Exam Style Questions

Simultaneous Equation (Three Unknowns)

Corbettmaths

Equipment needed: Calculator and Pen

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Video Tutorial

www.corbettmaths.com/contents

Video 393

Answers and Video Solutions

1. Solve the simultaneous equations

$$x - y + 3z = 5 \quad — \text{①}$$
$$x + y + 6z = 12 \quad — \text{②}$$
$$3x - 2y + 2z = 10 \quad — \text{③}$$

① + ② → ④

$$2x + 9z = 17 \quad — \text{④}$$

2 × ① + ③ → ⑤

$$2x + 2y + 12z = 24$$
add $\underline{3x - 2y + 2z = 10}$

$$5x + 14z = 34 \quad — \text{⑤}$$

5 × ④ − 2 × ⑤

$$10x + 45z = 85$$
$$10x + 28z = 68$$
$$\overline{17z = 17}$$

$$z = 1$$

Sub z = 1 into ⑤

$$5x + 14 = 34$$
$$5x = 20$$
$$x = 4$$

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$x =$ ......4...... $y =$ ......2...... $z =$ ......1......

(5)

Sub x = 4 & z = 1 into ①

$$4 - y + 3 = 5$$
$$7 - y = 5$$
$$y = 2$$

check in ②

$$4 + 2 + 6 = 12 ✓$$

2. Solve the simultaneous equations

$$2x + 3y + 5z = 21 \quad -①$$
$$3x + 6y + 15z = 51 \quad -②$$
$$5x + 4y + 10z = 37 \quad -③$$

$2 \times ① - ② \rightarrow ④$

$$4x + 6y + 10z = 42$$
sub $\dfrac{3x + 6y + 15z = 51}{}$

$$x - 5z = -9 \quad -④$$

$2 \times ② - 3 \times ③ \rightarrow ⑤$

$$6x + 12y + 30z = 102$$
sub $\dfrac{15x + 12y + 30z = 111}{}$

$$-9x \qquad\qquad = -9 \quad -⑤$$

$$x = 1$$

sub $x = 1$ into ④

$$1 - 5z = -9$$
$$-5z = -10$$
$$z = 2$$

$x = \underline{\ 1\ }$ $\quad y = \underline{\ 3\ }$ $\quad z = \underline{\ 2\ }$

(5)

sub $x = 1$ & $z = 2$ into ③

$$5 + 4y + 20 = 37$$

check in ①

$$2 + 9 + 10 = 21 \checkmark$$

$$4y = 12$$
$$y = 3$$

3. Solve the simultaneous equations

$$2x + 4y - z = 15 \quad -①$$

$$3x + 8y + z = 44 \quad ②$$

$$x + 2y + 2z = 15 \quad -③$$

① + ② → ④

$$5x + 12y = 59 \quad -④$$

2 × ② - ③ → ⑤

$$6x + 16y + 2z = 88$$

sub $\quad x + 2y + 2z = 15$

$$5x + 14y = 73 \quad -⑤$$

⑤ - ④ → ⑥

$$5x + 14y = 73$$

sub $\quad 5x + 12y = 59$

$$2y = 14$$

$$y = 7$$

$$x = \underline{-5} \quad y = \underline{7} \quad z = \underline{3}$$

(5)

Sub $y = 7$ into ⑤

$$5x + 98 = 73$$

$$5x = -25$$

$$x = -5$$

sub $x = -5$ & $y = 7$ into ①

$$-10 + 28 - z = 15$$

$$18 - z = 15$$

$$z = 3$$

check in ②

$$-15 + 56 + 3 = 44$$

✓

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4. Solve the simultaneous equations

$$10x + 60y + 10z = 25 \quad -①$$

$$5x + 40y + 20z = 40 \quad -②$$

$$20x + 20y + 40z = 30 \quad -③$$

$2 \times ① - ② \rightarrow ④$

$$20x + 170y + 20z = 50$$

sub

$$5x + 40y + 20z = 40$$

___

$$15x + 80y = 10 \rightarrow ④$$

$2 \times ③ - 2 ② \rightarrow 5$

~~40x + 80y written~~

$$20x + 20y + 40z = 30$$

$$10x + 80y + 40z = 80$$

sub

___

$$10x - 60y = -50 \rightarrow ⑤$$

$2 \times ④ - 3 \times ⑤$

$$30x + 160y = 20$$

sub $$30x - 180y = -150$$

___

$$340y = 170$$

$$y = \tfrac{1}{2}$$

$x = \underline{-2}$ $y = \underline{0.5}$ $z = \underline{1.5}$

(5)

