1st February **Higher Plus 5-a-day**

Corbettmaths

The circle $x^2 + y^2 = 25$ has tangents at the points A and B. $r = 5$

The point A has coordinates (0, 5)
The point B has coordinates (3, –4)

The tangents meet at the point P.

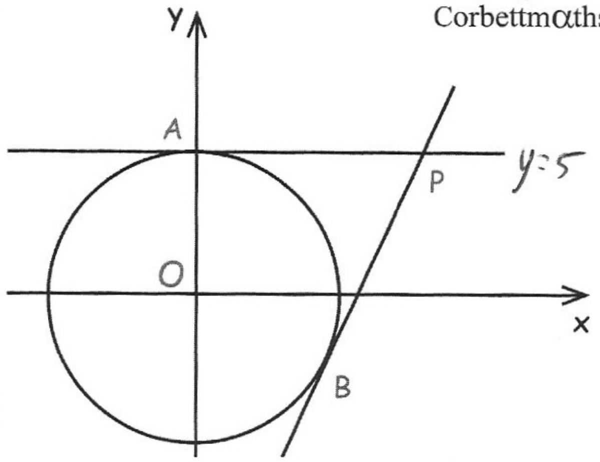
Work out the coordinates of the point P.

gradient of OB is $-\frac{4}{3}$

$y = \frac{3}{4}x + c$

$-4 = \frac{9}{4} + c$

$c = -\frac{25}{4}$

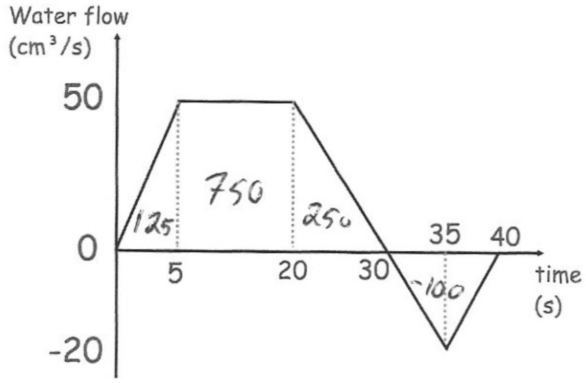


$y = \frac{3}{4}x - \frac{25}{4}$

$5 = \frac{3}{4}x - \frac{25}{4}$

$\frac{3}{4}x = 11\frac{1}{4}$ $x = 15$

(15, 5)



The graph above shows information on how an empty container is being filled with water.

What happens between 30 and 40 seconds?

Water is poured out

How much water is in the container after 40 seconds?

$125 + 750 + 250 - 100$

$= 1025\ cm^3$

The first five terms of a quadratic sequence are:

30 36 46 60 78
6 10 14 18
4 4 4

$a = 2$
$b = 0$
$c = 28$

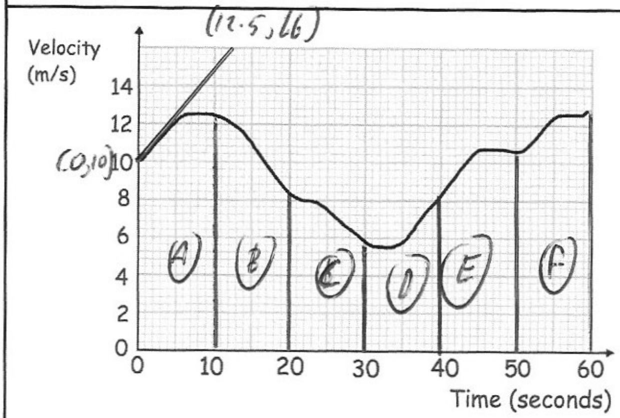
Work out the nth term

$2n^2 + 28$

Corbettmaths

Rationalise the denominator

$$\frac{2}{\sqrt{3}} \begin{matrix} \times\sqrt{3} \\ \times\sqrt{3} \end{matrix} = \frac{2\sqrt{3}}{3}$$



Here is a velocity time graph for the first 60 seconds of a journey.

$A: \frac{1}{2}(10 + 12.4) \times 10 = 112$

$B: \frac{1}{2}(12.4 + 8.4) \times 10 = 104$

$C: \frac{1}{2}(8.4 + 5.8) \times 10 = 71$

Calculate an estimate for the acceleration after 5 seconds

$$\frac{16-10}{12.5-0} = 0.48\ m/s^2$$

Calculate an estimate for the total distance travelled in the 60 seconds.

$D = \frac{1}{2}(5.8 + 8) \times 10 = 69$

$E: \frac{1}{2}(8 + 10.6) \times 10 = 93$

$F = \frac{1}{2}(10.6 + 12.8) \times 10 = 117$

566 m

Solve the simultaneous equations

$$y = x + 3$$
$$x^2 + y^2 = 29$$

$x^2 + (x+3)^2 = 29$

$x^2 + x^2 + 6x + 9 = 29$

$2x^2 + 6x - 20 = 0$

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$x = -5$, $y = -2$ or $x = 2$, $y = 5$

Length, L cm	Frequency
$0 \le L < 4$	7
$4 \le L < 8$	10
$8 \le L < 12$	53
$16 \le L < 20$	25
$20 \le L < 24$	5

✱

100

Work out an estimate of the median length 50th value

$$8 + \frac{33}{53} \times 4$$

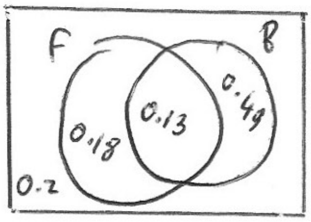
10.49 cm

3rd February **Higher Plus 5-a-day**

Corbettmαths

$P(F) = 0.31$
$P(B) = 0.62$
$P(F \cup B) = 0.8$

Draw a fully labelled Venn diagram to represent this information.



Find $P(F' \cap B)$

0.49

The line l is a tangent to the circle $x^2 + y^2 = 68$ at the point P.

P is the point (2, 8) gradient of OP = 4

gradient of tangent = $-\frac{1}{4}$

Work out the equation of the line l

$y = -\frac{1}{4}x + c$

$8 = -\frac{1}{2} + c$

$c = 8.5$

$\boxed{y = -\frac{1}{4}x + 8.5}$

The gravitational force, F, between two objects is inversely proportional to the square of the distance, d, between them.

$F \propto \frac{1}{d^2}$

When F = 4, d = 3.

$F = \frac{k}{d^2}$

Find F when d = 6.

$4 = \frac{k}{9}$

$k = 36$

$F = \frac{36}{d^2}$

$F = \frac{36}{6^2} = \frac{36}{36} = 1$

$F = 1$

Find the set of values of x that satisfy both

$2x - 6 > 6 - 6x$

$8x > 12$

$x > 1.5$

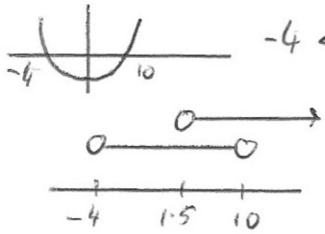
and

$x^2 - 6x + 2 < 42$

$x^2 - 6x - 40 < 0$

$(x - 10)(x + 4) = 0$

$x = 10 \quad x = -4$



$-4 < x < 10$

$1.5 < x < 10$

Corbettmaths

Write
$$(\sqrt{2}+\sqrt{6})^2$$
in the form
$$a+b\sqrt{3}$$

$(\sqrt{2}+\sqrt{6})(\sqrt{2}+\sqrt{6})$

$2+\sqrt{12}+\sqrt{12}+6$

$8+2\sqrt{12}$

$\sqrt{12} = \sqrt{4}\times\sqrt{3}$

$= 2\sqrt{3}$

$8 + 2(2\sqrt{3})$

$8+4\sqrt{3}$

Make c the subject of
$$x = \frac{y^2+c}{y-c}$$

$x(y-c) = y^2 + c$

$xy - cx = y^2 + c$

$xy - y^2 = c + cx$

$c(1+x) = xy - y^2$

$c = \dfrac{xy-y^2}{1+x}$

Find the nth term of the sequence

12 14 18 24 32 ...

2 4 6 8

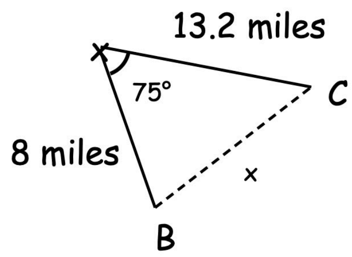
2 2 2

$a=1$ $b=-1$ $c=12$

$n^2 - n + 12$

Helicopter A and Helicopter B both take off from the same location.
Helicopter A flies 8 miles on a bearing of 172°.
Helicopter B flies 13.2 miles on a bearing of 097°.

How far is helicopter A from B?



