1st November **Higher Plus 5-a-day**

Corbettmaths

Josie writes down the number a "googol."
She has written the digit one followed by 100 zeros.

Write a googol in standard form.

1×10^{100}

$(3y + 5)(y - 2) + ay + b \equiv 3y^2 + y - 4$

Find the values of a and b

$3y^2 - y - 10 + ay + b$

$a = 2$

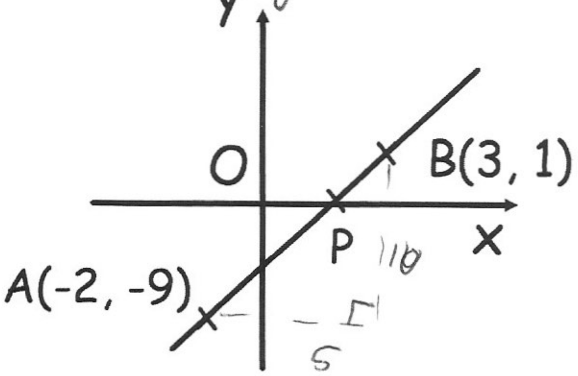
$b = 6$

A straight line passes through A(−2, −9) and B(3, 1).

$y = 2x + c$

The line crosses the x-axis at the point P.

$y = 2x - 5$



Find the coordinates of P

$0 = 2x - 5$

$5 = 2x$

$x = 2.5$

$(2.5, 0)$

Find the equation of the line perpendicular to AB that passes through P

$y = -\frac{1}{2}x + c$

$0 = -\frac{1}{2} \times \frac{5}{2} + c$

$0 = -\frac{5}{4} + c$

$c = \frac{5}{4}$

$y = -\frac{1}{2}x + \frac{5}{4}$

Write in the form $a\sqrt{b}$

$\sqrt{27} + \frac{18}{\sqrt{3}} \; \frac{\times\sqrt{3}}{\times\sqrt{3}}$

$\sqrt{9} \times \sqrt{3}$

$3\sqrt{3}$

$3\sqrt{3} + \frac{18\sqrt{3}}{3}$

$3\sqrt{3} + 6\sqrt{3}$

$9\sqrt{3}$

2nd November **Higher Plus 5-a-day** Corbettmaths

Helen says that the cosine of an angle is −1.

Write down three possible angles

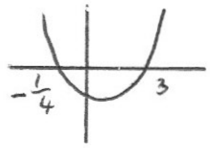
$180°$

$540°$

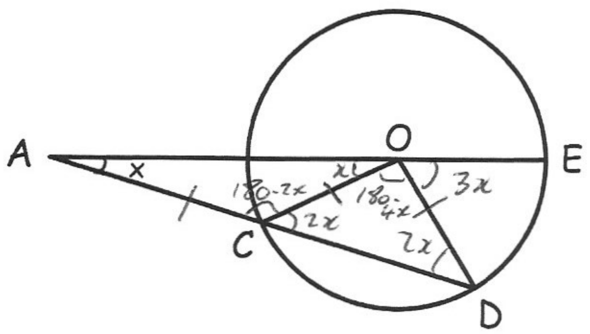
$900°$

Solve $(4x+1)(x-3)$

$4x^2 - 11x - 3 \le 0$



$-\frac{1}{4} \le x \le 3$



O is the circle of a circle that passes through the points C, D and E.

AOE is a straight line
AC = OD
Angle OAC = x

Prove angle DOE = 3x

$\angle AOC = x° = \angle OAC$
as $\triangle OAC$ is isosceles

$\angle OCA = 180 - 2x$
as angles in a triangle add to 180°.

$\angle OCD = 2x$
as angles in a straight line add to 180°

$\angle ODC = 2x$
as isosceles triangle ($\triangle OCD$)

$\therefore \angle COD = 180 - 4x$ (angles in a $\triangle$)

$\therefore \angle DOE = 3x$ as AOE is a straight line.

The universal set contains the whole numbers 1 to n.
n is an odd number greater than 200.
O is the set of odd numbers
P is the set of prime numbers

How many numbers are in O∪P?

$\underbrace{\frac{n+1}{2}}_{\text{odd numbers}} + \underbrace{1}_{\text{two}}$

3rd November **Higher Plus 5-a-day**

Corbettmaths

$4x = 5y$ $\quad x : y = 5 : 4$

$2y = 7z$ $\quad y : z = 7 : 2$

x, y and z are positive numbers.

Write the ratio x:y:z

$x \quad y \quad z$

$5 : 4$

$\quad 7 : 2$

$\cancel{35} : 28 : 8$

$35 : 28 : 8$

Given

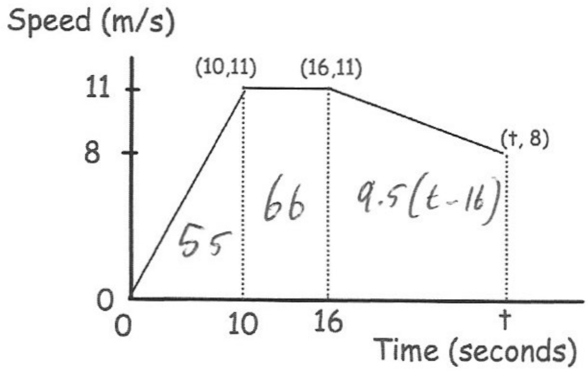
$$y = \frac{3\sqrt{2}}{2}$$

Write an expression for y^3

$\frac{3\sqrt{2}}{2} \times \frac{3\sqrt{2}}{2} \times \frac{3\sqrt{2}}{2} = \frac{27\sqrt{8}}{8}$

$= \frac{54\sqrt{2}}{8}$

$= \frac{27\sqrt{2}}{4}$



The average speed from 0 to t seconds was 8.725m/s

$s = \frac{d}{t} = \frac{121 + 9.5t - 152}{t}$

$8.725t = -31 + 9.5t$

Find t

$0.775t = 31$

$t = 40$

Find the acceleration for the first 10 seconds of the journey.

$1.1 m/s^2$

Find the nth term of the sequence

9 10 13 18 25 …

1 3 5 7

2 2 2

$a = 1$

$b = -2$

$c = 10$

$n^2 - 2n + 10$

4th November **Higher Plus 5-a-day**

Corbettmaths

A circle has equation

$$x^2 + y^2 = \frac{9}{4}$$

$\sqrt{\frac{9}{4}} = \frac{3}{2}$

Write down the length of its radius

$\frac{3}{2}$

Shown is part of $y = x^2 - 2x - 3$

By drawing an appropriate straight line, use your graph to find estimates for the solutions of $x^2 - 3x - 1 = 0$

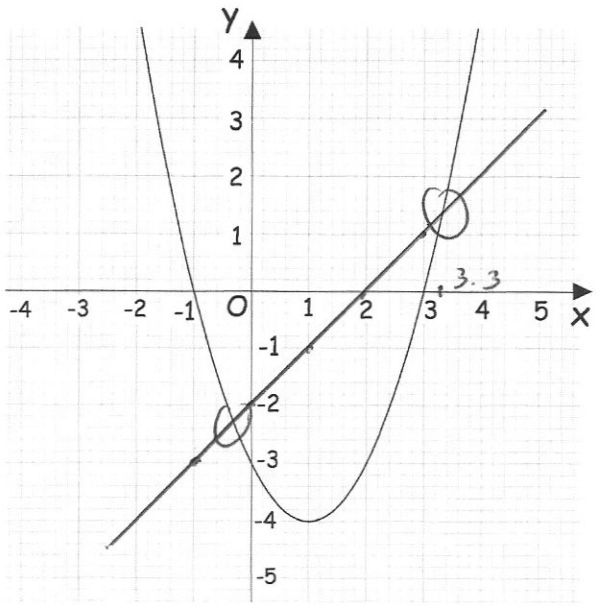
$y = x^2 - 2x - 3$

$0 = x^2 - 3x - 1$

$y = x - 2$

$x = -0.3$

& $x = 3.3$



There are x apples in a crate.
4 of the apples are bad.

Fiona chooses two apples from the crate, without replacement.
The probability she selects two bad apples is 1/11

$\frac{4}{x} \times \frac{3}{x-1} = \frac{1}{11}$

$\frac{12}{x^2 - x} = \frac{1}{11}$

$132 = x^2 - x$

$x^2 - x - 132 = 0$

Prove

$x^2 - x - 132 = 0$

Find x, the number of apples in the crate.

