \frac{0}{2} \right)$$

$$\underline{\underline{(0, 0)}} \quad (1)$$

(b) Work out the length of the radius of the C



$$r^2 = 5^2 + 12^2$$

$$r^2 = 25 + 144$$

$$r^2 = 169$$

$$r = 13$$

$$\underline{\underline{13}} \quad (1)$$

(c) Write down the equation of C

$$x^2 + y^2 = 169$$

or

$$\underline{\underline{x^2 + y^2 = 13^2}} \quad (1)$$

(d) Find the equation of the tangent to the circle, C, at the point D.

$$\text{gradient of } OD = \frac{5}{12}$$

$$\text{gradient of tangent} = -\frac{12}{5}$$

$$\begin{matrix} (-12, -5) \\ x \quad y \end{matrix}$$

$$y = -\frac{12}{5}x + c$$

$$-5 = \frac{144}{5} + c$$

$$c = -\frac{169}{5}$$

$$y = -\frac{12}{5}x - \frac{169}{5}$$

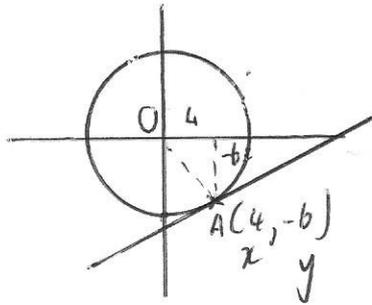
$$\underline{\underline{y = -\frac{12}{5}x - \frac{169}{5}}} \quad (4)$$

9.

The point A (4, -6) lies on the circle with equation $x^2 + y^2 = 52$



Find the coordinates of the point where the tangent to the circle at A crosses the x-axis.



$$\begin{aligned} \text{gradient of } OA &= -\frac{6}{4} \\ &= -\frac{3}{2} \end{aligned}$$

$$\text{gradient of tangent} = \frac{2}{3}$$

$$y = \frac{2}{3}x + c$$

$$-6 = \frac{2}{3}(4) + c$$

$$-6 = \frac{8}{3} + c$$

$$c = -\frac{26}{3}$$

$$y = \frac{2}{3}x - \frac{26}{3}$$

Crosses x-axis when $y=0$

$$0 = \frac{2}{3}x - \frac{26}{3}$$

$$\frac{26}{3} = \frac{2}{3}x$$

$$26 = 2x$$

$$x = 13$$

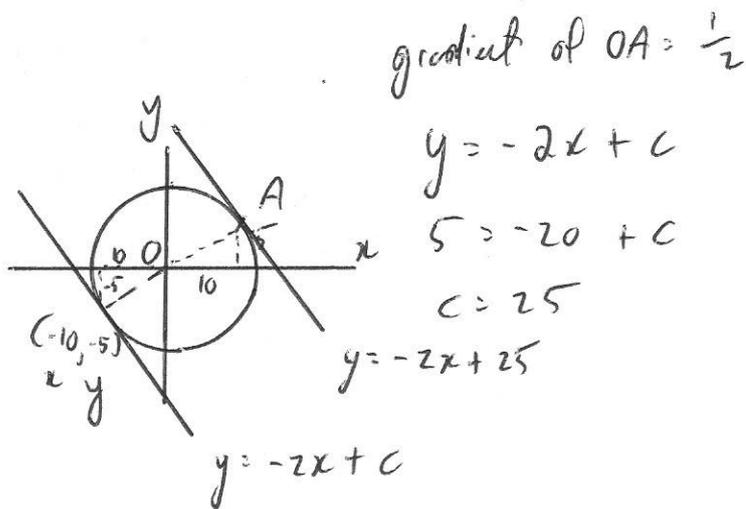
$$\underline{\underline{(13, 0)}}$$

(4)

10. The point A (10, 5) lies on the circle $x^2 + y^2 = 125$



(a) Work out the equation of the tangent to the circle at the point A.



$$\underline{y = -2x + 25}$$

(3)

The tangent to the circle at the point B is parallel to the tangent to the circle at point A.