Sub $y = \tfrac{1}{2}$ into ④

$$15x + 40 = 10$$

$$15x = -30$$

$$x = -2$$

sub $x = -2$ & $y = \tfrac{1}{2}$ into ①

$$-20 + 30 + 10z = 25$$

$$10 + 10z = 25$$

$$10z = 15$$

$$z = 1.5$$

check in ②

$$-10 + 20 + 30 = 40 \checkmark$$

5.   Solve the simultaneous equations

$$x + y + z = 1 \quad - \text{①}$$
$$4x - 3y + 4z = 32 \quad - \text{②}$$
$$x - 10y - 2z = 27 \quad - \text{③}$$

$$\text{①} - \text{③} \rightarrow \text{④}$$

$$x + y + z = 1$$
sub $x - 10y - 2z = 27$
_____
$$11y + 3z = -26 \rightarrow \text{④}$$

$$4 \times \text{①} - \text{②} \rightarrow \text{⑤}$$

$$4x + 4y + 4z = 4$$
sut $4x - 3y + 4z = 32$
_____
$$7y = -28$$
$$y = -4$$

sub $y = -4$ into ④

$$-44 + 3z = -26$$
$$3z = 18$$
$$z = 6$$

sub $y = -4$ & $z = 6$ into ①

$$x - 4 + 6 = 1$$
$$x + 2 = 1$$
$$x = -1$$

$$x = \underline{-1} \quad y = \underline{-4} \quad z = \underline{6}$$

(5)

Check in ②

ANSWER

$$-4 + 12 + 24 = 32 \checkmark$$

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6. Solve the simultaneous equations

$$6x + 8y - 2z = 750 \quad -①$$
$$18x - 2y + 4z = 1100 \quad -②$$
$$4x - 4y + 2z = 100 \quad -③$$

$2 \times ① - ③$

$36x - 4y + 8z = 2200$
$4x - 4y + 2z = 100$
_____
$32x + 6z = 2100 \quad -④$

$2 \times ③ + ①$

$8x - 8y + 4z = 200$
add $6x + 8y - 2z = 750$
_____
$14x + 2z = 950 \quad -⑤$

Ⓜ $3 \times ⑤ - ④$

$42x + 6z = 2850$
$32x + 6z = 2100$
_____
$10x = 750$
$x = 75$

sub $x = 75$ into ⑤

$1050 + 2z = 950$
$2z = -100$
$z = -56$

$x = \underline{75} \quad y = \underline{25} \quad z = \underline{-50}$

(5)

sub $x = 75$ & $z = -50$ into ①

$450 + 8y + 100 = 750$
$8y = 200$
$y = 25$

7.   Solve the simultaneous equations

$$7x + 5y + 4z = 23 \quad - \text{①}$$
$$21x - 10y + 6z = -4 \quad - \text{②}$$
$$7x + 15y - 2z = -15 \quad - \text{③}$$

$3 \times \text{①} \Rightarrow \quad 21x + 15y + 12z = 69 \quad - \text{④}$

$3 \times \text{③} \Rightarrow \quad 21x + 45y - 6z = -45 \quad - \text{⑤}$

$\text{④} - \text{②} \Rightarrow \text{⑥}$

$$21x + 15y + 12z = 69$$
$$\text{sub} \quad 21x - 10y + 6z = -4$$
$$\overline{\qquad 25y + 6z = 73 \quad - \text{⑥}}$$

$\text{⑤} - \text{④} \Rightarrow \text{⑦}$

$$21x + 45y - 6z = -45$$
$$\text{sub} \quad 21x + 15y + 12z = 69$$
$$\overline{\qquad 30y - 18z = -114 \quad - \text{⑦}}$$

$3 \times \text{⑥} \Rightarrow \text{⑧}$

$$75y + 18z = 219 \quad - \text{⑧}$$
$$\text{add} \quad 30y - 18z = -114$$
$$\overline{\qquad 105y = 105}$$

$$y = 1$$

$x = \underline{\quad -2 \quad}$ $y = \underline{\quad 1 \quad}$ $z = \underline{\quad 8 \quad}$