$(x-12)(x+11) = 0$

$\underline{x = 12}$ $\quad x = -11$ ✗

Corbettmaths

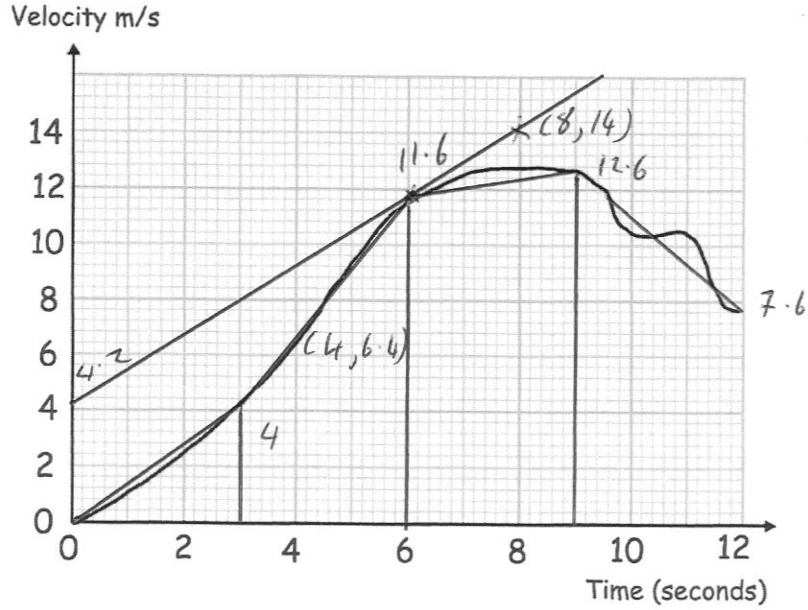
Work out

$$27^{\frac{2}{3}} \times 2^{-3}$$

$9 \times \frac{1}{8} = \frac{9}{8}$

giving your answer as a decimal

1.125



A) $\frac{1}{2} \times 3 \times 4 = 6$

B) $\frac{1}{2}(4 + 11.6) \times 3 = 23.4$

C) $\frac{1}{2}(11.6 + 12.6) \times 3 = 36.3$

D) $\frac{1}{2}(12.6 + 7.6) \times 3 = 30.3$

Here is a velocity/time graph for the first 12 seconds for a particle

Calculate an estimate for the acceleration of the particle at 6 seconds.

$\frac{\text{rise}}{\text{run}} = \frac{9.8}{8}$

1.225 m/s²

Calculate an estimate for the distance travelled by the particle in the first 12 seconds

96 m

Calculate an estimate for the average velocity of the particle over the first 12 seconds.

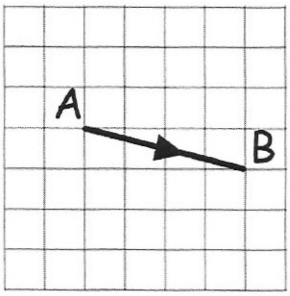
$\frac{96}{12} = 8$ m/s

Calculate an estimate for the average acceleration over the first 4 seconds.

$\frac{\text{rise}}{\text{run}} = \frac{6.4}{4} = 1.6$ m/s²

6th November **Higher Plus 5-a-day**

Corbettmaths



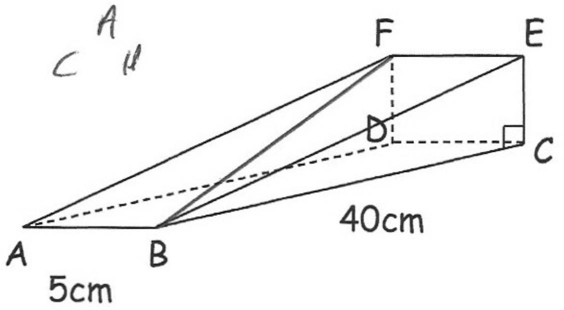
$\begin{pmatrix} 3 \\ 12 \end{pmatrix}$

$\overrightarrow{AB} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

Write down a vector that is perpendicular to AB and three times the length.

A triangular prism is shown below.

A
C H



Angle CBE = 8°

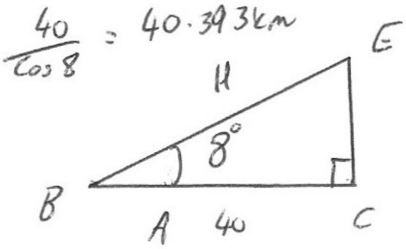
$BF^2 = BE^2 + EF^2$

$x^2 = 40.3931^2 + 5^2$

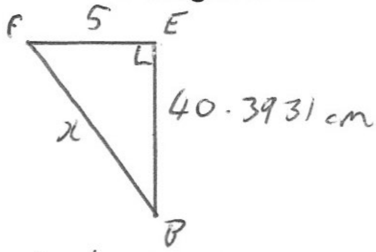
$x^2 = 1656.60276$

Calculate the length of BE

$\frac{40}{\cos 8} = 40.3934\text{cm}$



Calculate the length of BF



$x = 40.7014$ cm

A bag contains 15 sweets.
9 sweets are red.
4 sweets are yellow.
2 sweets are green.
Two sweets are taken from the bag without replacement.

$P(RR) = \frac{9}{15} \times \frac{8}{14}$

$= \frac{12}{35}$

$P(YY) = \frac{4}{15} \times \frac{3}{14} = \frac{2}{35}$

Work out the probability that the two sweets are same colour.

$P(GG) = \frac{2}{15} \times \frac{1}{14} = \frac{1}{105}$

$43/105$

Solve the simultaneous equations

$x - 7 = 2y$ $\quad x = 2y + 7$

$x^2 + 4y^2 = 37$

$(2y+7)(2y+7) + 4y^2 = 37$

$4y^2 + 28y + 49 + 4y^2 = 37$

$8y^2 + 28y + 12 = 0$

$2y^2 + 7y + 3 = 0$

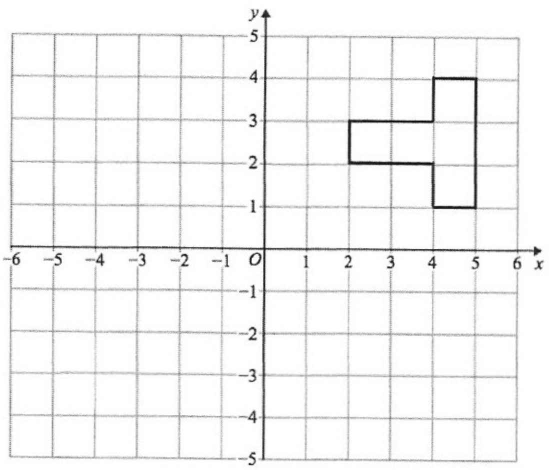
$(2y+1)(y+3) = 0$

$y = -\frac{1}{2}$ or $y = -3$

$x = 6$ $\quad x = 1$

7th November **Higher Plus 5-a-day**

Corbettmαths

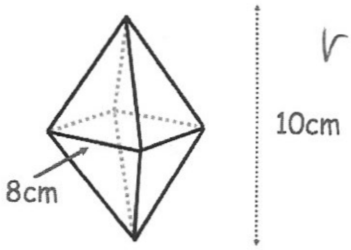


Describe fully a single transformation for which there is only one invariant point.

Rotation 180° about $(2, 2)$

Describe fully a single transformation for which there are three invariant points

reflection in $y = x$



$V = \frac{1}{3}Ah$

$= \frac{1}{3} \times 64 \times 5$

$= 106.\dot{6}$

Find the volume of the composite shape, made from two congruent square based pyramids.

$213.\dot{3}\ cm^3$

Solve

$$\frac{2}{x-5} - \frac{2}{x-4} = 1$$

$$\frac{2(x-4) - 2(x-5)}{(x-5)(x-4)} = 1$$

$$\frac{\cancel{2x} - 8 - \cancel{2x} + 10}{x^2 - 9x + 20} = 1$$

$2 = x^2 - 9x + 20$

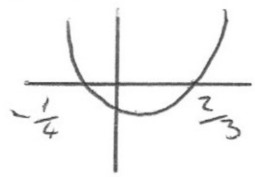
$0 = x^2 - 9x + 18$

$(x-6)(x-3) = 0$

$x = 3$ or $x = 6$

Solve $(3x-2)(4x+1)$

$12x^2 - 5x - 2 \geq 0$



$x \leq -\frac{1}{4}$

or

$x \geq \frac{2}{3}$

8th November **Higher Plus 5-a-day**

Corbettmaths

Show

$2x^3 + 3x^2 - 4x + 7 = 0$

has a solution between –3 and –2

$f(-2) = 11$

$f(-3) = -8$

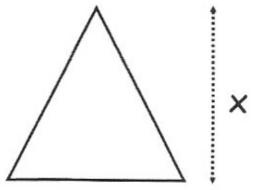
since there is a change of sign there is a root.

A particle travels 140m in 6.4 seconds. Both measurements are given to 2 significant figures.