(b) Work out the equation of the tangent to the circle at point B.

$$y = -2x + c$$

$$-5 = -2(-10) + c$$

$$-5 = 20 + c$$

$$c = -25$$

$$\underline{y = -2x - 25}$$

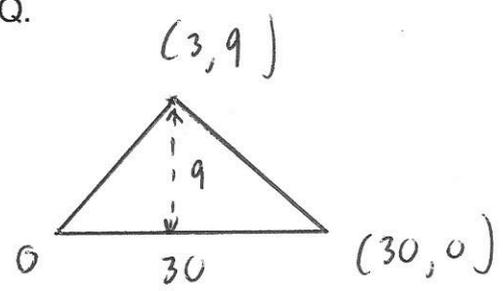
(3)

11. The line l is a tangent to the circle $x^2 + y^2 = 90$ at the point P.
 P is the point $(3, 9)$
 The line l crosses the x-axis at the point Q.

Work out the area of triangle OPQ.

gradient of OP = 3
 $y = -\frac{1}{3}x + c$
 $9 = -1 + c$
 $c = 10$
 $y = -\frac{1}{3}x + 10$

$0 = -\frac{1}{3}x + 10$
 $x = 30$
 Q(30, 0)

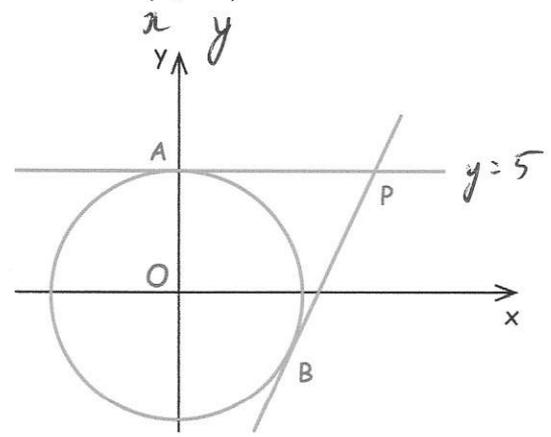


$\frac{1}{2} \times 30 \times 9 = 135$

135

(5)

12. The circle $x^2 + y^2 = 25$ has tangents at the points A and B.
 The point A has coordinates $(0, 5)$
 The point B has coordinates $(3, -4)$



The tangents meet at the point P.

Work out the coordinates of the point P.

gradient of OB = $-\frac{4}{3}$

$y = \frac{3}{4}x - 6.25$
 $y = 5$

$y = \frac{3}{4}x + c$
 $-4 = \frac{3}{4}(3) + c$
 $c = -\frac{25}{4}$

$\frac{3}{4}x - 6.25 = 5$
 $\frac{3}{4}x = 11.25$
 $x = 15$

(15, 5)

(5)

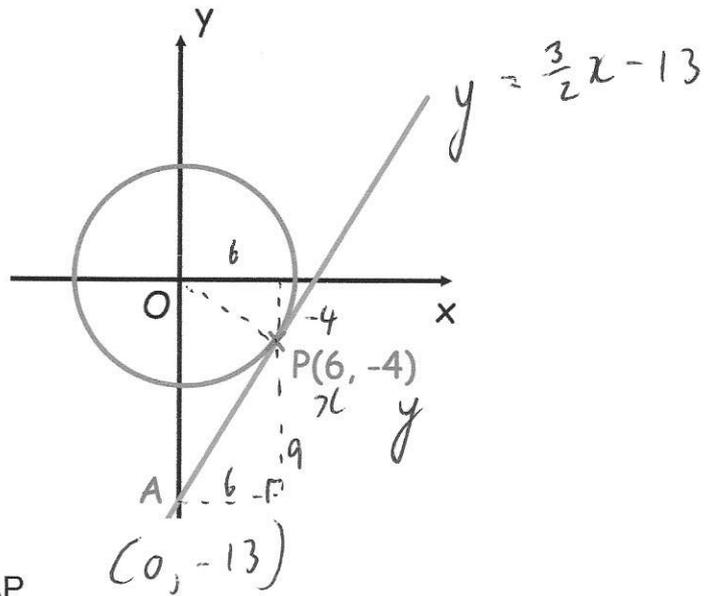
13. A circle is drawn with centre $(0, 0)$.
 The point $P(6, -4)$ lie on the circle.



