Name:

Exam Style Questions

Functions

Corbettmaths

Equipment needed: Ruler, Pencil, Calculator and Pen

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Video Tutorial

www.corbettmaths.com/contents

Videos 369, 370

Answers and Video Solutions

1. $f(x) = 3x - 7$

(a) Work out the value of $f(5)$

$f(5) = 3 \times 5 - 7$

$= 15 - 7$

$= 8$

8

.....

(1)

(b) Work out the value of $f(-2)$

$f(-2) = 3 \times (-2) - 7$

$= -6 - 7$

$= -13$

-13

.....

(1)

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2. $f(x) = x^3 - 6x + 2$

Work out the value of $f(5)$

$f(5) = 5^3 - 6 \times 5 + 2$

$= 125 - 30 + 2$

$= 95 + 2$

97

.....

(1)

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3. $f(x) = 4x - 10$

$g(x) = x + 9$

Work out the value of $fg(12)$

$g(12) = x + 9$

$= 12 + 9$

$= 21$

$f(21) = 4 \times 21 - 10$

$= 84 - 10$

$= 74$

74

.....

(2)

4. $f(x) = 5x$

Write down the expression for $f^{-1}(x)$

$y = 5x$

$\div 5 \quad \div 5$

$\frac{y}{5} = x \qquad x = \frac{y}{5}$

$f^{-1}(x) = \frac{x}{5}$

.....

(1)

5. Given $f(x) = \frac{2x+1}{3}$

(a) Calculate the value of $f(7)$

$f(7) = \frac{2 \times 7 + 1}{3}$

$= \frac{15}{3}$

5

.....

(1)

(b) Find $f^{-1}(x)$

$y = \frac{2x+1}{3}$

$3y = 2x + 1$

$3y - 1 = 2x$

$x = \frac{3y-1}{2}$

$f^{-1}(x) = \frac{3x-1}{2}$

.....

(2)

6. $f(x) = \frac{48}{\sqrt{x}}$

Find $f(9)$

$f(9) = \frac{48}{\sqrt{9}}$

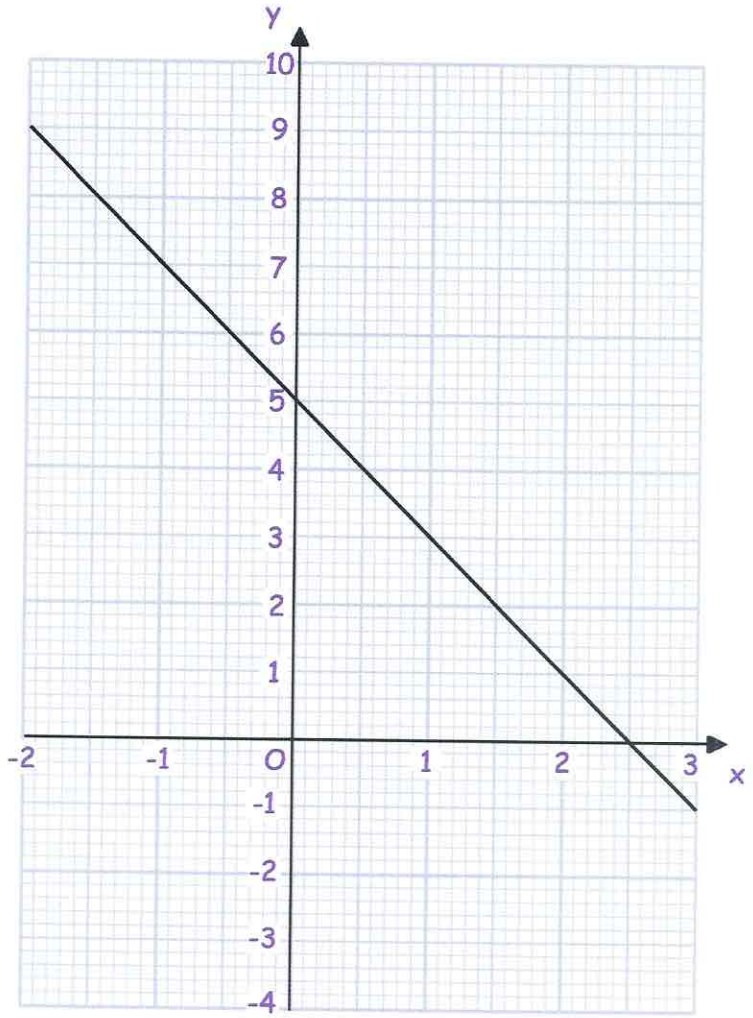
$= \frac{48}{3}$

16

.....