Find the upper bound for the speed of the particle

$$\text{Max speed} = \frac{\text{Max distance}}{\text{min time}}$$

$$= \frac{145}{6.35} = 22.834\ldots \text{ m/s}$$



$\sqrt{12}$ cm

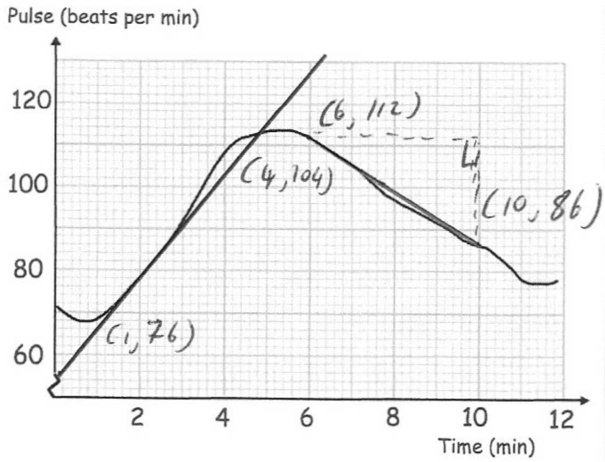
$\frac{1}{2} \times \sqrt{12} \times x = 2\sqrt{6}$

$\sqrt{12} \times x = 4\sqrt{6}$

$2\sqrt{3} \times x = 4\sqrt{6}$

$x = 2\sqrt{2}$ cm

The triangle has an area of $2\sqrt{6}$ cm²
Find the height of the triangle, x.
Give your answer as a simplified surd.



Work out the rate at which the pulse is increasing at two minutes.
Include units.

$$\frac{104-76}{4-1} = \frac{28}{3} = 9.\dot{3} \text{ beats per min per min}$$

or beats per min²

Work out the average rate at which the pulse is decreasing between six minutes and ten minutes.
Include units.

$$\frac{86-112}{4} = -6.5$$

6.5 beats per min per min

or beats per min².

9th November

Higher Plus 5-a-day

Corbettmaths

Solve

$(2x-3)^2 = 6x^2 - 5x + 5$

$(2x-3)(2x-3) = 6x^2 - 5x + 5$

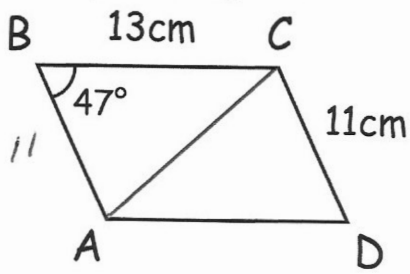
$4x^2 - 12x + 9 = 6x^2 - 5x + 5$

$0 = 2x^2 + 7x - 4$

$0 = (2x-1)(x+4)$

$x = \frac{1}{2}$ or $x = -4$

ABCD is a parallelogram.

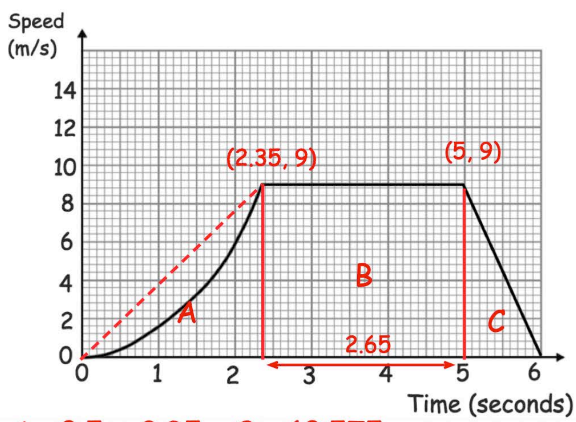


Find the area of the parallelogram.

$\frac{1}{2} \times 11 \times 13 \times \sin 47 = 52.2917\ldots$

$104.58 cm^2$

The graph below shows the speed of a bicycle.



A: 0.5 x 2.35 x 9 = 10.575m

B: 2.65 x 9 = 23.85m

C: 0.5 x 1 x 9 = 4.5m

Estimate the total distance travelled.

Answer may vary due to how area is found

10.575 + 23.85 + 4.5 = 38.925m

Is your estimate an underestimate or overestimate?

Overestimate - as the chord (A) is above the curve.

Write $x^2 + 6x + 21$ in the form $(x + a)^2 + b$

$(x+3)^2 - 9 + 21$

$(x+3)^2 + 12$

Find the turning point of the graph $y = x^2 + 6x + 21$

$(-3, 12)$

10th November **Higher Plus 5-a-day**

Corbettmaths

The square of w is 3

Write down the value of w^3

$w^2 = 3$

$w = \pm\sqrt{3}$

$w^3 = \pm\sqrt{27}$

$w^3 = \pm 3\sqrt{3}$

$3\sqrt{3}$ or $-3\sqrt{3}$

Given

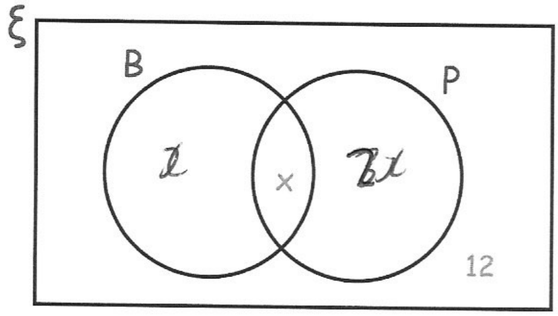
$$f(x) = 4x + 1$$

$$g(x) = 5 - 2x$$

Find gf(5) -37

$f(5) = 4 \times 5 + 1 = 21$

$g(21) = 5 - 2 \times 21 = -37$



There are 100 students in a college

B = number of students studying biology

P = number of students studying physics

A half of the students who study biology also study physics.
The number of students who study physics is 50% more than those studying biology.

Find x. $4x + 12 = 100$

$4x = 88$

$x = 22$

$y = x^2 + 4x + 1$ is translated 2 units left.

$y = f(x+2)$

Find the equation of the new curve.

$y = (x+2)^2 + 4(x+2) + 1$

$y = x^2 + 4x + 4 + 4x + 8 + 1$

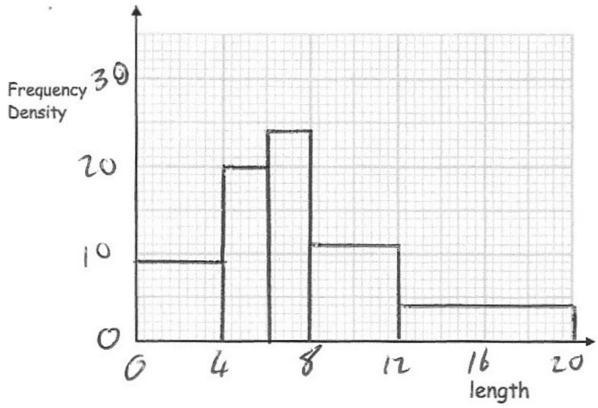
$y = x^2 + 8x + 13$

Corbettmaths

The lengths of 200 fish in a pond, *l* centimetres, are recorded below.

Length, l	Frequency	fd
$0 < l \leq 4$	36	9
$4 < l \leq 6$	40	20
$6 < l \leq 8$	48	24
$8 < l \leq 12$	44	11
$12 < l \leq 20$	32	4

Draw a histogram for this data.



Work out an estimate for the fraction of the fish that have a length between 5cm and 11cm.

$20 + 48 + 33 = 101$

$\frac{101}{200}$

The function f is such that
$f(x) = kx + 3$

The function g is such that
$g(x) = 2x - 4$

Given that $gf(2) = 34$

Work out the value of k

$f(2) = 2k + 3$

$g(2k+3) = 2(2k+3) - 4$

$= 4k + 6 - 4$

$= 4k + 2$

$4k + 2 = 34$

$4k = 32$

$\boxed{k = 8}$

Find the coordinates of the points where the circle $x^2 + y^2 = 8$ and the line $y = 2x + 2$ intersect.

$x^2 + (2x+2)^2 = 8$

$x^2 + 4x^2 + 8x + 4 = 8$

$5x^2 + 8x - 4 = 0$

$(5x - 2)(x + 2) = 0$

$x = 0.4$ $\quad$ $x = -2$

$y = 2.8$ $\quad$ $y = -2$

$(0.4, 2.8)$ $\quad$ $(-2, -2)$

or

$(\frac{2}{5}, 2\frac{4}{5})$

12th November **Higher Plus 5-a-day**

Corbettmaths

Panashe picks a five digit odd number.
The second digit is less than 8.
The fourth digit is a positive square number
The first digit is a prime number.

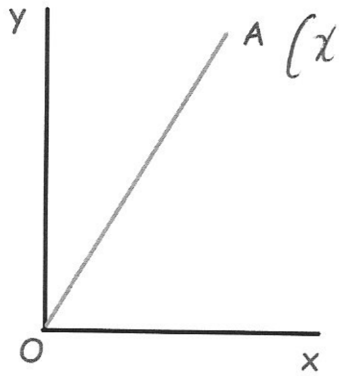
How many different numbers could she pick?