$x^2 = 8^2 + 13.2^2 - (2 \times 8 \times 13.2 \times \text{Cos } 75)$

$x^2 = 183.577417674$

$x = 13.549...$

13.55 miles

13.55 miles

Find the minimum point of the graph
$y = x^2 - 9x - 20$

$y = (x-4.5)^2 - 20.25 - 20$

$y = (x-4.5)^2 - 40.25$

$(4.5, -40.25)$

5th February **Higher Plus 5-a-day**

Corbettmaths

Make w the subject

$$g = \frac{w}{w-5}$$

$g(w-5) = w$

$gw - 5g = w$

$gw - w = 5g$

$w(g-1) = 5g$

$w = \frac{5g}{g-1}$

Find the reciprocal of $1.2\dot{3}\dot{5}$

$x = 1.235\,35\ldots$

$10x = 12.3535\ldots$

$1000x = 1235.3535\ldots$

$990x = 1223$

$x = \frac{1223}{990}$

reciprocal $= \frac{990}{1223}$

Given

$$f(x) = \frac{8x-1}{5}$$

find

$$f^{-1}(x)$$

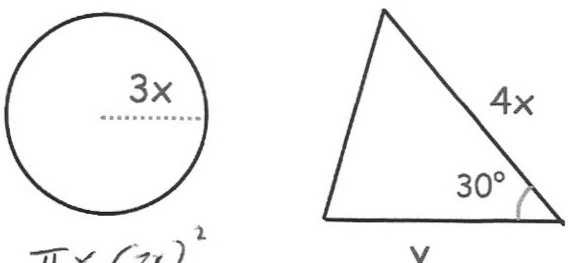
$y = \frac{8x-1}{5}$

$5y = 8x - 1$

$5y + 1 = 8x$

$x = \frac{5y+1}{8}$

$f^{-1}(x) = \frac{5x+1}{8}$



$\pi \times (3x)^2$

$= 9\pi x^2$

$\frac{1}{2}(4x)y \sin 30 = xy$

The areas of the circle and triangle are equal.
Express y in terms of x.

$xy = 9\pi x^2$

$y = 9\pi x$

In a crate, containing only red and green apples, the ratio of green to red apples is 2:1
An apple is picked at random and removed from the crate.
A second apple is then picked at random.

if 3 apples : $P(GG) = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$

The probability of picking two green apples is y.
Find the error interval for y.

$\frac{1}{3} \leq y < \frac{4}{9}$

if very large number of apples $P(GG) \rightarrow \frac{4}{9}$

6th February **Higher Plus 5-a-day**

Corbettmaths

Write down the exact value of:

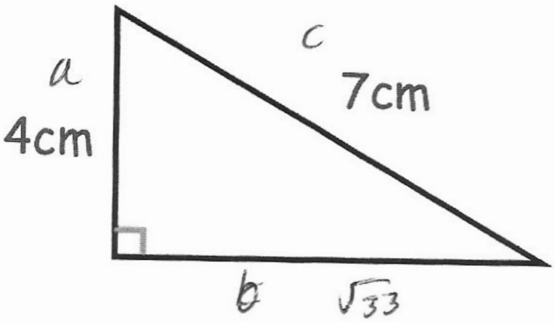
tan 0°

0

Write down the exact value of:

tan 60°

$\sqrt{3}$



Find the area of the triangle.
Give your answer in surd form and as simply as possible

$a^2 + b^2 = c^2$

$16 + b^2 = 49$

$b^2 = 33$

$b = \sqrt{33}$

Area

$\frac{1}{2} \times \sqrt{33} \times 4$

$= 2\sqrt{33}\ cm^2$

Given

$f(x) = 2x + 3$

$g(x) = 4x^2$

Find $fg(x)$

$fg(x) = 2(4x^2) + 3$

$= 8x^2 + 3$

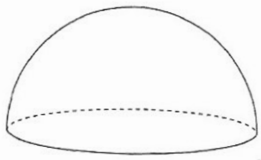
P	O	L	Y	G	O	N

A tile is selected at random, it is **not replaced** and then another tile is selected.

Work out the probability that both cards are O.

$P(OO) = \frac{2}{7} \times \frac{1}{6} = \frac{2}{42}$

$\frac{1}{21}$



$V = \frac{1}{2}\left(\frac{4}{3} \times \pi \times r^3\right)$

$= \frac{1}{2}\left(\frac{4}{3} \times \pi \times 3^3\right)$

$= 18\pi\ cm^3$

$m = d \times V$

$= 6.13 \times 18\pi = 346.64g$

The solid hemisphere shown has a radius of 3cm.
The hemisphere is made from a material with density 6.13g/cm³.
Calculate the mass of the hemisphere.

7th February **Higher Plus 5-a-day**

Corbettmaths

Given

$$2x^2 + cx + 13 \equiv d(x+4)^2 + e$$

Find c, d and e

$d(x^2 + 8x + 16) + e$

$dx^2 + \underset{16}{8d}x + \underset{32}{16d} + e \equiv 2x^2 + cx + 13$

$d = 2$

$c = 16$

$e = -19$

Using all of the 5 cards below once, how many different $\boxed{\text{odd}}$ numbers greater than 40000 can be made?

6 ways to arrange 3 cards

8	4	2	7	3

8 _ _ _ 7 — 6 possible

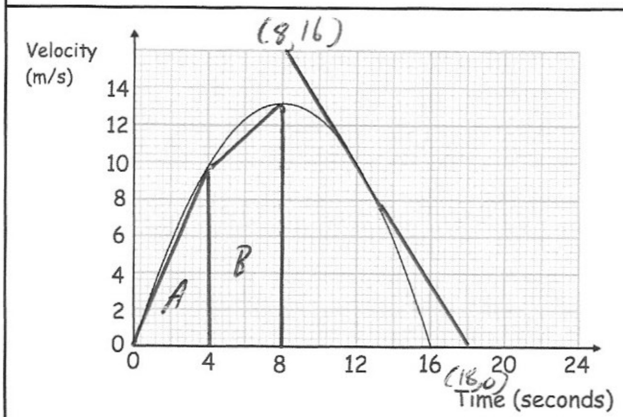
8 _ _ _ 3 — 6 possible

7 _ _ _ 3 — 6 possible

4 _ _ _ 3 — 6 possible

4 _ _ _ 7 — 6 possible

30



Here is a velocity-time graph of a bicycle.

Estimate the distance travelled in the first 8 seconds.

A : $\frac{1}{2} \times 4 \times 9.6 = 19.2$ m

B : $\frac{1}{2}(9.6 + 13.2) \times 4 = 45.6$ m

64.8m

Estimate the deceleration at 12 seconds.

$\frac{\text{rise}}{\text{run}} = \frac{-16}{10} = -1.6$

1.6 m/s^2

The set of values for x that satisfies a quadratic inequality is
x < –3 or x > 6

-3 6

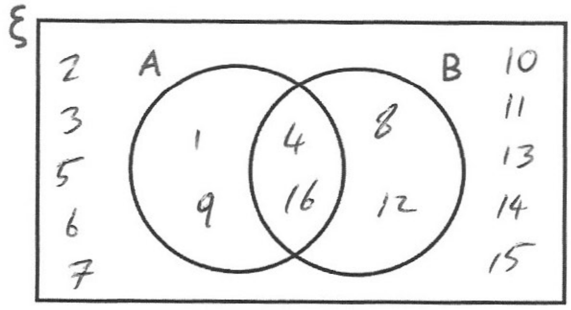
Write down a possible quadratic inequality.

$(x+3)(x-6)$

$x^2 - 3x - 18 > 0$

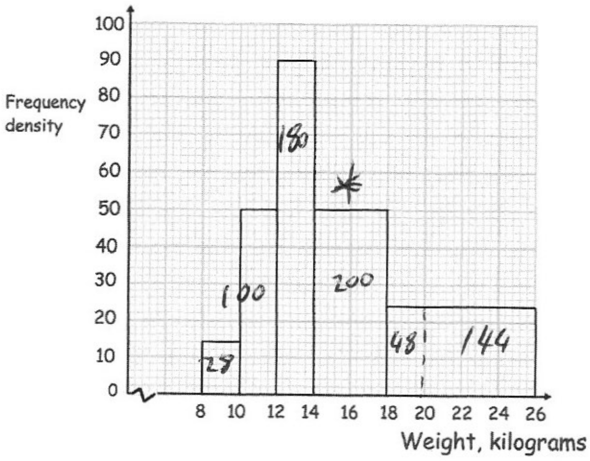
8th February **Higher Plus 5-a-day**

Corbettmαths

A circle has equation $x^2 + y^2 = 8$ $r = \sqrt{8}$ Find the area of the circle.	$\pi \times (\sqrt{8})^2$ $= 8\pi$
x : y = 2 : 9 y : z = 4 : 1 Write z in terms of x $x : y : z$ $2 : 9$ $4 : 1$ $8 : 36 : 9$ $x : z : 8 : 9$	$9x = 8z$ $z = \frac{9}{8}x$
Prove $-3x^2 - 2x + (2x+1)^2$ is never negative	$-3x^2 - 2x + 4x^2 + 4x + 1$ $= x^2 + 2x + 1$ $= (x+1)^2 - 1 + 1$ $= (x+1)^2$ min value is 0 when $x = -1$
ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16} A = square numbers 1 4 9 16 B = multiples of 4. 4 8 12 16	
Which is less likely? P(square number given multiple of 4) P(multiple of 4 given square number)	$\frac{2}{4} = \frac{1}{2}$ $\frac{2}{4} = \frac{1}{2}$ they have the same probability.