(1)

7. The graph of $y = f(x)$ is drawn on the grid.



(a) Write down the value of $f(-1)$

7

.....

(1)

$f(a) = 2$

(b) Write down the value of a

1.5

.....

(1)

(c) Write down the value of $ff(1)$

$f(1) = 3$

$f(3) = -1$

-1

.....

(1)

8. $f(x) = 3x - 5$

$g(x) = x^3$

(a) Work out the value of $gf(1)$

$f(1) = 3 - 5$

$= -2$

$g(-2) = (-2)^3$

-8

(2)

(c) Find $g^{-1}(x)$

$y = x^3$

$\sqrt[3]{y} = x$

$g^{-1}(x) = \sqrt[3]{x}$

(1)

(c) Find $f^{-1}(55)$

$y = 3x - 5$

$y + 5 = 3x$

$x = \frac{y+5}{3}$

$f^{-1}(x) = \frac{x+5}{3}$

$f^{-1}(55) = \frac{60}{3}$

20

(2)

9. $f(x) = x^2$ and $g(x) = x - 3$

Find $fg(x)$

$fg(x) = (x-3)^2$

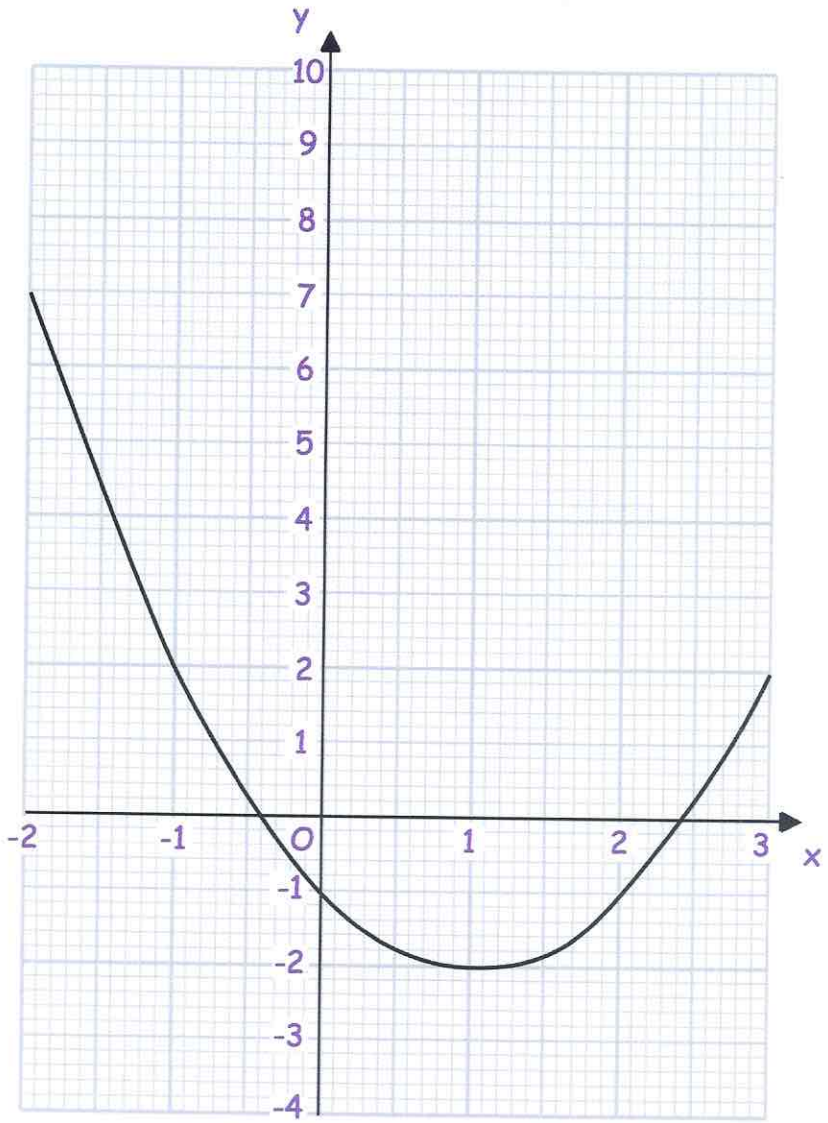
$= (x-3)(x-3)$

$= x^2 - 6x + 9$

$fg(x) = x^2 - 6x + 9$

(2)

9. Shown below is the graph of $y = f(x)$



(a) Write down the value of $f(1)$

-2

.....