1st 2nd 3rd 4th 5th

$4 \times 8 \times 10 \times 3 \times 5$

2, 3, 5, 7 — 0 to 8 — 1, 4, 9 — 1, 3, 5, 7, 9

4800



A $(x, 3x)$

The line OA has a gradient of 3
The length of OA is $12\sqrt{10}$

Work out the coordinates of A

$\sqrt{(x^2) + (3x)^2} = 12\sqrt{10}$

$x^2 + 9x^2 = 1440$

$10x^2 = 1440$

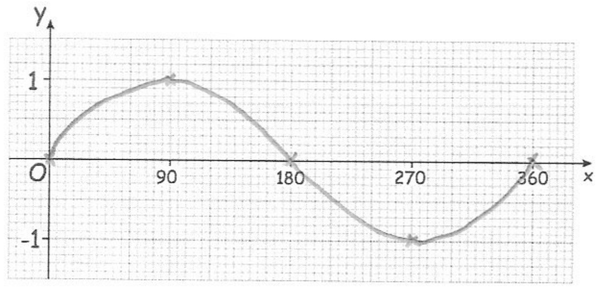
$x^2 = 144$

$x = \pm 12$

$x = 12$ given diagram.

$(12, 36)$

Sketch the graph of y = sin x for $0 \leq x \leq 360$.



The area of a rectangle is $\sqrt{125}$ cm²
The length of the rectangle is $(2+\sqrt{5})$ cm.
Calculate the width of the rectangle.

Express your answer in the form $a + b\sqrt{5}$, where a and b are integers.

$\frac{A}{l} = w$

$\frac{\sqrt{125}}{2+\sqrt{5}} = \frac{5\sqrt{5}}{2+\sqrt{5}} \times \frac{(2-\sqrt{5})}{(2-\sqrt{5})}$

$\frac{10\sqrt{5} - 25}{4-5} = \frac{10\sqrt{5} - 25}{-1}$

$= 25 - 10\sqrt{5}$ cm

13th November **Higher Plus 5-a-day**

Corbettmaths

Given that
$(x+3)(x+a)(x+7) \equiv x^3 + 15x^2 + 71x + 105$

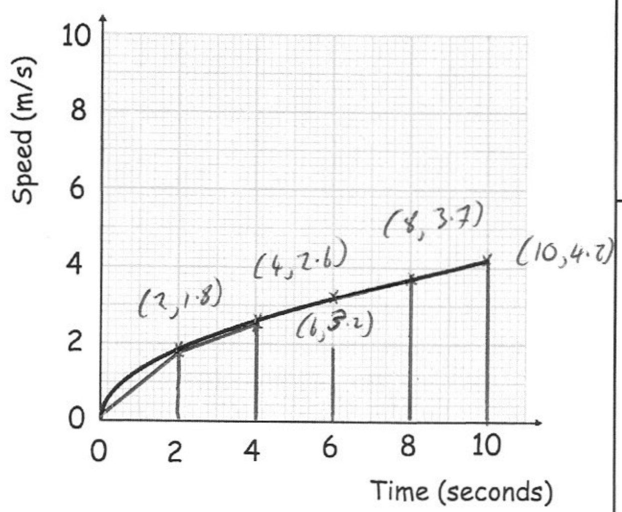
Find the value of a

$3 \times a \times 7 = 105$

$21a = 105$

$a = 5$

Henry walks for 10 seconds.
The speed-time graph shows information about his journey.



Find an estimate for how far Henry walks during the 10 seconds.

A: $\frac{1}{2} \times 2 \times 1.8 = 1.8$

B: $\frac{1}{2}(1.8+2.6) \times 2 = 4.4$

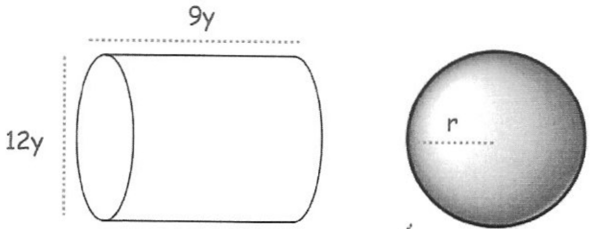
C: $\frac{1}{2}(2.6+3.2) \times 2 = 5.8$ $\approx 26.8\,m$

D: $\frac{1}{2}(3.2+3.7) \times 2 = 6.9$

E: $\frac{1}{2}(3.7+4.2) \times 2 = 7.9$

Is your answer an underestimate or an overestimate? Explain your answer.

Underestimate as the lines of the triangles / trapezia are under the curve



$\pi \times (6y)^2 \times 9y$ $\frac{4}{3}\pi r^3$

The cylinder and sphere have the same volume.
Express r in terms of y

$324\pi y^3 = \frac{4}{3}\pi r^3$

$972y^3 = 4r^3$

$243y^3 = r^3$ $r = \sqrt[3]{243y^3}$

$r = \sqrt[3]{243}\,y$

S is a geometric sequence

The first three terms of S are (x + 14), x and (2x – 21), where x is positive.

Find the value of x.

$x = 14$

$\frac{x}{x+14} \times \frac{2x-21}{x}$

$x^2 = (2x-21)(x+14)$

$x^2 = 2x^2 + 7x - 294$

$0 = x^2 + 7x - 294$

$(x+21)(x-14) = 0$

14th November **Higher Plus 5-a-day**

Corbettmαths

Solve the simultaneous equations x + y = 1 16x² + y² = 65	$x = 1 - y$ $16(1-y)^2 + y^2 = 65$ $16(1 - 2y + y^2) + y^2 = 65$ $16 - 32y + 16y^2 + y^2 = 65$ $17y^2 - 32y - 49 = 0$ $(17y - 49)(y + 1) = 0$ $y = \frac{49}{17}$ or $y = -1$ $x = \frac{-32}{17}$ $x = 2$
Work out in its simplest form $(4 + \sqrt{5})(4 - \sqrt{5})$ $= 16 - 4\sqrt{5} + 4\sqrt{5} - 5$ $= 11$	
Work out $\left(1\frac{9}{16}\right)^{-\frac{3}{2}}$ $\left(\frac{25}{16}\right)^{-\frac{3}{2}}$	$\left(\frac{16}{25}\right)^{\frac{3}{2}}$ $\sqrt{16} = 4$ $4^3 = 64$ $\sqrt{25} = 5$ $5^3 = 125$ $\frac{64}{125}$
The approximate solution to an equation is found by using the iterative process $x_{n+1} = \frac{(x_n)^3 - 7}{10}$ using $x_1 = -1$	Find $x_2 = \frac{(-1)^3 - 7}{10} = -\frac{4}{5}$
Work out the solution to 4 decimal places $x_3 = -0.7512$ $x_4 = -0.7423903242$ $x_5 = -0.7409113523$ $x_6 = -0.740673124$8	$x_7 = -0.7406330816$ $x_8 = -0.7406264916$ $x_9 = -0.7406254072$ -0.7406

15th November

Show

$x^4 - 7x^3 = 6$

has a solution between 7 and 8

$x^4 - 7x^3 - 6 = 0$

let

$f(x) = x^4 - 7x^3 - 6$

$f(7) = -6$

$f(8) = 506$

since f(x) is continuous and there is a change of sign, there is a solution.

Prove that the product of two consecutive even numbers is a multiple of 4.

$2n(2n+2)$

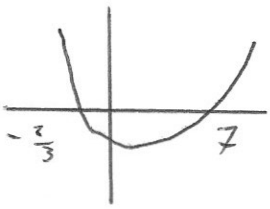
$4n^2 + 4n$

$4(n^2+n)$

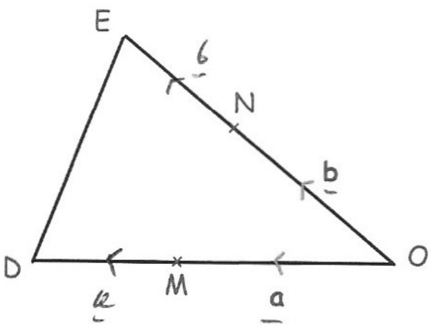
$\therefore$ multiple of 4

Solve $3x^2 - 19x - 14 < 0$

$(x-7)(3x+2)$



$-\frac{2}{3} < x < 7$



ODE is a triangle
M is the midpoint of OD
N is the midpoint of OE

$\overrightarrow{OM} = a$

$\overrightarrow{ON} = b$

Show DE is parallel to MN

$\overrightarrow{DE} = -2\underline{a} + 2\underline{b}$

$\overrightarrow{MN} = -\underline{a} + \underline{b}$

$\therefore$

$\overrightarrow{DE} = 2\overrightarrow{MN}$

so $\overrightarrow{DE}$ & $\overrightarrow{MN}$ are parallel

16th November **Higher Plus 5-a-day**

Corbettmαths

Make a the subject

$$\frac{a}{n} - \frac{w}{c} = \frac{a-w}{p}$$

$$\frac{ac - nw}{cn} = \frac{a-w}{p}$$

$p(ac - nw) = cn(a-w)$

$acp - npw = acn - cnw$

$acp - acn = npw - cnw$

$a(cp - cn) = npw - cnw$

$a = \frac{npw - cnw}{cp - cn} = \frac{nw(p-c)}{c(p-n)}$

w is directly proportional to the square of y.