9th February **Higher Plus 5-a-day**

Corbettmaths



The histogram shows the weights of 700 dogs.

Find an estimate of the percentage of the dogs that weighed over 20kg?

$\frac{144}{700} \times 100 = 20.571\%$

Calculate an estimate of the median

350^{th}

$14 + \frac{42}{200} \times 4$

$= 14.84\,kg$

The number of bacteria on a petri dish is measured every hour and is modelled by the formula below.
N = number of bacteria
t = time (in hours)

$$N = A \times 2.71^{0.2t}$$

At the beginning of the experiment there were 80 bacteria.

Show A = 80

$80 = A \times 2.71^{0}$

$\therefore A = 80$

How many hours would it take for there to be at least 200 bacteria on the petri dish?

4hrs $80 \times 2.71^{0.8} = 177.6..$

5hr $80 \times 2.71^{1} = 216.8$

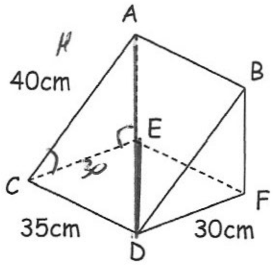
5 hours

How many bacteria would there be after one day?

$80 \times 2.71^{(0.2\times 24)}$

9579 (or 9580)

Here is a triangular prism



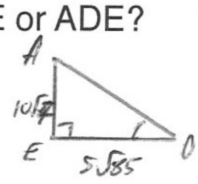
$\cos ACE = \frac{30}{40}$

$ACE = 41.41°$

Which angle is larger, ACE or ADE?

$ED = \sqrt{30^2 + 35^2} = 5\sqrt{85}$

$AE = \sqrt{40^2 - 30^2} = 10\sqrt{7}$



$\tan ADE = \frac{10\sqrt{7}}{5\sqrt{85}} = 0.57...$

$ADE = 29.85°$

$\boxed{ACE}$

10th February	Higher Plus 5-a-day

Corbettmaths

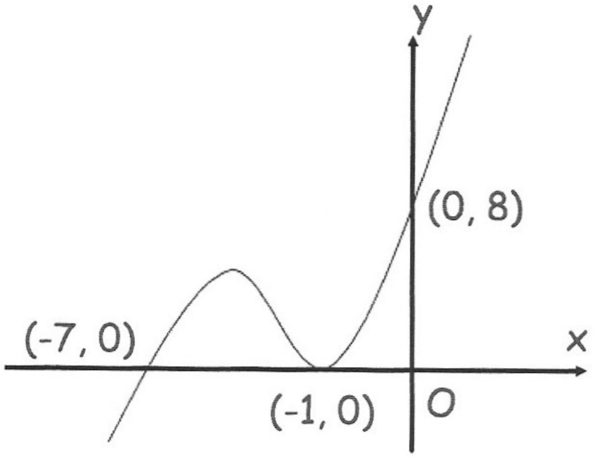
Prove that the product of two odd numbers is always odd.

$(2m+1)(2n+1)$

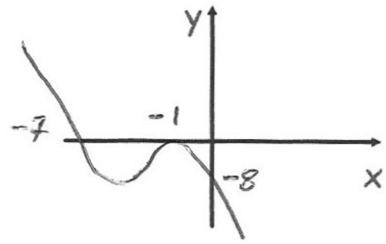
$4mn + 2m + 2n + 1$

$2(2mn + m + n) + 1$

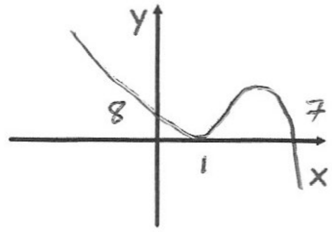
even + 1 = odd



Sketch y = –f(x)



Sketch y = f(–x)



Find the 20th term in the quadratic sequence

5 6 9 14 21

1 3 5 7

2 2 2

$2a = 2$

$\boxed{a = 1}$

$3a + b = 1$

$3 + b = 1$

$\boxed{b = -2}$

$a + b + c = 5$

$1 - 2 + c = 5$

$\boxed{c = 6}$

$n^2 - 2n + 6$

$20^2 - (2 \times 20) + 6 = 366$

Find x

Give your answers to 2 decimal places

$$\frac{7}{x+3} + \frac{1}{x-1} = 1$$

$\frac{7x - 7 + x + 3}{(x+3)(x-1)} = 1$

$8x - 4 = x^2 + 2x - 3$

$x^2 - 6x + 1 = 0$

$a = 1$

$b = -6$

$c = 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = 5.83$ or $x = 0.17$

11th February **Higher Plus 5-a-day**

Corbettmaths

a : b = 7 : 8

Work out (a + 3b) : 9b

$8a = 7b$

$a = \frac{7}{8}b$

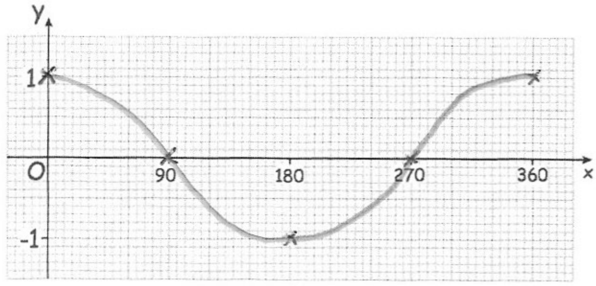
$\frac{7}{8}b + 3b : 9b$

$\frac{31}{8}b : 9b$

$\frac{31}{8} : 9$

$31 : 72$

Sketch the graph of y = cos x for
$0 \leq x \leq 360$.



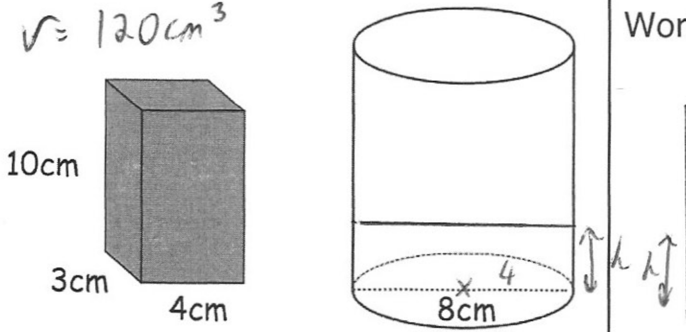
Simplify

$\sqrt{75} + \sqrt{48}$

$(\sqrt{25} \times \sqrt{3}) + (\sqrt{16} \times \sqrt{3})$

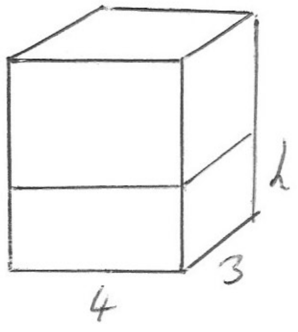
$5\sqrt{3} + 4\sqrt{3} = 9\sqrt{3}$

$V = 120 cm^3$



The cuboid is full of liquid.
Some of the liquid is poured into the cylinder.
The height, h, of liquid in both containers is the same.

Work out h.



$(\pi \times 4^2) \times h + (4 \times 3 \times h) = 120$

$16\pi h + 12h = 120$

$h(16\pi + 12) = 120$

$h = 1.927... cm$

12th February **Higher Plus 5-a-day**

Corbettmaths

Make a the subject

$$\frac{1}{a}-\frac{1}{b}=\frac{1}{c}$$

$$\frac{b-a}{ab}=\frac{1}{c}$$

$$c(b-a)=ab$$

$$bc-ac=ab$$

$$bc=ab+ac$$

$$bc=a(b+c)$$

$$a=\frac{bc}{b+c}$$

Solve $x^2 - 4x - 11 = 0$
using completing the square.

$$(x-2)^2-4-11=0$$

$$(x-2)^2-15=0$$

$$(x-2)^2=15$$

$$x-2=\pm\sqrt{15}$$

$$x=2\pm\sqrt{15}$$

$$x=2+\sqrt{15}$$

or $x=2-\sqrt{15}$

Here are the first 5 terms of a quadratic sequence

9 17 29 45 65

8 12 16 20

4 4 4

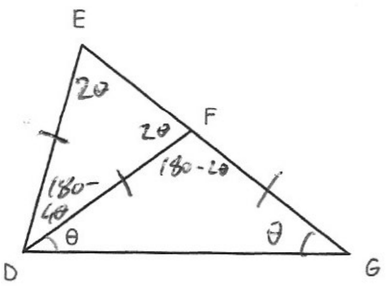
Find an expression, in terms of n, for the nth term of this quadratic sequence.