(1)

(b) Write down the estimates for the roots of $f(x) = 0$

-0.4 and 2.4

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(1)

10. The functions $f(x)$ and $g(x)$ are given by the following:

$$f(x) = 3x - 1$$
$$g(x) = 2x + 4$$

(a) Calculate the value of $fg(2)$

$g(2) = 4 + 4$
$= 8$

$f(8) = 24 - 1$

23

(2)

(b) Calculate the value of $ff(3)$

$f(3) = 8$

$f(8) = 23$

23

(2)

(c) Find $gf(x)$

$gf(x) = 2(3x - 1) + 4$
$= 6x - 2 + 4$
$= 6x + 2$

$gf(x) = 6x + 2$

(2)

11. The functions $f(x)$, $g(x)$ and $h(x)$ are given by the following:

$$f(x) = x^2 - 3$$

$$g(x) = 2x + 1$$

$$h(x) = \frac{x}{2}$$

(a) Find $fg(x)$

$fg(x) = (2x+1)^2 - 3$

$= 4x^2 + 4x + 1 - 3$

$= 4x^2 + 4x - 2$

$4x^2 + 4x - 2$

(2)

(b) Find $gh(x)$

$gh(x) = 2\left(\frac{x}{2}\right) + 1$

$= x + 1$

$x + 1$

(2)

(c) Find $g^{-1}(x)$

$y = 2x + 1$

$y - 1 = 2x$

$\frac{y-1}{2} = x$

$g^{-1}(x) = \frac{x-1}{2}$

(1)

12. The function f is such that $f(x) = 4x - 7$

(a) Solve $f(x) = 17$

$4x - 7 = 17$

$4x = 24$

$x = 6$

6

.....

(2)

(b) Find $f^{-1}(x)$

$y = 4x - 7$

$y + 7 = 4x$

$\frac{y+7}{4} = x$

$f^{-1}(x) = \frac{x+7}{4}$

.....

(2)

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13. Given $f(x) = x^2 + 2$ and $g(x) = x + 14$

Find the values of a such that $f(a) = g(a)$

$f(a) = a^2 + 2 \qquad g(a) = a + 14$

$a^2 + 2 = a + 14$

$a^2 - a - 12 = 0$

$(a - 4)(a + 3) = 0$

$a = 4$ or $a = -3$

$a = 4$ or $a = -3$

.....

(3)

14. The functions $f(x)$ and $g(x)$ are given by the following:

$$f(x) = 8 - 3x$$

$$g(x) = 4x$$

(a) Calculate the value of $gf(3)$

$f(3) = 8 - 9$

$= -1$

$g(-1) = -4$

-4

(2)

(b) Solve the equation $gf(x) = 80$

$gf(x) = 4(8 - 3x)$

$= 32 - 12x$

$32 - 12x = 80$

$-48 = 12x$

$x = -4$

-4

(4)

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15. $f(x) = \dfrac{ax + 9}{2}$

Given $f(3) = 54$, find the value of a

$f(3) = \dfrac{3a + 9}{2}$

$\dfrac{3a + 9}{2} = 54$

$3a + 9 = 108$

$3a = 99$

$a = 33$

33

(3)

16. $f(x) = 2x^3 - 1$

Find $f^{-1}(x)$

$y = 2x^3 - 1$

$y + 1 = 2x^3$

$\frac{y+1}{2} = x^3$

$x = \sqrt[3]{\frac{y+1}{2}}$

$\sqrt[3]{\frac{x+1}{2}}$

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(3)

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17. $g(x) = ax + 2b$

$g(3) = 18$

$g(7) = 50$

Work out the values of a and b

$g(3) = 3a + 2b$

$g(7) = 7a + 2b$

$7a + 2b = 50$

$3a + 2b = 18$

$4a = 32$

$a = 8$

$g(3) = 24 + 2b$

$24 + 2b = 18$

$2b = -6$

$b = -3$

$a =$ 8 .....