$w = ky^2$

y is inversely proportional to the cube root of x.

w = 306 when y = 6.

$306 = k \times 36$

$k = 8.5$

When x = 125, y = 48

$w = 8.5y^2$

$y = \frac{k}{\sqrt[3]{x}}$ $48 = \frac{k}{5}$ $k = 240$ $y = \frac{240}{\sqrt[3]{x}}$

Given that w, y and x are positive, find the value of x when w = 12240000.

$12240000 = 8.5y^2$

$y = 1200$

$1200 = \frac{240}{\sqrt[3]{x}}$ $\sqrt[3]{x} = 0.2$ $x = 0.008$

The maximum load that an elevator can safely lift is 2000kg to the nearest 100kg.
The average adult is 80kg to the nearest 5 kg.
Bethan says that the maximum number of people that can safely use the elevator is 26, since $2050 \div 77.5 = 26.45...$

Explain why Bethan is incorrect.

For safety she should use 1950kg and 82.5kg

Sketch the graph of

f(x) = $-x^2 - 2x + 10$

showing the coordinates of the turning points and the coordinates of any intercepts with the coordinate axes.

y - axis

(0, 10)

turning point

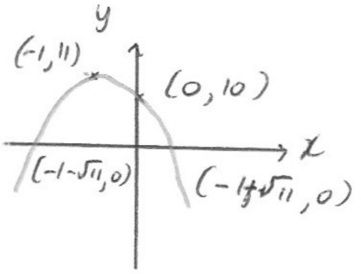
$y = -f(x) = x^2 + 2x - 10$

$(x+1)^2 - 1 - 10$

$(x+1)^2 - 11$

turning point of $y = -f(x)$ is $(-1, -11)$

turning point of $y = f(x)$ is $(-1, 11)$



y-axis

$0 = -x^2 - 2x + 10$

$x^2 + 2x - 10 = 0$

$(x+1)^2 - 11 = 0$

$(x+1)^2 = 11$

$x + 1 = \pm\sqrt{11}$

$x = -1 \pm \sqrt{11}$

$(-1+\sqrt{11}, 0)$

$(-1-\sqrt{11}, 0)$

17th November **Higher Plus 5-a-day**

Corbettmaths

Simplify this ratio fully

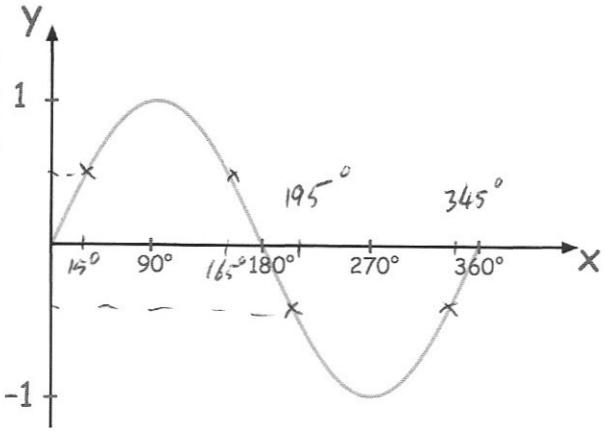
$$\sqrt{45} : \sqrt{180} : \sqrt{320}$$

$\sqrt{9} \times \sqrt{5} : \sqrt{36} \times \sqrt{5} : \sqrt{64} \times \sqrt{5}$

$3\sqrt{5} : 6\sqrt{5} : 8\sqrt{5}$

$3 : 6 : 8$

Here is a sketch of $y = sin x$ for $0° \leq x \leq 360°$



Given that $sin 195° = -0.2588$

Solve $sin x = 0.2588$ for $0° \leq x \leq 360°$

$x = 15°$ or $x = 165°$

Solve $x = 4 - y$

x + y − 4 = 0
$y^2 - 5 = 4x$

$y^2 - 5 = 4(4 - y)$

$y^2 - 5 = 16 - 4y$

$y^2 + 4y - 21 = 0$

$(y + 7)(y - 3) = 0$

$y = -7$ or $y = 3$

$x = 11$ or $x = 1$

A sphere has radius c
A hemisphere has radius d.

The volume of the hemisphere is twice the volume of the sphere.

$2\left(\frac{4}{3}\pi c^3\right) = \frac{2}{3}\pi d^3$

$\frac{8}{3}\pi c^3 = \frac{2}{3}\pi d^3$

Work out the value of

$$\frac{d}{c}$$

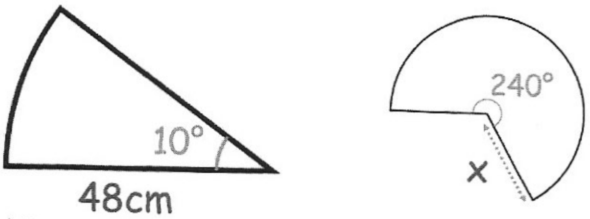
$4c^3 = d^3$

$\sqrt[3]{4}\, c = d$

$\frac{d}{c} = \sqrt[3]{4}$ (or 1.5874)

18th November — **Higher Plus 5-a-day**

Corbettmaths



$\frac{10}{360} \times \pi \times 48^2 = 64\pi$

The two sectors have the same area.
Find x

$\frac{240}{360} \times \cancel{\pi} \times x^2 = 64\cancel{\pi}$

$x^2 = 96$

$x = 4\sqrt{6}$ cm (9.798cm)

The curve C has equation
$y = x^2 + ax + b$

The minimum point of C has coordinates (−4, 6)

Find a and b

$a = 8$
$b = 22$

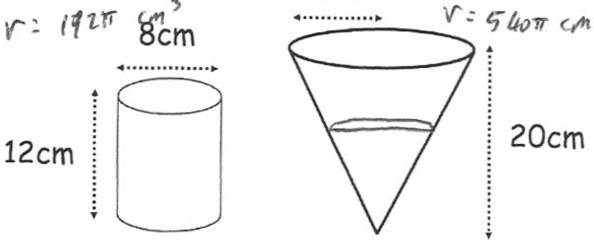
$y = (x+4)^2 + 6$

$y = x^2 + 8x + 16 + 6$

$y = x^2 + 8x + 22$

$\pi \times 4^2 \times 12$
$V = 192\pi$ cm^3

Cone $\frac{1}{3} \times \pi \times 9^2 \times 20$
$V = 540\pi$ cm^3



Shown above are two empty containers.

The cylinder is filled with water at a constant rate.
It takes 30 seconds to fill the container.

The cone is filled with water at the same constant rate.

How long does it take to fill the cone?

192π cm^3 in 30 seconds

540π cm^3 in 84.375 seconds

The cone is emptied and then the water from the cylinder is poured into the cone.
How many centimetres below the top of the cone does the water reach?

Similar shapes $540\pi \div 192\pi = 2.8125$

$\sqrt[3]{2.8125} = 1.411554\ldots$

$20 \div 1.411554\ldots = 14.16878\ldots$

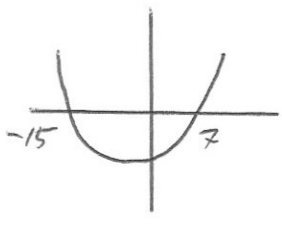
$20 - 14.16878\ldots = 5.8312$ cm

Solve $3x^2 + 5x - 105 < 2x^2 - 3x$

$x^2 + 8x - 105 < 0$

$(x + 15)(x - 7)$

$\boxed{-15 < x < 7}$

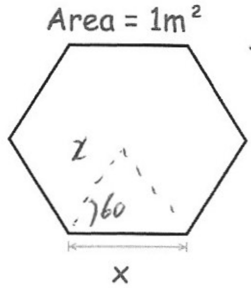


Factorise fully

$$(x+2)^4 - (x-3)(x+2)^2$$

$(x+2)^2\left((x+2)^2 - (x-3)\right)$

$(x+2)^2(x^2+3x+7)$



$\frac{1}{2} \times x \times x \times \sin 60 = \frac{1}{6}$

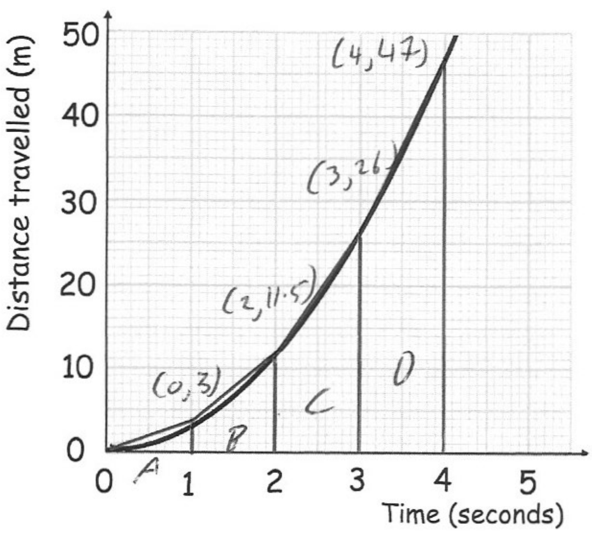
$x^2 = \frac{2\sqrt{3}}{9}$

$x = 0.62m$

Shown is a regular hexagon.
Find x

62.04cm

The graph shows information about part of a dog's journey.