$2a=4$ $\quad 3a+b=8$ $\quad a+b+c=9$

$a=2$ $\quad 6+b=8$ $\quad 2+2+c=9$

$b=2$ $\quad c=5$

$$2n^2+2n+5$$



DE = DF = FG
$\angle FDG = \theta$
Prove that $\angle EDF = 180 - 4\theta$

$\angle FGD = \theta$ (isosceles)
$\angle DFG = 180-2\theta$ (angles in a triangle)
$\angle EFD = 2\theta$ (straight line)
$\angle FED = 2\theta$ (isosceles)
$\angle EDF = 180-4\theta$ (angles in a triangle)

The minimum point of a quadratic graph in the form $y = x^2 + ax + b$ is (6, 3).

Find a and b.

$$y=(x-6)^2+3$$

$$y=x^2-12x+36+3$$

$$y=x^2-12x+39$$

$a=-12$

$b=39$

13th February **Higher Plus 5-a-day**

Corbettmaths

4 blue socks and 6 black socks are in a drawer.
Anju takes out two socks at random.

Work out the probability that Anju takes out two socks are different colours.

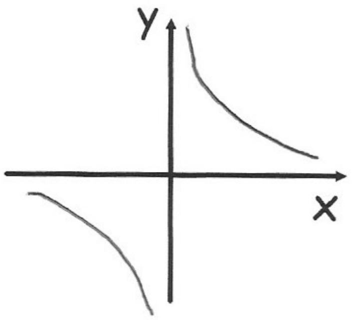
$P(\text{Blue}/\text{black}) = \frac{4}{10} \times \frac{6}{9} = \frac{24}{90}$

$P(\text{black}/\text{blue}) = \frac{6}{10} \times \frac{4}{9} = \frac{24}{90}$

$\frac{48}{90} = \frac{8}{15}$

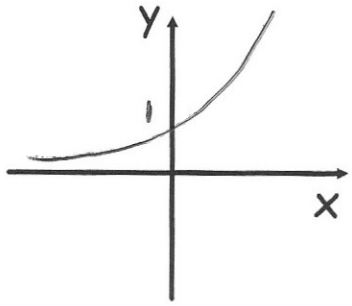
Sketch

$$y = \frac{1}{x}$$



Sketch

$$y = 4^x$$



Ornaments A and B are mathematically similar. They are solid and both made from copper and zinc.

Ornament A has a height of 5cm and volume of 30cm³

$\times 3.6^3$ $\times 3.6$

Ornament B has a height of 18cm.

$1399.68\,cm^3$

The ornaments are made from copper and zinc in the ratio 3:2

The density of copper is 8.96g/cm³
The density of zinc is 7.13g/cm³

Work out the difference in mass between ornament A and ornament B.

$\boxed{A}$ $18cm^3$ of copper $12cm^3$ of Zinc

d m v $\times 8.96$ $\times 7.13$

$161.28g$ $85.56g$

$246.84g$

$\boxed{B}$ $839.808cm^3$ of copper $559.872\,cm^3$ of Zinc

$\times 8.96$ $\times 7.13$

$7524.67968g$ $3991.88736g$

$11516.56704g$

$11269.7g$ to 1dp

Simplify fully

$$\frac{4x^2 - 25}{6x^2 - 11x - 10}$$

$$\frac{(2x+5)\cancel{(2x-5)}}{(3x+2)\cancel{(2x-5)}}$$

$$\frac{2x+5}{3x+2}$$

Find the value of x

$$2^x \times 4^{x+3} = 16$$

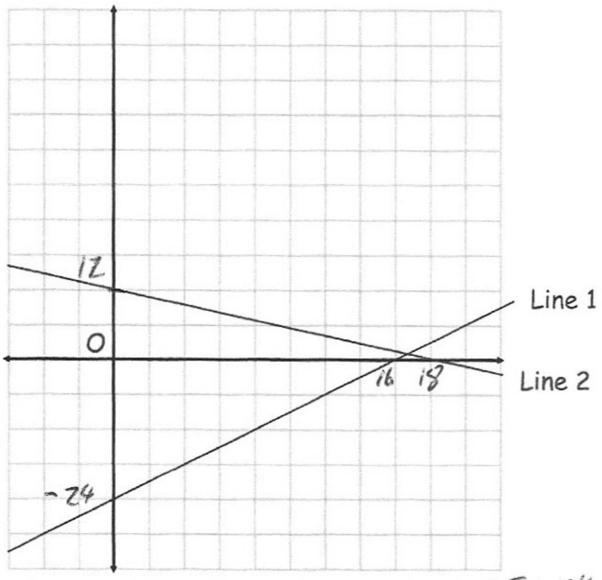
$$2^x \times (2^2)^{x+3} = 2^4$$

$$2^x \times 2^{2x+6} = 2^4$$

$$3x + 6 = 4$$

$$3x = -2$$

$$x = -2/3$$



$0 = 1.5x - 24$

$x = 16$

Line 1 has equation $y = \frac{3}{2}x - 24$

Find the equation of Line 2

$$y = -\frac{2}{3}x + 12$$

Are the lines perpendicular?

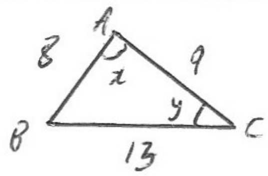
yes $m_1 \times m_2 = -1$

$$3/2 \times -2/3 = -1$$

A triangle has sides of 8cm, 9cm and 13cm.

Calculate the difference between the smallest and largest angles.

$99.59 - 37.36 = 62.23°$



Cosine rule

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Angle A = 99.59°

Angle C = 37.36°

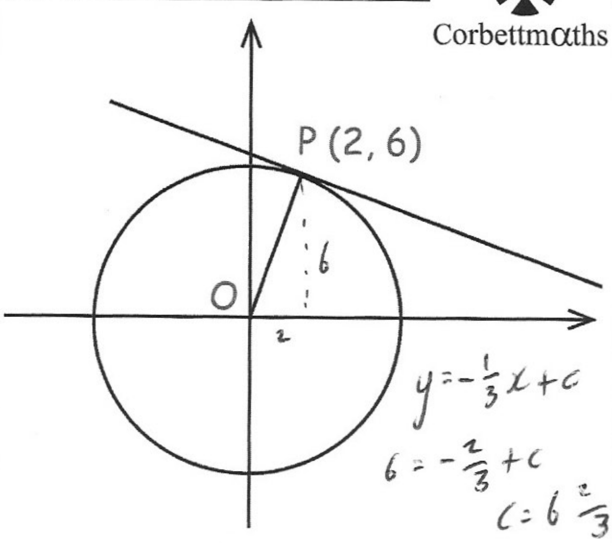
The diagram shows the circle $x^2 + y^2 = 40$ with a tangent at the point (2, 6)

Find the gradient of the line OP

3

Find the gradient of the tangent

$-\frac{1}{3}$



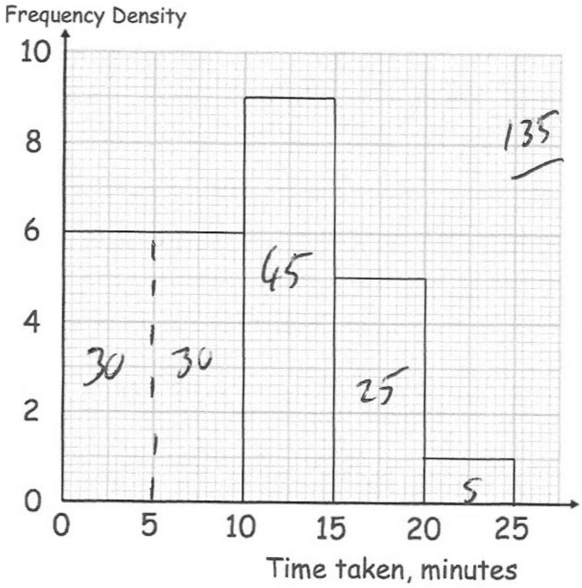
$y = -\frac{1}{3}x + c$

$6 = -\frac{2}{3} + c$

$c = 6\frac{2}{3}$

Find the equation of the tangent

$y = -\frac{1}{3}x + 6\frac{2}{3}$



The histogram shows the time taken to travel to school by 135 students.

Two students are chosen at random.

Work out an estimate to the probability that both students take more than 5 minutes to travel to school.

$P(\text{more than 5 mins}) = \frac{105}{135}$

$\frac{105}{135} \times \frac{104}{134} = \frac{364}{603}$

Prove that if two consecutive integers are squared, that the sum always gives a remainder of 1 when divided by 4.

$n^2 + (n+1)^2$

$= n^2 + n^2 + 2n + 1$

$= 2n^2 + 2n + 1$

$= \underbrace{2}_{\text{even}}\underbrace{(n^2 + n)}_{\text{even}} + 1$

$n^2 + n$
$n(n+1) \rightarrow$ even as product of two consecutive numbers.

even x even = multiple of 4

(multiple of 4 plus 1) leaves a remainder of 1 when divided by 4.