$b =$ −3 .....

(3)

18. The function f is such that $f(x) = 2x - 1$

The function g is such that $g(x) = \frac{x}{8}$

(a) Find $fg^{-1}(x)$

$g^{-1}(x) = 8x$

$fg^{-1}(x) = 2(8x) - 1$

$= 16x - 1$

$16x - 1$

.....

(3)

(b) Find $gf^{-1}(x)$

$y = 2x - 1$

$y + 1 = 2x$

$\frac{y+1}{2} = x$

$f^{-1}(x) = \frac{x+1}{2}$

$gf^{-1}(x) = \left(\frac{x+1}{2}\right) \div 8$

$gf^{-1}(x) = \frac{x+1}{16}$

$\frac{x+1}{16}$

.....

(3)

19. $f(x) = 8 + 3x$

$g(x) = \sqrt[3]{x+4}$

$h(x)$ is the composite function $gf(x)$

Find $h^{-1}(x)$

$gf(x) = \sqrt[3]{8+3x+4}$

$= \sqrt[3]{3x+12}$

$h(x) = \sqrt[3]{3x+12}$

$h^{-1}(x) = \frac{x^3 - 12}{3}$

$y = \sqrt[3]{3x+12}$

$y^3 = 3x + 12$

$y^3 - 12 = 3x$

$\frac{y^3 - 12}{3} = x$

$\frac{x^3 - 12}{3}$

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(4)

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20. Given $f(x) = x^2 + 3x - 5$

Express $f(2x-1)$ in the form $ax^2 + bx + c$

$f(2x-1) = (2x-1)^2 + 3(2x-1) - 5$

$= 4x^2 - 4x + 1 + 6x - 3 - 5$

$= 4x^2 + 2x - 7$

$4x^2 + 2x - 7$

.....

(3)

21. $f(x) = x^2 + 4x - 1$

$g(x) = 3x - 1$

Solve $fg(x) = 5x^2 + 2x + 3$

$fg(x) = (3x-1)^2 + 4(3x-1) - 1$

$= 9x^2 - 6x + 1 + 12x - 4 - 1$

$= 9x^2 + 6x - 4$

$9x^2 + 6x - 4 = 5x^2 + 2x + 3$

$4x^2 + 4x - 8 = 0 \longrightarrow x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$x = -2$ or $x = 1$

$x = -2$ or $x = 1$

(5)

22. $f(x) = \dfrac{x - 9}{3x + 1}$

Find $f^{-1}(x)$

$y = \dfrac{x-9}{3x+1}$

$(3x + 1)y = x - 9$

$3xy + y = x - 9$

$y + 9 = x - 3xy$

$y + 9 = x(1 - 3y)$

$x = \dfrac{y+9}{1-3y}$

$f^{-1}(x) = \dfrac{x+9}{1-3x}$

$f^{-1}(x) = \dfrac{x+9}{1-3x}$

(4)

23. The function f is such that $f(x) = kx + 3$

The function g is such that $g(x) = 2x - 4$

Given that $gf(2) = 34$

work out the value of k

$gf(x) = 2(kx + 3) - 4$

$= 2kx + 2$

$gf(2) = 4k + 2$

$4k + 2 = 34$

$4k = 32$

$k = 8$

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(3)

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24. For all values of x,

$f(x) = x^2 + 4$

$g(x) = x - 9$

Solve $fg(x) = gf(x)$

$fg(x) = (x - 9)^2 + 4$

$= x^2 - 18x + 81 + 4$

$= x^2 - 18x + 85$

$gf(x) = x^2 + 4 - 9$

$= x^2 - 5$

$x^2 - 18x + 85 = x^2 - 5$

$-18x + 85 = -5$

$-18x = -90$

$x = 5$

$x = 5$

.....

(4)

25. $f(x) = x^2 + 2x + 1$

Show that $f(x+2) - f(x) = 4x + 8$

$f(x+2) = (x+2)^2 + 2(x+2) + 1$

$= x^2 + 4x + 4 + 2x + 4 + 1$

$= x^2 + 6x + 9$

$x^2 + 6x + 9 - (x^2 + 2x + 1)$

$= 4x + 8$

QED

(4)