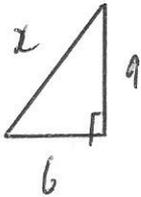
The tangent to the circle at P crosses the y -axis at the point A .

$$\begin{aligned} \text{gradient of } OP &= \frac{-4}{6} \\ &= -\frac{2}{3} \end{aligned}$$

$$\begin{aligned} y &= \frac{3}{2}x + c \\ -4 &= \frac{3}{2}(6) + c \\ c &= -13 \end{aligned}$$



Work out the length of AP .



$$x^2 = 6^2 + 9^2$$

$$x^2 = 36 + 81$$

$$x^2 = 117$$

$$x = \sqrt{117}$$

$$x = 10.8166\dots$$

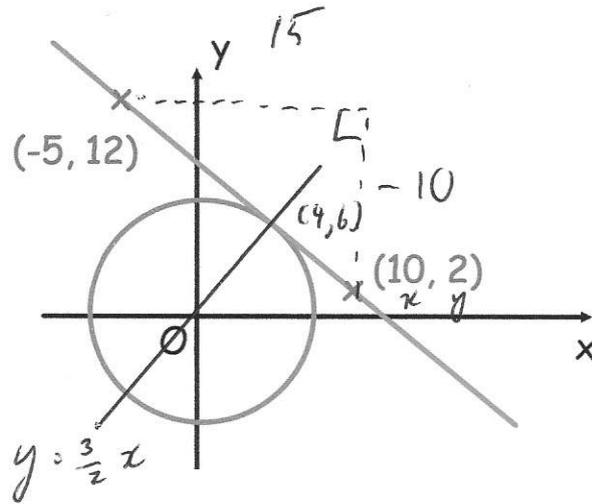
$$10.817$$

.....
 to 3dp (5)

14. A circle is drawn with centre (0, 0).



A tangent to the circle passes through the points (-5, 12) and (10, 2)



(a) Find the equation of the tangent.

$$m = \frac{-10}{15}$$

$$= -\frac{2}{3}$$

$$2 = -\frac{20}{3} + c$$

$$c = \frac{26}{3}$$

$$y = -\frac{2}{3}x + c$$

$$2 = -\frac{2}{3}(10) + c$$

$$y = -\frac{2}{3}x + \frac{26}{3}$$

(2)

(b) Find the equation of the circle.

$$y = \frac{3}{2}x \quad \text{and} \quad y = -\frac{2}{3}x + \frac{26}{3}$$

x6

$$\frac{3}{2}x = -\frac{2}{3}x + \frac{26}{3}$$

$$9x = -4x + 52$$

$$13x = 52$$

$$x = 4$$

$$y = \frac{3}{2}(4)$$

$$y = 6$$

$$x^2 + y^2 = r^2$$

$$4^2 + 6^2 = r^2$$

$$r^2 = 52$$

$$x^2 + y^2 = 52$$

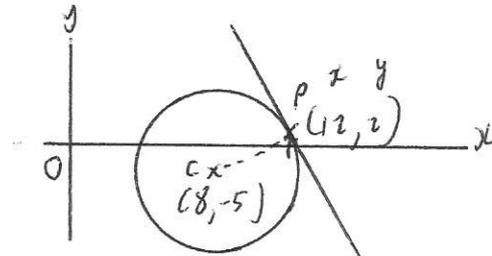
(4)

15. A circle is drawn with centre $(8, -5)$

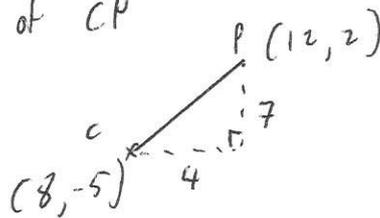


The circle passes through the point $P(12, 2)$

Find the equation of the tangent to the circle at P .



gradient of CP



$$m = \frac{7}{4}$$

$$y = -\frac{4}{7}x + c$$

$$2 = -\frac{4}{7}(12) + c$$

$$2 = -\frac{48}{7} + c$$

$$c = \frac{62}{7}$$

$$y = -\frac{4}{7}x + \frac{62}{7} \quad (5)$$