16th February **Higher Plus 5-a-day**

Corbettmaths

The cosine rule is

$a^2 = b^2 + c^2 - 2bc \cos A$

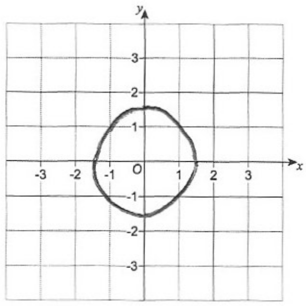
Make cos A the subject.

$2bc \cos A = b^2 + c^2 - a^2$

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Sketch $x^2 + y^2 = 2.25$

$r = 1.5$



A bag contains 7 red sweets and 5 green sweets.
Kelly removes 3 sweets, one at a time, without replacement.

$1 - P(\text{same})$

Find the probability that she does not choose 3 sweets that are the same colour.

$P(RRR) = \frac{7}{12} \times \frac{6}{11} \times \frac{5}{10} = \frac{7}{44}$

$P(GGG) = \frac{5}{12} \times \frac{4}{11} \times \frac{3}{10} = \frac{1}{22}$

$1 - \frac{1}{22} - \frac{7}{44} = \frac{35}{44}$

Show that the equation $x^3 + x = 20$ has a solution between 2 and 3.

$x^3 + x - 20 = 0$

let $f(x) = x^3 + x - 20$

$f(2) = -10 \quad f(3) = 10$

As $f(x)$ is continuous and there is a change of sign, there must be a solution between $x = 2$ and $x = 3$

Starting with $x_0 = 2$
use the iterative formula
$x_{n+1} = \sqrt[3]{20 - x_n}$
four times to find an estimate for the solution of $x^3 + x = 20$ that lies between 2 and 3.

$x_0 = 2$

$x_1 = 2.6207...$

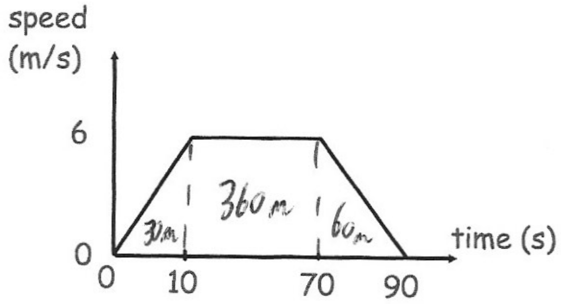
$x_2 = 2.59026...$

$x_3 = 2.591775...$

$x_4 = 2.591700574$

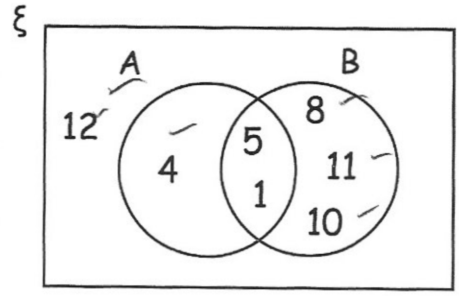
17th February **Higher Plus 5-a-day**

Corbettmaths



Calculate the total distance travelled.

$30 + 360 + 60 = 450\text{m}$

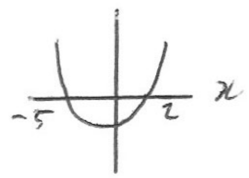


Find $P(A' \cup B')$

$\frac{5}{7}$

Solve the inequality

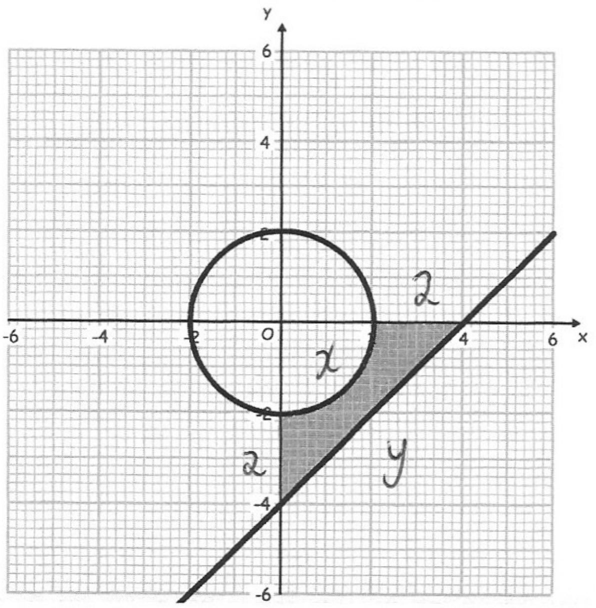
$x^2 + 3x \leq 10$ $\quad x^2 + 3x - 10 \leq 0$



$(x+5)(x-2)$

$x = -5$ or $x = 2$

$-5 \leq x \leq 2$



Find the perimeter of the shaded region.

$4^2 + 4^2 = y^2$

$y^2 = 32$

$y = \sqrt{32}$

$y = 4\sqrt{2}$

$x = \frac{1}{4} \times \pi \times 4 = \pi$

$P = 2 + 2 + \pi + 4\sqrt{2}$

$= 4 + \pi + 4\sqrt{2}$

$= 12.798$

18th February | **Higher Plus 5-a-day**

Corbettmaths

Given

$$f(x) = \frac{2+x}{3}$$

find

$f(11)$

$f(11) = \frac{2+11}{3}$

$= \frac{13}{3}$

Given

$$f(a) = 0$$

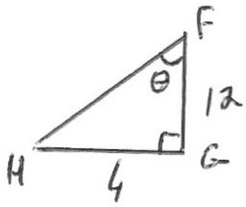
find a

$\frac{2+x}{3} = 0$

$2 + x = 0$

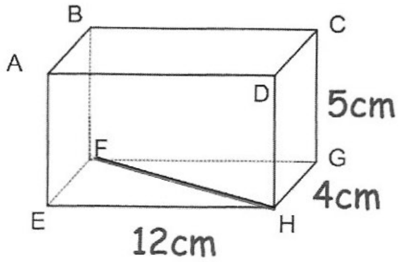
$x = -2$

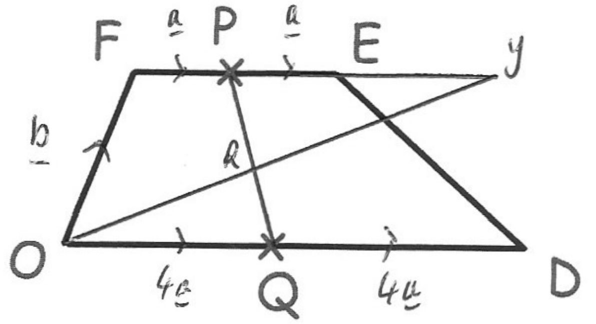
Calculate angle HFG



$\tan\theta = \frac{4}{12}$

$\theta = 18.43°$





ODEF is a trapezium
P is the midpoint of FE
Q is the midpoint of OD

$\overrightarrow{FE} = 2a \quad \overrightarrow{OF} = b \quad \overrightarrow{OD} = 8a$

Find in terms of **a** and **b**

$\overrightarrow{OP}$

$\underline{a} + \underline{b}$

$\overrightarrow{PQ} = \overrightarrow{PF} + \overrightarrow{FO} + \overrightarrow{OQ}$

$= -\underline{a} + (-\underline{b}) + 4\underline{a}$

$= 3\underline{a} - \underline{b}$

R is the midpoint of PQ $= 2\overrightarrow{PR}$

$\overrightarrow{OR} = \overrightarrow{OF} + \overrightarrow{FP} + \overrightarrow{PR}$

$= \underline{b} + \underline{a} + \frac{1}{2}(3\underline{a} - \underline{b})$

$= 2.5\underline{a} + 0.5\underline{b}$

The lines OR and FE are extended and meet at the point Y.