Work out an estimate of the distance travelled by the dog over the first 4 seconds.

$A: \frac{1}{2} \times 1 \times 3 = 1.5$

$B: \frac{1}{2}(3+11.5) \times 1 = 7.25$

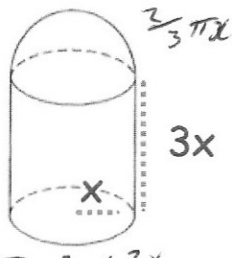
$C: \frac{1}{2}(11.5+26) \times 1 = 18.75$

$D: \frac{1}{2}(26+47) \times 1 = 36.5$

$\approx \boxed{64m}$

Is your answer an underestimate or an overestimate?

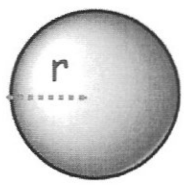
Overestimate



$\frac{2}{3}\pi x^3$

$\pi x^2 \times 3x$

$3\pi x^3$



$\frac{4}{3}\pi r^3$

Both shapes have the same volume.
Express r in terms of x.

$\frac{11}{3}\pi x^3 = \frac{4}{3}\pi r^3$

$11x^3 = 4r^3$

$r^3 = \frac{11}{4}x^3$

$r = \sqrt[3]{\frac{11}{4}x^3}$ or $r = \sqrt[3]{\frac{11}{4}}x$

20th November | Higher Plus 5-a-day | Corbettmaths

Write down the exact value of

sin 150°

$\frac{1}{2}$

$\sin 30 = \sin 150$

Find the value of y

$$2^y \times 4^{y+3} = 16$$

$2^y \times (2^2)^{y+3} = 2^4$

$2^y \times 2^{2y+6} = 2^4$

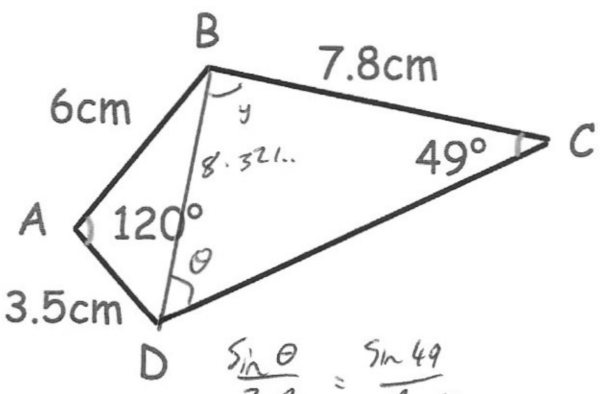
$3y + 6 = 4$

$3y = -2$

$y = -\frac{2}{3}$

$\triangle ABD = \frac{1}{2} \times 6 \times 3.5 \times \sin 120$

$= 9.09326674$



$\frac{\sin\theta}{7.8} = \frac{\sin 49}{8.321..}$

$\theta = 45°$

$y = 86°$

Calculate the length BD

$BD^2 = 6^2 + 3.5^2 - 2 \times 6 \times 3.5 \times \cos 120$

$BD^2 = 69.25$

$BD = 8.32165...$ cm

Calculate the area of ABCD

$\triangle BCD = \frac{1}{2} \times 8.32165.. \times 7.8 \times \sin 86$

$= 32.37539...$

$\triangle ABD + \triangle BCD = 41.469 \text{ cm}^2$

Charlotte and Ben invest money for 4 years.
Charlotte's bank paid 4% interest for the first year and then 1% compound interest for the other years.
Ben's bank pays the same percentage compound interest each year.

Charlotte 1.04×1.01^3

$= 1.07151304$

They invest the same amount of money and have the same amount of money at the end of 4 years.
Work out the percentage interest that Ben's bank pays.

$\sqrt[4]{1.07151304} = 1.0174178...$

1.742%

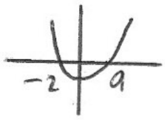
Corbettmaths

Solve

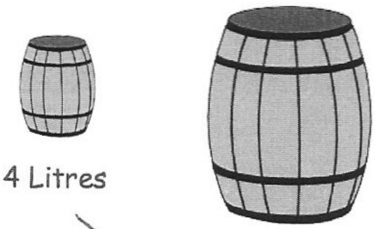
$x^2 > 7x + 18$

$x^2 - 7x - 18 > 0$

$(x-9)(x+2)$



$x < -2$ or $x > 9$



4 Litres

108 Litres

$\times 27$

$\sqrt[3]{27} = 3$

$3^2 = 9$

Two barrels are mathematically similar.
It takes 80ml of paint to paint the smaller barrel.
How much paint is needed for the larger barrel?

$80 \times 9 = 720$ ml

Age (A years)	Frequency
20 < A ≤ 25	145
25 < A ≤ 30	200
30 < A ≤ 35	94
35 < A ≤ 40	141
40 < A ≤ 45	294
45 < A ≤ 50	326 *
	1200

Calculate an estimate of the upper quartile.

$\frac{3}{4} \times 1200 = 900^{th}$

$45 + \frac{26}{326} \times 5 = 45.399$ years

Shown is a right angled triangle.
Find the possible value of x.

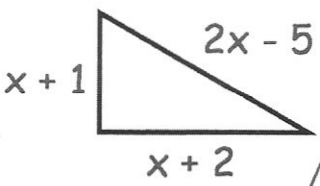
$(x+1)^2 + (x+2)^2 = (2x-5)^2$

$(x^2 + 2x + 1) + (x^2 + 4x + 4) = 4x^2 - 20x + 25$

$2x^2 + 6x + 5 = 4x^2 - 20x + 25$

$2x^2 - 26x + 20 = 0$

$x^2 - 13x + 10 = 0$



$a = 1$

$b = -13$

$c = 10$

$x = \frac{13 \pm \sqrt{129}}{2}$

$\boxed{x = 12.179 \text{ cm}}$ ✓

or

$x = 0.821$ cm ✗

Solve the simultaneous equations

$x - y = 3$

$x^2 + y^2 = 89$

$x = 3 + y$

$(y+3)^2 + y^2 = 89$

$y^2 + 6y + 9 + y^2 = 89$

$2y^2 + 6y - 80 = 0$

$y^2 + 3y - 40 = 0$

$y = -8$ or $y = 5$

$x = -5$ or $x = 8$

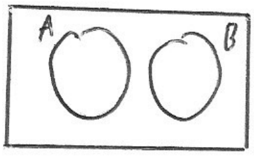
$(-5, -8)$ or $(8, 5)$

22nd November **Higher Plus 5-a-day**

Corbettmaths

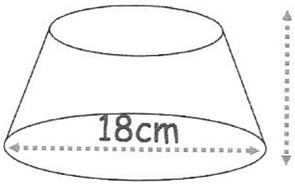
The events A and B are mutually exclusive.

P(A) = 0.7
P(B) = 0.2



Find P(A∩B)

$P(A \cap B) = 0$



Shown is a frustum of a cone that had a perpendicular height of 40cm

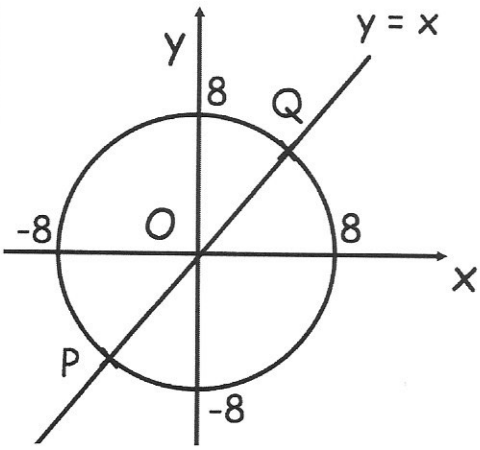
Calculate the volume of the frustum

$v = \frac{1}{3} \times \pi \times 9^2 \times 40 = 3392.92...$

$v = \frac{1}{3} \times \pi \times 4.5^2 \times 20 = 424.115...$

2968.8 cm^3

A straight line y = x intersects a circle at the points P and Q.



Find the coordinates of points P and Q.

$x^2 + y^2 = 64 \qquad y = x$

$x^2 + x^2 = 64$

$2x^2 = 64$

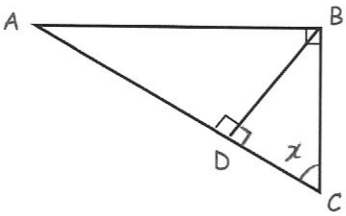
$x^2 = 32$

$x = \pm\sqrt{32}$

$x = 4\sqrt{2}$ or $x = -4\sqrt{2}$

$y = 4\sqrt{2} \qquad y = -4\sqrt{2}$

$P(-4\sqrt{2}, -4\sqrt{2}) \quad Q(4\sqrt{2}, 4\sqrt{2})$



$\angle BDC = 90°$
(straight line)

let $BCD = x$

$\angle DBC = 90 - x$
(angles in $\triangle BCD$)

$\angle CAB = 90 - x$
(angles in $\triangle ABC$)

ABC and ABD are right angled triangles.
ADC is a straight line.