(5)

sub $y = 1$ into ⑥

$$25 + 6z = 73$$
$$6z = 48$$
$$z = 8$$

sub $y = 1$ e $z = 8$ into ①

$$7x + 5 + 32 = 23$$
$$7x = -14$$
$$x = -2$$

check in ②

$$-42 - 10 + 48 = -4$$
✓

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8. Solve the simultaneous equations

$$y - x + 2z = 2.1 \qquad — \; ①$$

$$3x - 2z - y + 2.5 = 0 \; — \; ②$$

$$8z + 10y + 5x = 4.5 \; — \; ③$$

$$-x + y + 2z = 2.1 \qquad 3x - y - 2z = -2.5 \qquad 5x + 10y + 8z = 4.5$$

$$① + ② = > ④$$

$$2x = -0.4$$
$$x = -0.2$$

$$④ × ② + ③ = > ⑤$$

$$12x - 4y - 8z = -10$$
$$\text{add} \quad 5x + 10y + 8z = 4.5$$
$$\overline{17x + 6y = -5.5 \; — ⑤}$$

sub $x = -0.2$ into ⑤

$$-3.4 + 6y = -5.5$$
$$6y = -2.1$$
$$y = -0.35$$

sub $x = -0.2$ & $y = -0.35$ into ①

$$0.2 - 0.35 + 2z = 2.1$$
$$-0.15 + 2z = 2.1$$
$$z = 1.125$$

check in ②

$$x = \underline{-0.2} \qquad y = \underline{-0.35} \qquad z = \underline{1.125}$$

(5)

$$-0.6 + 0.35 - 2.25 = -2.5$$
$$-2.5 = -2.5 \qquad ✓$$

9. Roshan sells vehicles for a company.

She receives:

£x commission for each car sold.

£y commission for each motorcycle sold.

£z commission for each van sold.

During January, Roshan sold: 12 cars, 6 motorcycles and 3 vans; she received a total of £1590 commission.

(i) Show that x, y and z satisfy the equation

$$4x + 2y + z = 530$$

$$12x + 6y + 3z = 1590 \qquad \div 3$$

$$4x + 2y + z = 530 \quad \checkmark$$

(1)

During February, Roshan sold: 10 cars, 5 motorcycles and 5 vans; she received a total of £1600 commission.

(ii) Show that x, y and z satisfy the equation

$$2x + y + z = 320$$

$$10x + 5y + 5z = 1600 \quad \div 5$$

$$2x + y + z = 320 \quad \checkmark$$

(1)

During March, Roshan sold: 18 cars, 4 motorcycles and 6 vans; she received a total of £1600 commission.

(iii) Show that x, y and z satisfy the equation

$$9x + 2y + 3z = 1175$$

$$18x + 4y + 6z = 2350 \qquad \div 2$$

$$9x + 2y + 3z = 1175 \quad \checkmark$$

(1)

(iv)   Solve the equations

$$4x + 2y + z = 530 \quad - ①$$

$$2x + y + z = 320 \quad - ②$$

$$9x + 2y + 3z = 1175 \quad - ③$$

to find the commission Roshan receives for selling each type of vehicle.

Show clearly each stage of your solution.

$① - ② = ④$

$$\begin{array}{r} 4x + 2y + z = 530 \\ \text{sub} \quad 2x + y + z = 320 \\ \hline 2x + y \qquad = 210 \quad - ④ \end{array}$$

$③ - 3 \times ② = ⑤$

$$\begin{array}{r} 9x + 2y + 3z = 1175 \\ \text{sub} \quad 6x + 3y + 3z = 960 \\ \hline 3x - y \qquad = 215 \quad - ⑤ \end{array}$$

$④ + ⑤ = ⑥$

$$\begin{array}{r} 2x + y = 210 \\ \text{add} \quad 3x - y = 215 \\ \hline 5x \qquad = 425 \quad ⑥ \\ x = 85 \end{array}$$

sub $x = 85$ into ④

$$170 + y = 210$$

$$y = 40$$

sub $x = 85$ & $y = 40$ into ①

$$340 + 80 + z = 530$$

$$420 + z = 530$$

$$z = 110$$

check $x = 85$, $y = 40$ & $z = 110$ in ③

$$765 + 80 + 330 = 1175 \quad \checkmark$$

Car £85

Motorcycle £40

Van £110

(8)

10. A football team sells three types of season ticket: adult, child and pensioner.

This year, the McManus family bought:

  4 adult season tickets at £x each.