$\overrightarrow{OY} = 5\underline{a} + \underline{b}$

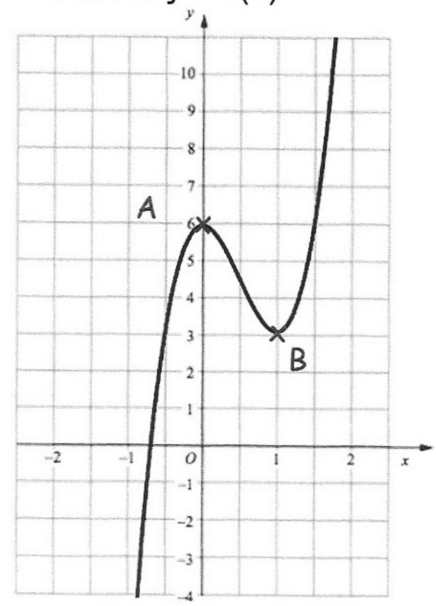
$\overrightarrow{QY} = \overrightarrow{QO} + \overrightarrow{OY}$

$= -4\underline{a} + 5\underline{a} + \underline{b}$

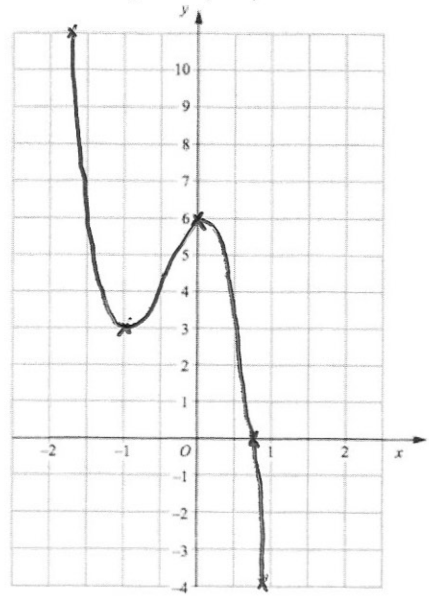
$= \underline{a} + \underline{b}$

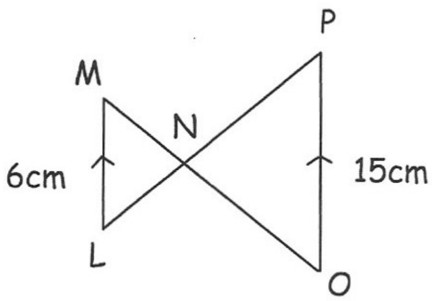
Corbettmaths

Shown is $y = f(x)$



Sketch $y = f(-x)$





Explain why triangles LMN and NOP are similar

$\angle MNL = \angle PNO$ vertically opposite

$\angle LMO = \angle POM$ alternate angles

$\angle LPO = \angle MLP$ alternate angles

AAA

There are 10 socks in a drawer.
5 are white
3 are black
2 are red

$P(WW) = \frac{5}{10} \times \frac{4}{9} = \frac{20}{90}$

$P(BB) = \frac{3}{10} \times \frac{2}{9} = \frac{6}{90}$

Heather takes two socks at random from the drawer.

$P(RR) = \frac{2}{10} \times \frac{1}{9} = \frac{2}{90}$

Work out the probability that Heather has picked 2 socks of the same colour.

$\frac{28}{90} = \frac{14}{45}$

$$f(x) = 3x - 1$$
$$g(x) = x^2 + 8$$

$3(x^2 + 8) - 1$

$3x^2 + 24 - 1$

Find
$$fg(x)$$

$3x^2 + 23$

20th February | **Higher Plus 5-a-day** | Corbettmaths

Make c the subject of

$$w = 6 + \frac{a}{c+2}$$

$w - 6 = \frac{a}{c+2}$

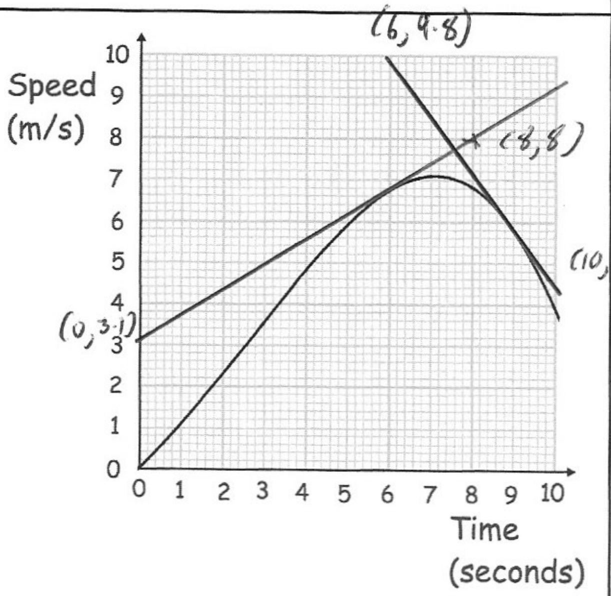
$(w-6)(c+2) = a$

$cw + 2w - 6c - 12 = a$

$cw - 6c = a + 12 - 2w$

$c(w-6) = a + 12 - 2w$

$c = \frac{a + 12 - 2w}{w - 6}$



Calculate an estimate for the acceleration at t = 6

$\frac{\text{rise}}{\text{run}} = \frac{8 - 3.1}{8} = 0.6125 \, m/s^2$

Calculate an estimate for the deceleration at t = 9

$\frac{\text{rise}}{\text{run}} = \frac{-5.4}{4} = -1.35 \, m/s^2$

$1.35 \, m/s^2$

* I have also got an answer of $1.8 \, m/s^2$ when I tried this question before.

Given

$f(x) = 5x + 1$

$g(x) = 8 - 2x$

$gf(x) = 8 - 2(5x + 1)$

$= 8 - 10x - 2$

$= 6 - 10x$

Solve

$gf(x) = 0$

$6 - 10x = 0$

$6 = 10x$

$x = \frac{6}{10} = \frac{3}{5}$ $\quad x = 0.6$

A scientist wants to estimate the number of frogs living near a lake.

On Friday she catches 250 frogs and tags them. She then releases the frogs.

On Saturday the scientist catches 80 frogs and 35 of them are tagged.

Estimate the number of frogs that live near the lake.

$\frac{250}{N} = \frac{35}{80}$

$35N = 20000$

$N = 571.428...$

571

(or 572)

* the gradient of the tangent is an approximate
In exams they allow a range of possible answers.

21st February **Higher Plus 5-a-day**

Corbettmaths

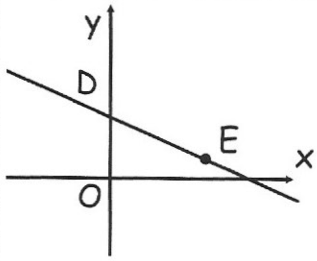
The volumes of two mathematically similar solids are in the ratio 8 : 125
The surface area of the smaller solid is 24 cm²

Work out the surface area of the larger solid.

V: 8:125
S 2:5
A 4:25

$24 \div 4 = 6$

$6 \times 25 = 150 \text{ cm}^2$



A straight line passes through the points D(0,10) and E (16, 2)

gradient of DE $= \frac{2-10}{16-0}$

$= \frac{-8}{16}$

$= -\frac{1}{2}$

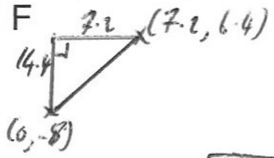
Find the equation of the line perpendicular to DE and passing through F(0, –8)

$y = 2x - 8$

$-\frac{1}{2}x + 10 = 2x - 8$ $\quad x = 7.2$

$18 = 2.5x$ $\quad y = 6.4$

Find the shortest distance between the line passing through DE and the point F



$7.2^2 + 14.4^2$

$= 259.2$

$\sqrt{259.2} = 16.0997$

$a = \frac{c}{w}$ $\quad$ Max a: $\frac{120.5}{41.205} = 2.924$

119.5 / 120.5

c = 120 correct to 3 significant figures.
w = 41.21 correct to 2 decimal places.

41.205 / 41.215

Min a $= \frac{119.5}{41.215} = 2.899..$

By considering bounds, work out the value of a to a suitable degree of accuracy.

2.9 to 1dp.

Find the minimum point of the graph
$y = x^2 - 11x + 1$

$y = (x-5.5)^2 - 30.25 + 1$

$y = (x-5.5)^2 - 29.25$

$(5.5, -29.25)$

22nd February **Higher Plus 5-a-day**

Corbettmaths

Write 0.0282828… as a simplified fraction

$x = 0.02828...$

$10x = 0.2828...$

$1000x = 28.2828...$

$990x = 28$

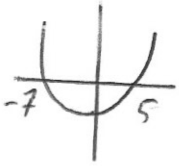
$x = \frac{28}{990} = \frac{14}{495}$

Solve the inequality

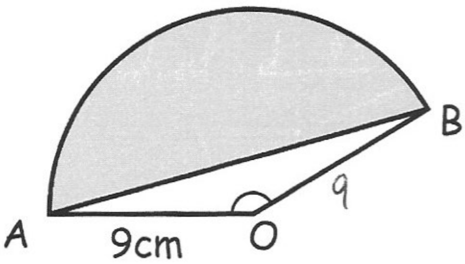
$x^2 + 2x - 35 > 0$

$(x+7)(x-5)$

$x = -7 \quad x = 5$



$x < -7$ or $x > 5$



Angle AOB = 150°

Find the area of the segment.