Prove ABC and BCD are similar triangles.

since AAA they are similar.

Corbettmαths

Work out $\left(\frac{8}{125}\right)^{-\frac{2}{3}}$ $\left(\frac{125}{8}\right)^{\frac{2}{3}}$	$\left(\frac{5}{2}\right)^{2} = \frac{25}{4}$ or $6\frac{1}{4}$
The Line L passes through (1, 6) and (2, 1). gradient of $L = -5$ $y = -5x + 11$ Find the equation of the line P, perpendicular to L, which passes through (6, −4) x y	$y = \frac{1}{5}x + c$ $-4 = \frac{6}{5} + c$ $c = -\frac{26}{5}$ $y = \frac{1}{5}x - 5\frac{1}{5}$
Find the coordinates of the point of intersection of the lines L and P $\frac{1}{5}x - 5.2 = -5x + 11$ $5.2x = 16.2$ $x = \frac{81}{26}$ $y =$	$\left(\frac{81}{26}, -\frac{119}{26}\right)$
Find the nth term of −10 −7 −2 5 … 3 5 7 2 2 $a = 1$ $b = 0$ $c = -11$	$n^2 - 11$
Given $2^{89} - 1$ is prime. Show that $2^{89} + 1$ is a multiple of 3	$2^{89} - 1$ is prime 2^{89} is multiple of 2 (even) but not 3. $\therefore$ 2^{89} is a multiple of 3

24th November **Higher Plus 5-a-day**

Corbettmaths

A curve with equation $y = 6^x$ crosses the y-axis at the point A.

A is $(0, 1)$

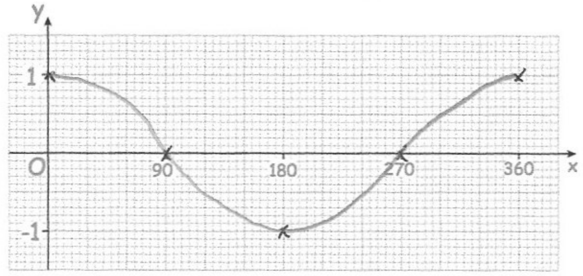
A circle with centre (0, 0) passes through the point A.

Write down the equation of the circle.

$r = 1$

$x^2 + y^2 = 1$

Sketch the graph of y = cos x for $0 \le x \le 360$.



Yasmin is making a scale drawing of Wales.

The distance from Conwy to Newport is 2×10^5 metres. 2×10^7 cm

Yasmin wants the distance between Conway and Newport to be 30cm on her drawing.

$(2 \times 10^7) \div 30 = 6.\dot{6} \times 10^5$

Work out the scale that Yasmin should use, in the form n : 1.

$6.\dot{6} \times 10^5 : 1$

The area of a circle is 20cm² to the nearest cm². $19.5 \;/\; 20.5$

Find the error interval for the circumference of the circle.

$\pi r^2 = 19.5$ $d = 4.982\ldots$ cm

$r = 2.4913\ldots$ cm $C = 15.65\ldots$ cm

$\pi r^2 = 20.5$ $d = 5.108\ldots$

$r = 2.55\ldots$ $C = 16.05\ldots$ cm

$15.65\ldots \text{cm} \le C < 16.05\ldots \text{cm}$

Rearrange $w = \dfrac{x^3 + 2}{2x^3 - y}$

to make x the subject

$w(2x^3 - y) = x^3 + 2$

$2wx^3 - wy = x^3 + 2$

$2wx^3 - x^3 = wy + 2$

$x^3(2w - 1) = wy + 2$

$x^3 = \dfrac{wy + 2}{2w - 1}$

$x = \sqrt[3]{\dfrac{wy + 2}{2w - 1}}$

25th November **Higher Plus 5-a-day**

Corbettmaths

Write $x^2 - 6x + 1$ in the form $(x + a)^2 + b$, where a and b are integers to the form.

$(x-3)^2 - 9 + 1$

$(x-3)^2 - 8$

Write 3.418181818… as a fraction.

$x = 3.41818...$

$10x = 34.1818...$

$1000x = 3418.1818...$

$990x = 3384$

$x = \frac{3384}{990}$

$x = \frac{188}{55}$ (or $3\frac{23}{55}$)

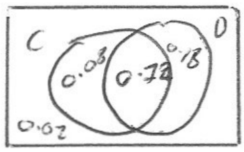
C and D are two independent events

$P(C) = 0.8$

$P(D') = 0.1$ $\quad p(D) = 0.9$

$P(C \cup D)$

$0.8 \times 0.9 = 0.72$



0.98

A scientist is carrying out an experiment to remove microplastics from water.
In an experiment 20,000 microplastics are added to a sample of clean water.

The number of microplastics, M, after t minutes is $M = 20000 \times 2^{-t}$

Calculate the number of microplastics in the water after 3 minutes.

20000×2^{-3}

$= 2500$

After how many complete minutes does it take for the number of microplastics to fall below 100?

20000×2^{-8}

78.125

8 minutes

26th November **Higher Plus 5-a-day**

Corbettmaths

n is the set of even numbers from 1 to 200

cubes: 8, 64

O is the set of odd numbers 0

P is the set of prime numbers 1 (two)

C is the set of cube numbers 2

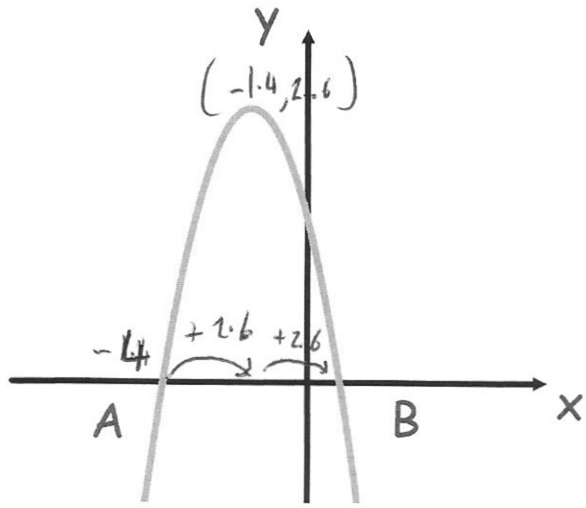
S is the set of square numbers 7 (inc. 64)

Square: 4, 16, 36, 64, 100, 144, 196

How many numbers are there from set n that are in the set

$O \cup P \cup C \cup S$

9



Shown is a graph $y = f(x)$ where $f(x)$ is a quadratic function.

The coordinates of point A are $(-4, 0)$

The maximum point is $(-1.4, 2.6)$

Write down the coordinates of point B

$(1.2, 0)$

The equation $f(x) = k$ has exactly one solution.

Write down the value of k

2.6

Mrs Jenkins runs a company that makes decorations for weddings.
She needs $18\sqrt{5}$ metres of ribbon in total.
Mrs Jenkins has 40 metres of ribbon.

Without using a calculator, work out if she has enough ribbon.

$18\sqrt{5} = \sqrt{1620}$

$\sqrt{1620} > \sqrt{1600}$

No, she does not have enough.

The point A has coordinates $(-6, 0)$ (x_1, y_1)
The point B has coordinates $(0, 3)$ (x_2, y_2)
The point C has coordinates $(9, -1)$ (x, y)

Find the equation of the line that passes through C and is perpendicular to AB.

gradient of AB $= \frac{3-0}{0--6} = \frac{1}{2}$

gradient of line $= -2$

$y = -2x + c$

$-1 = -18 + c$

$c = 17$

$y = -2x + 17$

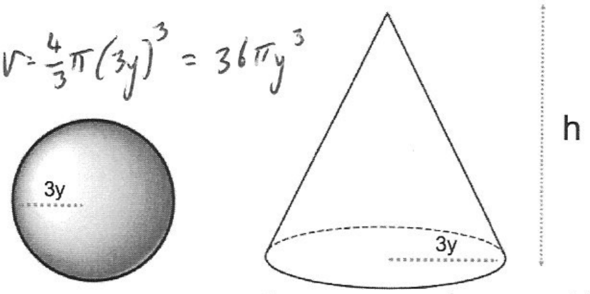
Corbettmaths

Express $x^2 + 8x + 40$ in the form $(x + a)^2 + b$

$(x+4)^2 - 16 + 40$

$(x+4)^2 + 24$

$V = \frac{4}{3}\pi(3y)^3 = 36\pi y^3$



$\frac{1}{3}\pi(3y^2)h = 3\pi y^2 h$

The volume of the cone is twice the volume of the sphere.