  6 child season tickets at £y each.

  2 pensioner tickets at £z each.

The total cost of the tickets is £5650

(i) Show that x, y and z satisfy the equation

$$2x + 3y + z = 2825$$

$4x + 6y + 2z = 5650 \qquad \div 2$

$2x + 3y + z = 2825 \quad \checkmark$

(1)

The cost of an two adult season ticket is £100 more than the total cost of two child season tickets **and** two pensioner season tickets.

(ii) Show that x, y and z satisfy the equation

$$x - y - z = 50$$

$2x - 2y - 2z = 100 \qquad \div 2$

$x - y - z = 50$

(1)

Last year, all season tickets cost £25 less than the current prices.

Last year, the Wilcox family bought 4 adult season tickets, 8 child season tickets and 4 pensioner tickets.

The total price was £6600

(iii) Show that x, y and z satisfy the equation

$$x + 2y + z = 1750$$

$4(x - 25) + 8(y - 25) + 4(z - 25) = 6600$

$4x - 100 + 8y - 200 + 4z - 100 = 6600$

$4x + 8y + 4z = 7000$

$x + 2y + z = 1750$

(2)

(iv)   Solve the equations

$$2x + 3y + z = 2825 \quad - \text{①}$$

$$x - y - z = 50 \quad - \text{②}$$

$$x + 2y + z = 1750 \quad - \text{③}$$

to find the cost of the current season tickets.

Show clearly each stage of your solution.

① + ② = ④

$$2x + 3y + z = 2825$$
Add $\quad x - y - z = 50$
$$\overline{ } $$
$$3x + 2y \qquad = 2875 \quad -\text{④}$$

② + ③ = ⑤

$$x - y - z = 50$$
Add $\quad x + 2y + z = 1750$
$$\overline{}$$
$$2x + y \qquad = 1800 \quad - \text{⑤}$$

2 × ⑤ - ④

$$4x + 2y = 3600$$
sub $\quad 3x + 2y = 2875$
$$\overline{}$$
$$x \qquad = 725$$

sub $x = 725$ into ⑤

$$1450 + y = 1800$$

$$y = 350$$

check in ③

$$725 + 700 + 325 = 1750 \checkmark$$

sub $y = 350$ & $x = 725$ into ②

$$725 - 350 - z = 50$$

$$375 - z = 50$$

$$z = 325$$

Adult ..... £725 .....

Child ..... £350 .....

Pensioner ..... £325 .....

(8)

11. Oisín is training for a race and as part of his training, Oisín completes three types of run: short, medium and long.

Let x, y and z represent the length, in kilometres, of a short, medium and long run respectively.

In September, Oisín completed 24 short runs, 18 medium runs and 6 long runs. The total distance ran during September was 450km

    (i) Show that x, y and z satisfy the equation

$$4x + \underline{2}y + z = 75$$

$$24x + 18y + 6z = 450$$
$$\div 6$$
$$4x + 3y + z = 75 \checkmark$$

(1)

In October, Oisín completed 5 short runs, 15 medium runs and 20 long runs. The total distance ran during October was 615km.

    (ii) Show that x, y and z satisfy the equation

$$x + 3y + 4z = 123$$

$$5x + 15y + 20z = 615 \quad \div 5$$
$$x + 3y + 4z = 123 \checkmark$$

(1)

In November, Oísin replaced the long run with an ultra run.
The ultra run is 40% longer than the long run.   ultra $= 1.4 \times z$
                                                         or $1.4z$
That month he ran 14 short runs, 4 medium runs and 10 ultra runs, covering a total distance of 420km.

    (iii) Show that x, y and z satisfy the equation

$$7x + 2y + 7z = 210$$

$$14x + 4y + 10(1.4z) = 420$$
$$14x + 4y + 14z = 420 \quad \div 2$$
$$7x + 2y + 7z = 210 \checkmark$$

(1)

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(iv)   Solve the equations

$$4x + 3y + z = 75 \quad — \text{①}$$

$$x + 3y + 4z = 123 \quad — \text{②}$$

$$7x + 2y + 7z = 210 \quad — \text{③}$$

to find the length of a short, medium and long run.