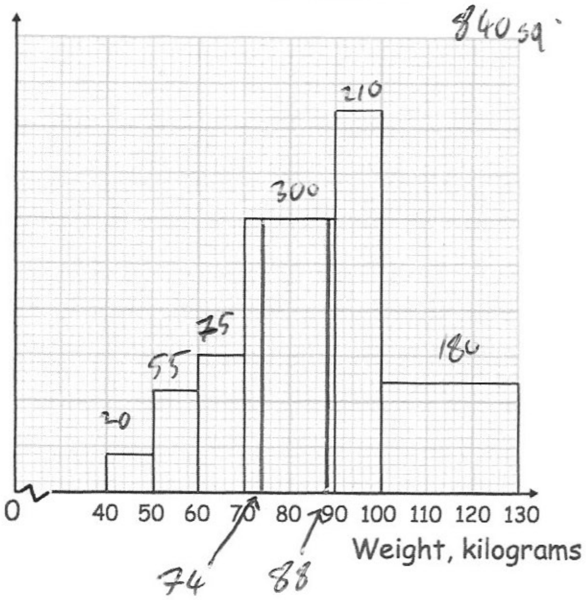
Sector AOB : $\frac{150}{360} \times \pi \times 9^2$

$= 106.0287521$

$\triangle AOB = \frac{1}{2} \times 9 \times 9 \times \sin 150 = 20.25$

$106.0287521 - 20.25$

$= 85.78 cm^2$



The histogram shows the weights of some athletes

Work out an estimate of the median

420 squares

$420 - 20 - 55 - 75 = 270$ squares

88 kg

Work out an estimate of the lower quartile

210 sq.

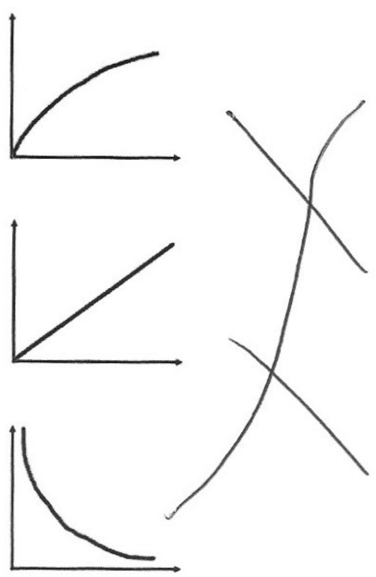
$210 - 20 - 55 - 75 = 60$ squares

74kg

23rd February **Higher Plus 5-a-day**

Corbettmaths

Match each graph to the correct relationship.



$y \propto \frac{1}{x}$

$y \propto \sqrt{x}$

$y \propto x$

Simplify fully

$\sqrt{275}$

$\sqrt{25} \times \sqrt{11}$

$5\sqrt{11}$

Write 18cos30° + 2tan60° in the form $a\sqrt{b}$

$18\left(\frac{\sqrt{3}}{2}\right) + 2\sqrt{3}$

$9\sqrt{3} + 2\sqrt{3}$

$11\sqrt{3}$

The population of an island is decreasing exponentially.

Osian has begun to monitor the population each year.
Year 6 - Population 3000
Year 8 - Poplulation 2000

What was the population in Year 2?

$3000 \xrightarrow{\times y} \xrightarrow{\times y} 2000$

$y^2 = \frac{2}{3}$ $\quad y = 0.8164.....$

$3000 \div 0.8164..^4 = 6750$

The area of a circle is 400cm² to 1 significant figure. $350 cm^2$

Work out the lower bound for the circumference of the circle

$\pi r^2 = 350$

$r = 10.555..$

$d = 21.11...$

$c = 66.319$ cm

24th February **Higher Plus 5-a-day**

Corbettmaths

Prove the product of three consecutive odd numbers is odd

$(2n+1)(2n+3)(2n+5)$

$8n^3 + 36n^2 + 46n + 15$

$2(4n^3 + 18n^2 + 23n) + 15$

even + odd = odd

Make c the subject of

$$\frac{1}{m} = \frac{a}{c} + \frac{1}{n}$$

$\frac{1}{m} = \frac{an + c}{cn}$

$\frac{cn}{m} = an + c$

$cn = amn + cm$

$cn - cm = amn$

$c(n - m) = amn$

$c = \frac{amn}{n - m}$

Martina has the following coins.

5p 5p 10p 20p

20p 20p 50p £1

$P(20\ 20\ 20)$

$\frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} = \frac{1}{56}$

Martina has to pay 60p for a car park ticket.
She selects 3 coins at random, without replacement, from her pocket.

Work out the probability that she has chosen the exact price of the ticket

$P(50\ 5\ 5) = \frac{1}{8} \times \frac{2}{7} \times \frac{1}{6} = \frac{1}{168}$

$\frac{1}{56} + \frac{1}{168} + \frac{1}{168} + \frac{1}{168} = \frac{1}{28}$

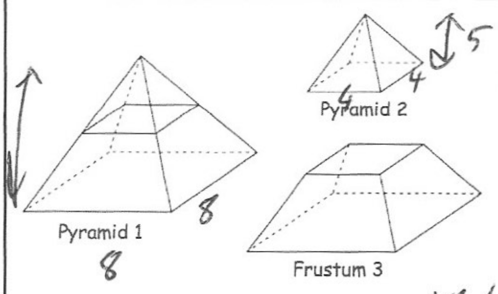
A solid square based pyramid 1 is divided into two parts: a square based pyramid 2 and a frustum 3, as shown.

Pyramid 1 has a base of side length 8cm.
Pyramid 2 has a base of side length 4cm.
The perpendicular height of pyramid 1 is 10cm.

$v = \frac{1}{3}Ah$

Frustum 3 is made from a material with a density of 4.2g/cm³

$d\ m\ v$



$v = 26.\dot{6}$

$v = 213.\dot{3}$

$v = 186.\dot{6}\ cm^3$

Work out the mass of the frustum.

$m = 4.2 \times 186.\dot{6}$

$= 784g$

25th February **Higher Plus 5-a-day**

Corbettmaths

Write $1.0\dot{3}\dot{5}$ as a fraction

$x = 1.03535\ldots$

$10x = 10.3535\ldots$

$1000x = 1035.353535\ldots$

$990x = 1025$

$x = \frac{1025}{990}$

$x = \frac{205}{198} = 1\frac{7}{198}$

Prove the sum of two consecutive odd numbers is even.

$(2n+1) + (2n+3)$

$4n+4$

$2(2n+2)$

$\therefore$ even

1 7 19

Pattern 1 Pattern 2 Pattern 3

$2a = 6$ $3a + b = 6$ $c = 1$

$a = 3$ $9 + b = 6$

$b = -3$

How many tiles are needed to make Pattern number 10?

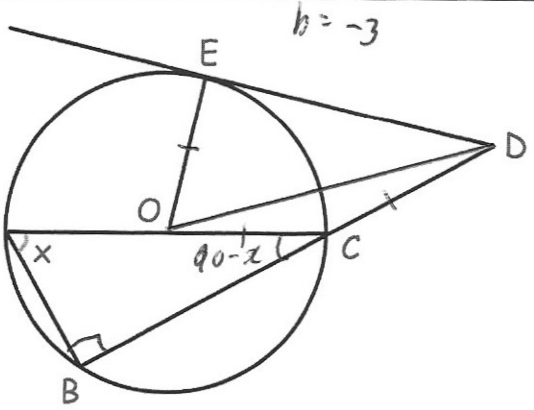
1 7 19 37

6 12 18

6 6

$3n^2 - 3n + 1$

$3 \times 10^2 - (3 \times 10) + 1 = 271$



AC is the diameter of a circle, centre O.
DE is the tangent to the circle.
BCD is a straight line.
AO = CD
Angle BAC = x

Express angle COD in terms of x.

$\angle DCO = 90 + x$

$180 - (90 + x) = 90 - x$

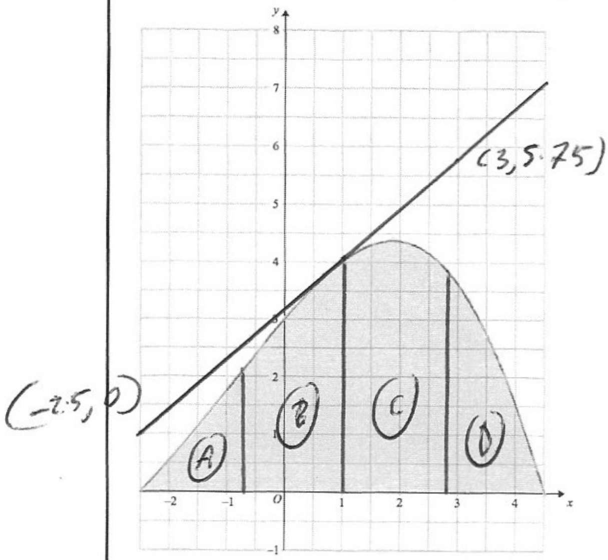
$\frac{90 - x}{2} = \left(45 - \frac{x}{2}\right)^\circ$

Work out

$27^{-\frac{2}{3}}$

$\frac{1}{27^{\frac{2}{3}}} = \frac{1}{9}$

$27^{\frac{2}{3}} = 9$



Calculate an estimate for the rate of change at x = 1

$\frac{rise}{run} = \frac{5.75}{6} = 0.958\dot{3}$

Calculate an estimate for area under the curve (shaded region)

A) $\frac{1}{2}(0+2.2)\times1.75 = 1.925$

B) $\frac{1}{2}(2.2+4)\times1.75 = 5.425$

C) $\frac{1}{2}(4+3.9)\times1.75 = 6.9125$

D) $\frac{1}{2}(3.9+0)\times1.75 = 3.4125$

$\boxed{17.675}$

Make c the subject

$$w = \frac{ac}{a-c}$$

$w(a-c) = ac$

$aw - cw = ac$

$aw = ac + cw$

$aw = c(a+w)$

$c = \frac{aw}{a+w}$

The population of island A is 4 times larger than island B.
It is expected that every year, the population of island A decreases by 5% and the population of island B increase by 5%

After how many years, will the population of island B be greater than island A?