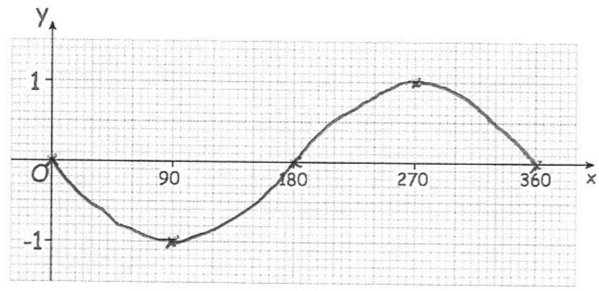
$2(36\pi y^3) = 3\pi y^2 h$

Express h in terms of y.

$72\cancel{\pi} y^{\cancel{3}} = 3\cancel{\pi} \cancel{y^2} h$

$24y = h$

$h = 24y$



Sketch the graph of $y = -\sin x$ for $0 \le x \le 360$.

Given that $a = \sqrt{3}$ and $b = \sqrt{48}$

find the value of a^2

$\sqrt{3} \times \sqrt{3} = 3$

show that $(a + b)^2 = 75$

$(\sqrt{3} + \sqrt{48})(\sqrt{3} + \sqrt{48})$

$3 + \sqrt{144} + \sqrt{144} + 48$

$3 + 12 + 12 + 48 = 75$ ✓

$\mathbf{c} = \begin{pmatrix} -4 \\ q \end{pmatrix}$ $\mathbf{d} = \begin{pmatrix} p \\ 2 \end{pmatrix}$

Given $3\mathbf{d} - \mathbf{c} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Work out the values of p and q

$3p + 4 = 1$

$6 - q = 0$

$3p = -3$ $q = 6$

$p = -1$

28th November **Higher Plus 5-a-day**

Corbettmaths

Solve the simultaneous equations

$$9^x \times 27^{2-y} = 3\sqrt{3}$$

$$3x + 2y = 3$$

$2x - 3y = -4.5$

$9x + 6y = 9$

$4x - 6y = -9$

$13x = 0$

$x = 0 \quad y = 1.5$

$(3^2)^x \times (3^3)^{2-y} = 3^{1.5}$

$3^{2x} \times 3^{6-3y} = 3^{1.5}$

$\therefore \; 2x + 6 - 3y = 1.5$

$2x - 3y = -4.5$

$\boxed{x = 0 \quad y = 1.5}$

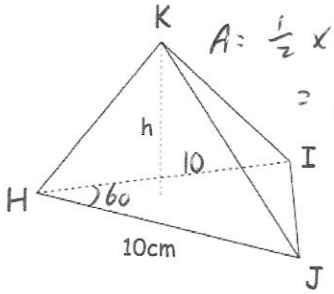
Expand and simplify

$(3 + \sqrt{8})(4 + \sqrt{2})$

$12 + 3\sqrt{2} + 4\sqrt{8} + \sqrt{16}$

$12 + 3\sqrt{2} + 8\sqrt{2} + 4$

$16 + 11\sqrt{2}$



$A = \frac{1}{2} \times 10 \times 10 \times \sin 60$

$= 25\sqrt{3}$

$V = \frac{1}{3} \times 25\sqrt{3} \times h$

$300 = \frac{1}{3} \times 25\sqrt{3} \times h$

$900 = 25\sqrt{3} \times h$

HIJK is a triangle based pyramid.
The base HIJ is an equilateral triangle with side 10cm.
The volume of the pyramid is 300cm³.
Calculate the perpendicular height, *h*, of the pyramid.

$36 = \sqrt{3} \times h$

$h = 12\sqrt{3}$ or 20.7846 cm

The point (–6, –7) is the turning point of the graph $y = x^2 + ax + b$

Find a and b

$(x+6)^2 - 7$

$x^2 + 12x + 36 - 7$

$x^2 + 12x + 29$

$y = x^2 + 12x + 29$

$a = 12 \quad b = 29$

C and D are two independent events

$P(C) = 0.6$

$P(D') = 0.3$ $\quad P(D) = 0.7$

Find $P(C \cap D)$

$0.6 \times 0.7 = 0.42$

29th November **Higher Plus 5-a-day**

Corbettmaths

If

$f(x) = 4x^{\frac{2}{3}} - x^{-1}$

find f(8)

$x^{-1} = \frac{1}{x}$

$8^{\frac{2}{3}} = 4$

$4 \times 4 - \frac{1}{8} = 16 - \frac{1}{8}$

$15\frac{7}{8}$

A group of scientists want to estimate the number of eels in a lake.
They catch and ring 200 eels.
They return the 200 eels to the lake.
They then catch 500 eels.
Of these, 18 are ringed.

Estimate the number of eels in the lake.

$\frac{200}{N} = \frac{18}{500}$

$18N = 100000$

$N = 5555.\dot{5}$

5556

(or 5555)

The first 5 triangular numbers are

1, 3, 6, 10, 15

2 3 4 5

1 1 1

$an^2 + bn + c$

$a = \frac{1}{2}$ $b = \frac{1}{2}$ $c = 0$

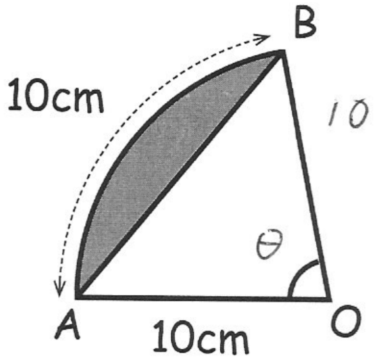
by considering the nth term, find the 100th triangular number

$\frac{1}{2}n^2 + \frac{1}{2}n$

$\frac{1}{2} \times 100^2 + \frac{1}{2} \times 100$

5050

OAB is a sector of a circle, centre O.
OA is 10cm and arc AB is also 10cm.



Find the size of angle AOB

$\frac{\theta}{360} \times \pi \times 20 = 10$

$\theta = 57.296°$

Find the area of the shaded segment.

Sector $\frac{57.29...}{360} \times \pi \times 10^2 = 50\text{ cm}^2$

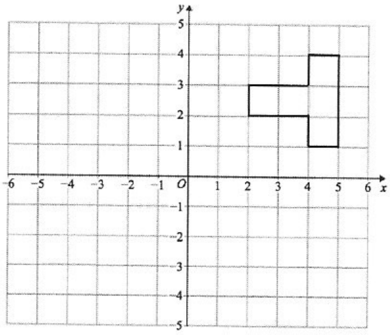
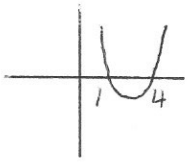
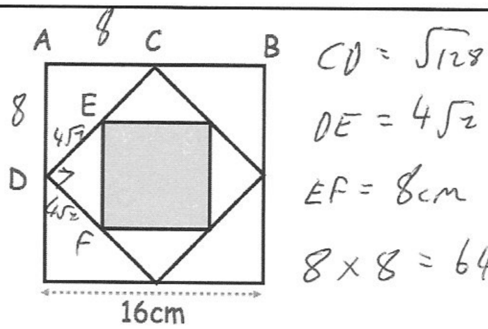
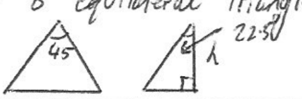
triangle

$\frac{1}{2} \times 10 \times 10 \times \sin 57.29... = 42.07...$

Segment : 7.926 cm^2

Higher Plus 5-a-day

Corbettmaths

	Describe fully a single transformation for which there are exactly two invariant points. Reflection $x = 3$
Solve $x^2 - 5x + 4 < 0$  $(x-1)(x-4)$	$1 < x < 4$
The midpoints of the sides of a square of side 16cm are joined to form another square. This process is then repeated to create the shaded square Find the area of the shaded square.	 $CD = \sqrt{128}$ $DE = 4\sqrt{2}$ $EF = 8cm$ $8 \times 8 = 64cm^2$
Solve $$\frac{4}{x+1} + \frac{2}{x-2} = 3$$ $\frac{4(x-2) + 2(x+1)}{(x+1)(x-2)} = 3$	$\frac{4x - 8 + 2x + 2}{x^2 - x - 2} = 3$ $\frac{6x - 6}{x^2 - x - 2} = 3$ $3x^2 - 3x - 6 = 6x - 6$ $0 = 3x^2 - 9x$ $0 = 3x(x-3)$ $x = 0$ or $x = 3$
An octagonal pyramid has a base that is a regular octagon with side length 20cm. The perpendicular height of the pyramid is 75cm. base - 8 equilateral triangles 	Find the volume of the pyramid $A = (\frac{1}{2} \times 20 \times 24.142\ldots) \times 8$ $A = 1931.37085$ $V = \frac{1}{3} \times 1931.37085\ldots \times 75$ $V = 48284.27125\ cm^3$

20   10 T^OA

$h = 24.142\ldots$