Show clearly each stage of your solution.

① - ② = ④ $4x + 3y + z = 75$

sub $x + 3y + 4z = 123$
_____
$3x \quad\quad -3z = -48 \quad — $④

$3 \times$③ $- 2 \times$② $=$ ⑤

$21x + 6y + 21z = 630$

sub $2x + 6y + 8z = 246$
_____
$19x \quad\quad + 13z = 384 \quad — $⑤

$13 \times$④ $+ 3 \times$⑤

$39x - 39z = -624$

Add $57x + 39z = 1152$
_____
$96x \quad\quad = 528$

$x = 5.5$

sub $x = 5.5$ into ④

$16.5 - 3z = -48$

$-3z = -64.5$

$z = 21.5$

sub $x = 5.5$ & $z = 21.5$ into ①

$22 + 3y + 21.5 = 75$

$3y = 31.5$

$y = 10.5$

check in ②

$5.5 + 31.5 + 86 = 123$

$123 = 123$ ✓

Short ....5.5........km

Medium ...10.5......km

Long ....21.5......km

(8)

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12. A concert venue sells three types of tickets: standing, seated and VIP.

For each concert

the venue sells a maximum of x standing tickets.
the venue sells a maximum of y seated tickets.
the venue sells a maximum of z VIP tickets.

The venue can sell a total of 18000 tickets for each concert.

Therefore, x, y and z satisfy the equation

$$x + y + z = 18000$$

For a recent pop concert, 75% of the standing tickets, 90% of the seated tickets and 45% of the VIP tickets **were sold**.
A total of 15600 tickets were sold for the pop concert.

(i) Show that x, y and z satisfy the equation

$$5x + 6y + 3z = 104000$$

$0.75x + 0.9y + 0.45z = 15600 \quad \times 100$

$75x + 90y + 45z = 1560000 \quad \div 15$

$5x + 6y + 3z = 104000 \checkmark$

(2)

For a recent rock concert, 4% of the standing tickets, $\dfrac{1}{5}$ of the seated tickets and 52% of the VIP tickets **were not sold**.
A total of 14640 tickets were sold for the rock concert.

(ii) Show that x, y and z satisfy the equation

$$6x + 5y + 3z = 91500$$

$0.96x + 0.8y + 0.48z = 14640 \quad \times 100$

$96x + 80y + 48z = 1464000 \quad \div 16$

$6x + 5y + 3z = 91500 \checkmark$

(2)

(iv) Solve the equations

$$x + y + z = 18000 \quad -①$$

$$5x + 6y + 3z = 104000 \quad -②$$

$$6x + 5y + 3z = 91500 \quad -③$$

to find the maximum number of standing, seated and VIP tickets available for each concert.

Show clearly each stage of your solution.

① × 3 = ④ $3x + 3y + 3z = 54000 \quad -④$

② - ④ = ⑤

$$\begin{array}{l} 5x + 6y + 3z = 104\,000 \\ \text{sub } 3x + 3y + 3z = 54000 \\ \hline 2x + 3y \qquad\qquad = 50000 \quad -⑤ \end{array}$$

③ - ④ = ⑥

$$\begin{array}{l} 6x + 5y + 3z = 91500 \\ \text{sub } 3x + 3y + 3z = 54000 \\ \hline 3x + 2y \qquad\qquad = 37500 \quad -⑥ \end{array}$$

3×⑥ - 2×⑤

$$\begin{array}{l} 9x + 6y = 112500 \\ \text{sub } 4x + 6y = 100000 \\ \hline 5x \quad = 12500 \end{array}$$

$$x = 2500$$

Sub $x = 2500$ into ⑤

$$5000 + 3y = 50000$$

$$3y = 45000$$

$$y = 15000$$

$$\begin{array}{l} \text{sub } x = 2500 \ \& \ y = 15000 \\ \text{into ①} \end{array}$$

$$2500 + 15000 + z = 18000$$

$$z = 500$$

check in ③ $15000 + 75000 + 1500 = 91500 \ ✓$

Standing ....2500....

Seated ....15000....

VIP ....500....

(8)