Years	A	B
10	$400\times0.95^{10} = 239.5$	$100\times1.05^{10} = 162.9$
13	$400\times0.95^{13} = 205.3$	$100\times1.05^{13} = 188.6$
14	$400\times0.95^{14} = 195.07$	$100\times1.05^{14} = 197.99$

14

27th February **Higher Plus 5-a-day**

Corbettmaths

Simplify fully

$$\frac{3x}{x^2+3x+2} + \frac{3}{x+1}$$

$$\frac{3x}{(x+1)(x+2)} + \frac{3}{x+1}$$

$$\frac{3x + 3(x+2)}{(x+1)(x+2)}$$

$$\frac{3x+3x+6}{(x+1)(x+2)} = \frac{6x+6}{(x+1)(x+2)} = \frac{6(\cancel{x+1})}{(\cancel{x+1})(x+2)}$$

$$\frac{6}{x+2}$$

a is directly proportional to $\sqrt{c}$
w is inversely proportional to a^3.

When c = 49, a = 35
When a = 2, w = 16.

Find the value of w when c = 4.

$a \propto \sqrt{c}$

$a = k\sqrt{c}$

$35 = k \times \sqrt{49}$

$35 = 7k$

$k = 5$

$a = 5\sqrt{c}$

$a = 5 \times 2 = 10$

$w \propto \frac{1}{a^3}$

$w = \frac{k}{a^3}$

$16 = \frac{k}{8}$

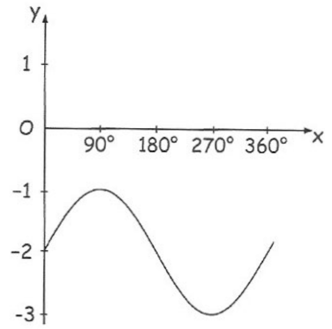
$k = 128$

$w = \frac{128}{a^3}$

$w = \frac{128}{10^3}$

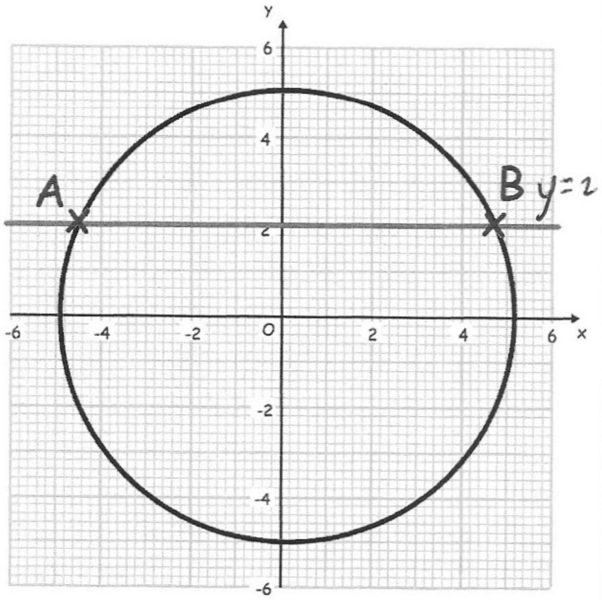
$= \frac{128}{1000}$

$= 0.128$



Write down the equation of the curve shown.

$y = \sin(x) - 2$



A circle has equation $x^2 + y^2 = 25$
A straight line, y = 2, meets the circle at the points A and B.

Find the coordinates of the points A and B. Give your answers in surd form.

$x^2 + 2^2 = 25$

$x^2 + 4 = 25$

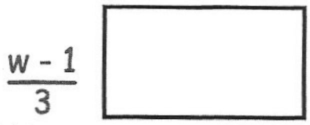
$x^2 = 21$

$x = \pm\sqrt{21}$

$(\sqrt{21}, 2)$ & $(-\sqrt{21}, 2)$

Corbettmaths

Write an expression for the perimeter of the rectangle



$\frac{w+3}{2} \times 2 = w+3$

$\frac{w-1}{3} \times 2$

$\frac{2(w-1)}{3}$

$\frac{2(w-1)}{3} + w + 3$

$\frac{2w-2}{3} + w + 3 = \frac{2}{3}w - \frac{2}{3} + w + 3$

$= \frac{5}{3}w + 2\frac{1}{3}$

$\frac{5}{3}w + \frac{7}{3}$

or $\frac{5w+7}{3}$

C, D and E are such that

C:D = 1:7

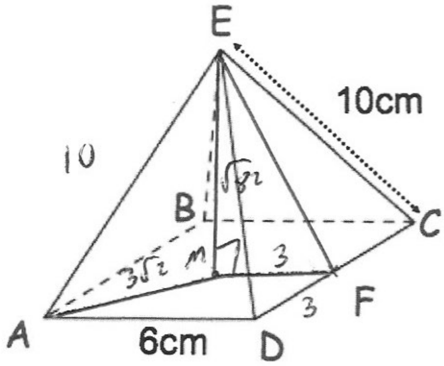
D is $\frac{2}{5}$ of E

D : E
2 : 5

Work out the ratio of C:E

C D E
1 : 7
2 : 5] ×7
×2
2 : 14 : 35

2 : 35



Shown is a square based pyramid, ABCDE.
F is the midpoint of CD

Find the length of EF

$AC^2 = 6^2 + 6^2 = 72$ $\quad A = \sqrt{72} = 6\sqrt{2}$

$AM = 3\sqrt{2}$

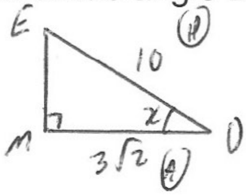
$EM = 10^2 - (3\sqrt{2})^2 = 82$

$EM = \sqrt{82}$

$EF^2 = 3^2 + (\sqrt{82})^2 = 91$

$EF = \sqrt{91}$ cm

Calculate angle BDE



$\cos x = \frac{3\sqrt{2}}{10}$

$x = 64.896°$

The set of values for x that satisfies a quadratic inequality is

–5 < x < –2

Write down a possible quadratic inequality.

$(x+2)(x+5) < 0$

$x^2 + 7x + 10 < 0$

The radius of a sphere is 4cm.
The radius of the base of a cone is also 4cm.
The volume of the sphere is twice the volume of the cone.

$2\left(\frac{1}{3} \times \pi \times 4^2 \times h\right) = \frac{4}{3} \times \pi \times 4^3$

$\frac{2}{3} \times \pi \times 16 \times h = \frac{4}{3} \times \pi \times 64$

Find the height of the cone.

$\frac{2}{3} \times 16 \times h = \frac{4}{3} \times 64$

$2 \times 16h = 4 \times 64$

$32h = 256$

$h = 8cm$

On Friday, 30% of the people visiting a leisure centre went swimming.
90% of the people who swam were members.
18% of the people who visited the leisure centre but did not go swimming were not members.

$0.3 \times 0.9 = 0.27$

0.7×0.82
$= 0.574$

Find what the percentage of the visitors to the leisure centre on Friday were members.

0.3 S — 0.9 M 27%; X
0.7 X — 0.82 M 57.4%; 0.18 X

$\boxed{84.4\%}$

Calculate the size of angle DCE

$CE^2 = 9^2 + 20^2 - 2 \times 9 \times 20 \times \cos 50$

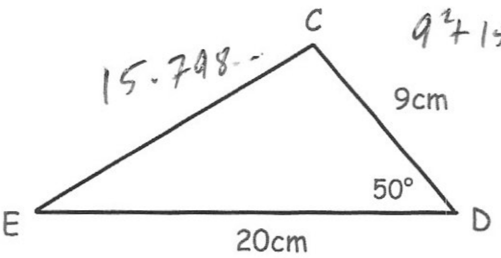
$CE = 15.798...$

$\frac{\sin 50}{15.798...} = \frac{\sin x}{20}$

$x = 75.82$ ~~x~~

or

$\boxed{x = 104.13°}$ ✓



$9^2 + 15.798..^2 < 20^2$

∴ obtuse

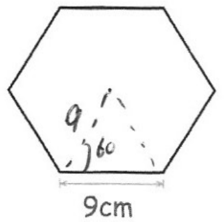
Express $3x^2 - 18x + 16$ in the form $a(x - b)^2 + c$

$3(x^2 - 6x) + 16$

$3[(x-3)^2 - 9] + 16$

$3(x-3)^2 - 27 + 16$

$3(x-3)^2 - 11$



$\frac{1}{2} \times 9 \times 9 \times \sin 60$

$= 35.074...$

$35.074... \times 6$

$= 210.44cm^2$

Calculate the area of the regular hexagon